CHAPTER 4

Motion in a Plane

TRY YOURSELF ANSWERS

1. The vector which have a sterling point or a point of application are called polar vector.

2. Given : $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

or $|\vec{A}+\vec{B}|^2 = |\vec{A}-\vec{B}|^2$

or $(\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B}) = (\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B})$ $\quad [\because |\vec{A}|^2 = \vec{A}\cdot\vec{B}]$

or $\vec{A}\cdot\vec{A}+\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{A}+\vec{B}\cdot\vec{B}$

$= \vec{A}\cdot\vec{A}-\vec{A}\cdot\vec{B}-\vec{B}\cdot\vec{A}+\vec{B}\cdot\vec{B}$

or $A^2+2\vec{A}\cdot\vec{B}+B^2 = A^2-2\vec{A}\cdot\vec{B}+B^2$ $\quad [\because \vec{B}\cdot\vec{A} = \vec{A}\cdot\vec{B}]$

or $4\vec{A}\cdot\vec{B} = 0$ or $4AB\cos\theta = 0$

As $\vec{A}$ and $\vec{B}$ are non-zero vectors, so $\cos\theta = 0$ or $\theta = 90°$.

3. Here $\vec{A} = \vec{B}+\vec{C}$

Let angle between $\vec{B}$ and $\vec{C}$ be θ. Then

$A^2 = B^2 + C^2 + 2BC\cos\theta$

$(5)^2 = 4^2 + 3^2 + 2(4)(3)\cos\theta$

or $0 = 24\cos\theta,\ \theta = \frac{\pi}{2}$

In the right angled triangle, let the angle between $\vec{A}$ and $\vec{C}$ be α.

$\therefore \quad \cos\alpha = \frac{C}{A} = \frac{3}{5} \quad \therefore \alpha = \cos^{-1}\left(\frac{3}{5}\right)$

4. Let $v = 80$ km h^{-1}, $v_x = 40$ km h^{-1}, then $v_y = ?$

As $v = \sqrt{v_x^2+v_y^2}$

$\therefore \quad v_y = \sqrt{v^2-v_x^2} = \sqrt{80^2-40^2} = \sqrt{6400-1600}$

$= \sqrt{4800} = 69.28$ km h^{-1}.

5. Here, $\vec{A} = a_x\hat{i}+a_y\hat{j}+a_z\hat{k}$

Let $\vec{B} = (\hat{i}-\hat{j})$

The component of $\vec{A}$ along $\vec{B}$ is

$= A\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|} \qquad \left(\because \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right)$

$= \frac{(a_x\hat{i}+a_y\hat{j}+a_z\hat{k})\cdot(\hat{i}-\hat{j})}{\sqrt{(1)^2+(-1)^2}} = \frac{a_x-a_y}{\sqrt{2}}$

6. Let $P = 3x$ N, $Q = 5x$ N,

$R = 35$ N, $\theta = 60° \Rightarrow R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

or $35 = \sqrt{(3x)^2+(5x)^2+2\times 3x\times 5x\cos 60°}$

or $35 = 7x$ or $x = \frac{35}{7} = 5$

$\therefore \quad P = 3\times 5 = 15$ N and $Q = 5\times 5 = 25$ N.

7. $\vec{A} = -4\hat{i}+3\hat{j},\ \vec{B} = 2\hat{i}+5\hat{j}$

$\vec{C} = \vec{A}\times\vec{B} = (-4\hat{i}+3\hat{j})\times(2\hat{i}+5\hat{j})$

$= -20\hat{k}-6\hat{k} = -26\hat{k}$

Hence $\vec{C}$ makes an angle of 180° with z-axis.

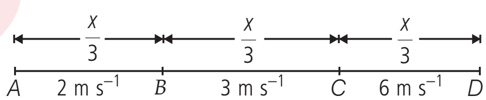
8. Area of the triangle $= \frac{1}{2}(\vec{A}\times\vec{B})$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ 1 & 0 & -1 \end{vmatrix} = \frac{1}{2}|[\hat{i}(3-0)-\hat{j}(-2-4)+\hat{k}(+3)]|$$

$= \frac{1}{2}|3\hat{i}+6\hat{j}+3\hat{k}|$

$= \frac{1}{2}\sqrt{9+36+9} = \frac{1}{2}\times\sqrt{54} = 3.7$ unit.

9. Let x be the length of whole journey.



Average velocity $= \frac{\text{Total displacement}}{\text{Total time taken}}$

$= \frac{x}{\frac{x/3}{2}+\frac{x/3}{3}+\frac{x/3}{6}} = \frac{1}{\frac{1}{6}+\frac{1}{9}+\frac{1}{18}}$

$= \frac{18}{3+2+1} = 3$ m s^{-1}

10. Given : $x = 7t + 4t^2$, $\quad y = 5t$

$\therefore \quad$ Velocity, $v_x = \frac{dx}{dt} = 7+8t$ m s^{-1}

$v_y = \frac{dy}{dt} = 5$ m s^{-1}

Acceleration, $a_x = \frac{dv_x}{dt} = 8$ m s^{-2} ...(i)

$a_y = \frac{dv_y}{dt} = 0$ ...(ii)

Equations (i) and (ii) show that acceleration does not depend on time *i.e.*, a particle moves with constant acceleration *a*.

$$a=\sqrt{(a_x)^2+(a_y)^2}=\sqrt{(8)^2+(0)^2}=8\ \text{m s}^{-2}$$

Therefore, acceleration of a particle after $t = 5$ s is a *i.e.*, 8 m s^{-2}

11. Here, $R = nH$

or $\dfrac{u^2\sin 2\theta}{g}=n\cdot\dfrac{u^2\sin^2\theta}{2g}$

or $\dfrac{u^2\times 2\sin\theta\cos\theta}{g}=n\cdot\dfrac{u^2\sin^2\theta}{2g}$ or $\tan\theta=\dfrac{4}{n}$

$\Rightarrow\quad \theta=\tan^{-1}\left(\dfrac{4}{n}\right).$

12. For maximum horizontal range, $\theta = 45°$

Velocity at highest point = Horizontal component of velocity $= u\cos 45° = u\times\dfrac{1}{\sqrt{2}}$.

13. Maximum height $H=\dfrac{u^2\sin^2\theta}{2g}$

or $80=\dfrac{u^2\sin^2\theta}{2\times 10}$ or $u^2\sin^2\theta = 1600$ or $u\sin\theta = 40$ m s^{-1}

Horizontal velocity = $u\cos\theta$

As per question, $u\cos\theta = at$

$u\cos\theta = 3\times 30 = 90$ m s^{-1}

$\therefore\quad \dfrac{u\sin\theta}{u\cos\theta}=\dfrac{40}{90}$ or $\tan\theta=\dfrac{4}{9}$ or $\theta=\tan^{-1}\left(\dfrac{4}{9}\right)$

14. Let u_1 and u_2 be the velocities of projection of two balls and θ_1, θ_2 be their angles of projection with the horizontal direction. At the highest point, the velocity of two balls is the same.

$\therefore\quad u_1\cos\theta_1 = u_2\cos\theta_2$

$$\frac{u_1}{u_2}=\frac{\cos\theta_2}{\cos\theta_1}=\frac{\cos 45°}{\cos 30°}=\frac{1/\sqrt{2}}{\sqrt{3}/2}=\frac{\sqrt{2}}{\sqrt{3}}$$

$$\frac{R_1}{R_2}=\frac{u_1^2\sin 2\theta_1/g}{u_2^2\sin 2\theta_2/g}=\frac{u_1^2\sin 2\times 30°}{u_2^2\sin 2\times 45°}$$

$$\frac{R_1}{R_2}=\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2\times\frac{\sqrt{3}}{2}\times\frac{1}{1}=\frac{1}{\sqrt{3}}$$

15. Given $\dfrac{\sqrt{3}u}{2}=u\cos\theta=$ speed at maximum height

$\therefore\quad \cos\theta=\dfrac{\sqrt{3}}{2}$ or $\theta = 30°$

Given $P\times H_{max} = R$

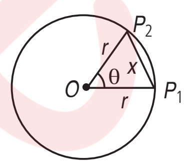
$$H_{max}=\frac{R\tan\theta}{4}$$

$\therefore\quad P=\dfrac{4}{\tan\theta}=\dfrac{4}{\tan 30°}=4\sqrt{3}$

16. The angle between velocity vector and acceleration vector in uniform circular motion is 90°

17. The direction of centripetal acceleration does not depend on the clockwise or anticlockwise sense of rotation of the body. It always acts along the radius towards the centre of the circle.

18.



According to cosine formula

$\cos\theta=\dfrac{r^2+r^2-x^2}{2r^2}$ or $2r^2\times\cos\theta=r^2+r^2-x^2$

or $x^2 = 2r^2 - 2r^2\cos\theta = 2r^2\,[1-\cos\theta]$

$$=2r^2\left[2\sin^2\frac{\theta}{2}\right]$$

Displacement from P_1 to P_2 is $x = 2r\sin\dfrac{\theta}{2}$