EXAM DRILL

Laws of Motion

CHAPTER 5

ANSWERS

1. **(a) :** Block of mass 1 kg moves along x-axis under a force, $F = kt$

$\therefore \quad F = ma = kt \Rightarrow 1 \times \frac{dv}{dt} = kt$

By integrating both sides, we get

$v = \frac{kt^2}{2}$ or $\frac{dx}{dt} = \frac{kt^2}{2}$ $\quad (\because k = 1\ \text{N s}^{-1})$

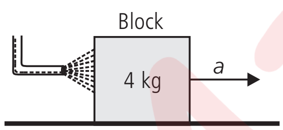
$\therefore \quad x = \frac{t^3}{6}$ $\quad$ (at $t = 6$ s)

$x = \frac{(6)^3}{6} = 36\ \text{m}$

2. **(d) :** The water jet striking the block at the rate of 2 kg s^{-1} at a speed of 10 m s^{-1} will exert a force on the block,

$F = v\frac{dm}{dt} = 10 \times 2 = 20\ \text{N}$

Under the action of this force of 20 N, the block of mass 4 kg will move with an acceleration given by



$a = \frac{F}{m} = \frac{20\ \text{N}}{4\ \text{kg}} = 5\ \text{m s}^{-2}$

3. **(b) :** From conservation of momentum

$Mv = m_1v_1 + m_2v_2$

Here, $M = 100$ kg, $v = 10^4$ m s^{-1}

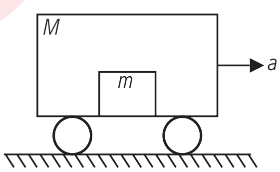
$m_1 = 10$ kg, $v_1 = 0$

$m_2 = 90$ kg, $v_2 = ?$

$\therefore \quad 100 \times 10^4 = 10 \times 0 + 90 \times v_2$

or $\quad v_2 = \frac{100 \times 10^4}{90} = 11.11 \times 10^3\ \text{m s}^{-1}$

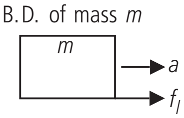
4. **(b) :** $m = 50$ kg, $\mu = 0.3$, $g = 10$ m s^{-2}



Suppose a is the common acceleration of the box and the train.

Block will be at rest if limiting friction (f_l) is equal to ma (a = maximum acceleration).

F.B.D. of mass m



$\therefore \quad f_l = ma \Rightarrow \mu N = ma$

$\Rightarrow \quad \mu mg = ma \Rightarrow a = \mu g$

$\therefore \quad a = 0.3 \times 10 = 3$ m s^{-2}.

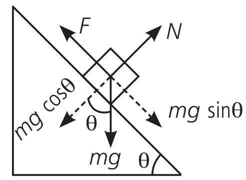
5. **(b) :** We know, friction , $f = \mu N$

On a horizontal surface, $N = mg$, and $f = ma$.

$\therefore \quad \mu = \frac{ma}{mg} = \frac{a}{g}$

6. **(d) :** $v \propto \frac{1}{m} \Rightarrow mv = \text{constant } P$

7. **(c) :** When a car is moving on a frictionless inclined plane,



$mg\cos\theta = N$

and $\quad F = mg\sin\theta$

or $\quad ma = mg\sin\theta$

$\Rightarrow \quad a = g\sin\theta$

8. **(a) :** Here, $m = 2$ kg, $\omega = 2\pi$ rad s^{-1}, $R = 1$ m

The centripetal force acting on the body is

$F = m\omega^2R = (2) \times (2\pi)^2 \times 1 = 8\pi^2$ N

9. **(c) :** Assertion is true but reason is false.

According to Newton's second law, acceleration $= \frac{\text{Force}}{\text{Mass}}$ *i.e.* if net external force on the body is zero $F = 0$. Then there is zero acceleration of a body.

10. **(a) :** Both assertion and reason are true and reason is the correct explanation of assertion.

The wings of the aeroplane push the external air backward and the aeroplane move forward by reaction of pushed air. At low altitudes, density of air is high and so the aeroplane gets sufficient force to move forward.

11. **(b) :** Both assertion and reason are true but reason is not the correct explanation of assertion.

According to law of inertia (Newton's first law), when cloth is pulled from a table, the cloth come in state of motion but dishes remains stationary due to inertia. Therefore when we pull the cloth from table the dishes remains stationary.

12. **(c) :** Assertion is true but reason is false.

The cyclist adopt a zig-zag path as it requires less force than if he move straight.

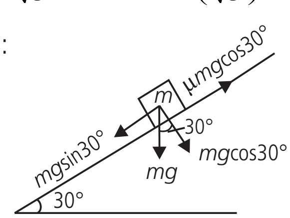
13. **(i)** **(a)** : $\mu_s = \tan 12°$

(ii) **(a)** : As $\tan\theta = \mu$

Here, $\quad \mu = \frac{1}{\sqrt{3}}$

$\therefore \quad \tan\theta = \frac{1}{\sqrt{3}}$ or $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$

(iii) **(a)** :



At the inclination of 30°, the mass m just starts sliding down. Let μ be the coefficient of static friction. Then

$$mg\sin 30° = \mu mg\cos 30°$$

or $\quad \mu = \tan 30° = \dfrac{1}{\sqrt{3}}$

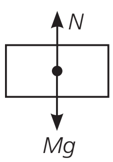
(iv) (d) : $\mu_k < \mu_s$

14. It states that the impulse applied to an object is equal to the total change in its momentum produced by the force.

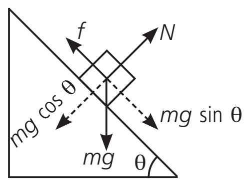
15. When the tyres are properly inflated, the area between the tyres and the ground reduces. This reduces rolling friction, which makes it easier to ride a bicycle with properly inflated tyres.

16. Action and reaction forces, even though equal and opposite act on two different bodies. Thus they don't cancel out each other. Further, when the weight of the body is double, coefficient of friction remains unchanged as it a constant for a particular pair of surface.

17. (a)



(b)



18. It can be defined as the motion of a body in circular path at a constant speed.

OR

(i) A cricket player lowers his hand while catching a ball.

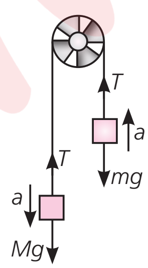
(ii) Chinawares are packed in straw paper before packing .

19. Let a be the acceleration with which the heavier mass M moves downwards and the lighter mass m moves upwards. Let T be the tension in the string. According to Newton's second law, the resultant downward force on mass M is

$$Ma = Mg - T \qquad \text{...(i)}$$

Resultant upward force on mass m is

$$ma = T - mg \qquad \text{...(ii)}$$



Adding equation (i) and (ii), we get

$$a(M + m) = (M - m)g$$

or $\quad a = \dfrac{(M-m)}{(M+m)}g$

Obviously, $a < g$ *i.e.*, the acceleration a of the two connected bodies is less than the acceleration due to gravity.

Dividing (i) by (ii), we get

$$\frac{Ma}{ma} = \frac{Mg-T}{T-mg} \text{ or } \frac{M}{m} = \frac{Mg-T}{T-mg}$$

or $\quad MT - Mmg = Mmg - mT$

or $\quad T(M + m) = 2Mmg$

or $\quad T = \dfrac{2Mm}{(M+m)}g$

20. To study about the motion of an inclined plane, Galileo conducted two experiments. In his first experiment, he observed the motion of an object down an inclined plane as well as up an inclined plane and along a horizontal plane. He could observe that, when the object came down through an inclined plane it accelerated, when it went up through an inclined plane, it retarded and on the horizontal plane, it neither accelerated nor retarded. From this, he concluded that in the absence of friction, the object will move with uniform velocity on the horizontal plane upto infinite distance.

In his second experiment, he used two inclined planes. He observed that, a ball released from the top of one inclined plane rolls down and climbs up the other, approximately to the same height. When the inclination of the second plane was reduced, still it was observed that the ball climbed up the same height, this time covering a larger distance. When the inclination of the second plane was reduced to zero, it was observed that the ball moved through an infinite distance in the absence of friction (idealised situation). These experiments laid down the basis of Newton's first law of motion.

21. Newton's laws of motion : Sir Isaac Newton made a systematic study of motion and extended the ideas of Galileo. He arrived at three laws of motion which are Newton's laws of motion.

These laws are as follows :

First law : Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by some external force to change that state.

Second law : The rate of change of linear momentum of a body is directly proportional to the applied force and the change takes place in the direction of the applied force.

Third law : To every action, there is always an equal and opposite reaction. Action and reaction occur in different bodies.

OR

A diagram for each body of the system showing all the forces exerted on the body by remaining parts of the system is called free-body diagram.

Use of free-body diagram : When a number of bodies are connected together by strings, rods, etc., it is convenient to consider each body separately and to write equation of motion for each body by taking into account all the forces acting on it and then equating the net force acting on the body to its mass times the acceleration produced.

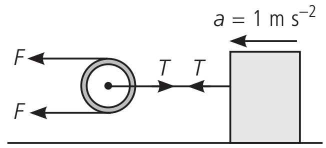
The equation of motion obtained for different bodies can be solved to determine unknown quantities.

22. Origin of friction : The force of friction is due to the atomic or molecular forces of attraction between the two surfaces at the points of actual contacts.

Due to the surface irregularities, the actual area of contact is much smaller than the apparent area of contact. The pressure at the points of contacts is very large. Molecular bonds are formed at these points. When one body is pulled over the other, the bonds break, the material is deformed and new bonds are formed. The local deformation sends vibrational waves into the bodies. These vibrations finally damp out and energy appears as heat. Hence a force is needed to start or maintain the motion.

23. As shown in figure, when force F is applied at the end of the rope, the tension in the lower part of the rope is also F.

If T is the tension in rope connecting the pulley and the block, then from Newton's third law, $T = 2F$



But $T = ma = 50 \times 1 = 50$ N

$\therefore \quad 2F = 50$ N or $F = 25$ N

OR

As the mass m_1 moves towards right through distance x, the mass m_2 moves down through distance $x/2$. Clearly, if the acceleration of m_1 is a, then that of m_2 will be $a/2$.

Applying Newton's second law to the motion of m_1 and m_2, we have

$$T_1 = m_1 a$$

and $m_2 g - T_2 = m_2 \cdot \dfrac{a}{2}$

Also $T_2 = 2T_1 = 2m_1 a$

$\therefore \quad m_2 g - 2m_1 a = m_2 \cdot \dfrac{a}{2}$

or $\quad 2m_2 g - 4m_1 a = m_2 a$ or $2m_2 g = (4m_1 + m_2)a$

$\therefore \quad$ Acceleration of $m_1 = a = \dfrac{2m_2 g}{4m_1 + m_2}$

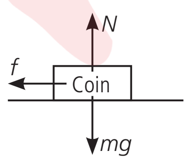
Acceleration of $m_2 = \dfrac{a}{2} = \dfrac{m_2 g}{4m_1 + m_2}$

24. Initial acceleration, $a = \dfrac{u}{m_0}\dfrac{dm}{dt} - g$

$\therefore \quad 20 = \dfrac{250}{500}\dfrac{dm}{dt} - 9.8$

or $\quad \dfrac{dm}{dt} = 2(20 + 9.8) = 59.6 \text{ kg s}^{-1}$.

25. From the diagram,



$$N = mg$$

and $\Sigma F_x = -f = ma_x$

But $f = \mu_k N = \mu_k mg$

$\Rightarrow \quad -\mu_k mg = ma_x$ or $a_x = -\mu_k g$ ...(i)

Now after travelling 20 m, the coin comes to rest, $u = 2$ m/s

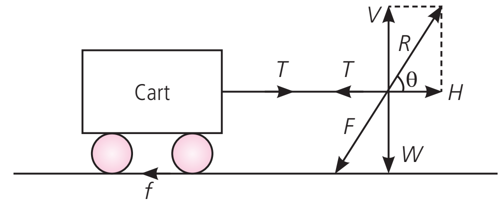
$\therefore \quad v^2 - u^2 = 2as$ or $a = \dfrac{v^2 - u^2}{2s}$

$$= \frac{0^2 - 4}{2 \times 20} = \frac{-4}{40} = \frac{-1}{10} \text{ m/s}^2$$

Substituting in (i)

$$\frac{-1}{10} = -\mu_k g \Rightarrow \mu_k = \frac{1}{98} = 0.01$$

26. Horse and cart problem : As shown in figure consider a cart connected to a horse by a string. The horse while pulling the cart produces a tension T in the string in the forward direction (action). The cart, in turn, pulls the horse by an equal force T in the opposite direction.



Initially, the horse presses the ground with a force F in an inclined direction. The reaction R of the ground acts on the horse in the opposite direction making on angle θ with the horizontal. The reaction R has two rectangular components :

(a) The vertical component V which balances the weight W of the horse.

(b) The horizontal component H which helps the horse to move forward.

Let f be be the force of friction. The net force on the horse in the forward direction is $H - T$ or $R \cos\theta - T$

If $R \cos\theta > T$, the horse accelerates forward in accordance with Newtons second law.

27. Kinetic or Dynamic Friction (μ_k) : Kinetic or dynamic friction is the opposing force that comes into play, when one body is actually moving over the surface of another body.

Kinetic friction is of two types :

(a) Sliding friction : The frictional force that comes into play when one body is actually sliding over the surface of the other body is called sliding friction.

(b) Rolling friction : The frictional force that comes into play when one body is actually rolling over the surface of the other body is called rolling friction.

Rolling friction is less than sliding friction.

Laws of kinetic friction :

(i) The force of kinetic friction (f_k) depends on the nature of the two surfaces in contact.

(ii) f_k is independent of the area of contact.

(iii) f_k is independent of the velocity of sliding provided this dosesn't affect the surface.

(iv) f_k is directly proportional to the normal reaction.

i.e., $f_k \propto R$, $f_k = \mu_k R$

$\mu_k = \dfrac{f_k}{R}$, where μ_k is the coefficient of kinetic friction. Coefficient of kinetic friction has no unit.

OR

Rolling friction or rolling resistance is the force resisting the motion when a body rolls over a surface. Similar to sliding friction, rolling friction is often expressed as a coefficient times the normal force. But the coefficient of rolling friction is generally much smaller than that of sliding friction. Thus for the same magnitude of normal reaction, rolling friction is always much lesser than the sliding friction.

When a wheel rolls over a surface it exerts a pressure on the surface due to its weight, which is sufficiently high because of the small area of contact of the wheel. Due to this high pressure, a small depression is formed on this area and a small elevation in front of it. The adhesive force between the surface in contact opposes the motion.

These two factors, create a retarding force on the motion of the object. This retarding force is called as rolling friction.
Rolling friction is directly proportional to the normal force and is inversely proportional to the radius of the rolling cylinder.

Thus $f_r \propto \frac{N}{R}$ or $f_r = \mu_r \frac{N}{R}$

where μ_r is the coefficient of rolling friction.

28. Friction helps us to walk, hold things, write on a piece of paper etc. It helps to apply brakes on vehicles, we are able to use nails and screws to join different machine parts because of friction. We are able to perform the simple tasks of our day to day life due to friction.

Methods of increasing friction :

(i) Treading of tyres is done to increase friction between the road and the tyres. Moreover, synthetic rubber is preferred over the natural rubber to manufacture tyres because of its larger coefficient of friction with the road.

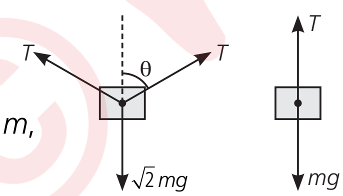
(ii) Sand is thrown on tracks covered with snow. This increases the force of friction between the wheels and the track and the driving becomes safer.

(iii) On a rainy day, we throw some sand on the slippery ground. This increases the friction between our feet and the ground. This reduces the chances of slipping.

29. (a) An inextensible string can also be considered as a spring with a very high force constant. The restoring force in such strings is known as tension, (T). It is customary to use a constant tension throughout the string.

In physics, tension is described as the pulling force transmitted axially by means of a string, a cable, chain , rope etc. It is a force along the length of a medium. It is important to note that tension is a pull force. It pull outwards along the length of the rope.

(b) The free body diagrams of the masses $\sqrt{2}m$ and m are shown in figure.



For the equilibrium of mass m,

$$mg = T$$

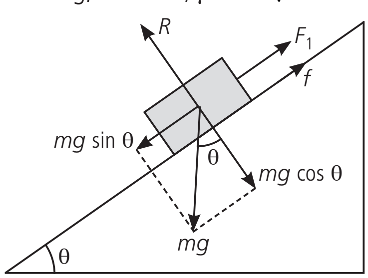
For the equilibrium of mass $\sqrt{2}m$,

$$\sqrt{2}mg = 2T\cos\theta$$

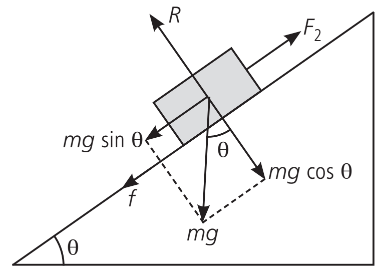
or $\sqrt{2} \times T = 2T\cos\theta$

or $\cos\theta = \frac{1}{\sqrt{2}}$ or $\theta = 45°$

30. Here $m = 200$ kg, $\theta = 30°$, $\mu = 1/\sqrt{3}$



(a)



(b)

From the above figures, force of friction is

$$f = \mu R = \mu\, mg\cos\theta = \frac{1}{\sqrt{3}} \times 200 \times 9.8\cos 30°$$

$$= \frac{1}{\sqrt{3}} \times 200 \times 9.8 \times \frac{\sqrt{3}}{2} = 980\text{ N}$$

Component of the weight acting down the inclined plane

$= mg\sin\theta = 200 \times 9.8\sin 30°$

$= 200 \times 9.8 \times 0.5 = 980$ N

(i) From figure (a), the least force required to prevent the mass from sliding down (when friction f acts upwards) is

$F_1 = mg\sin\theta - f = 980 - 980 = 0$

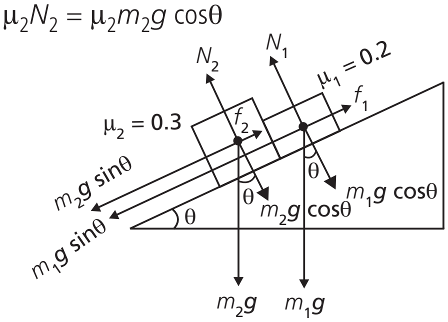
(ii) From figure (b), the greatest force required to prevent the mass from sliding up (when friction f acts downwards) is

$F_2 = mg\sin\theta + f = 980 + 980 = 1960$ N.

31. Here, $m_1 = 2$ kg, $m_2 = 4$ kg, $\mu_1 = 0.2$, $\mu_2 = 0.3$, $\theta = 30°$

Frictional force, $f_1 = \mu_1 N_1 = \mu_1 m_1 g\cos\theta$

and $f_2 = \mu_2 N_2 = \mu_2 m_2 g\cos\theta$



Total friction force up the plane $= f_1 + f_2$

Total force down the plane $= m_1 g\sin\theta + m_2 g\sin\theta$

Net force down the plane $= g\sin\theta(m_1 + m_2) - (f_1 + f_2)$

$$\therefore \text{ Acceleration} = \frac{g\sin\theta(m_1 + m_2) - (f_1 + f_2)}{m_1 + m_2}$$

$$= \frac{g\sin\theta(m_1 + m_2) - (\mu_1 m_1 + \mu_2 m_2)g\cos\theta}{m_1 + m_2}$$

$$= \frac{10 \times \sin 30°(2+4) - (0.2 \times 2 + 0.3 \times 4) \times 10 \times \cos 30°}{2+4}$$

$$= \frac{30 - 13.84}{6} = \frac{16.16}{6} = 2.69\text{ m/s}^2$$

32. (i) Absolute units of force :

(a) In S.I. the absolute unit of force is newton (symbol N).

One newton = 1 kilogram × 1 m s^{-2}

or 1 N = 1 kg × 1 m s^{-2} = 1 kg m s^{-2}

One newton force is that much force which produces an acceleration of 1 m s^{-2} in a body of mass 1 kg.

(b) In c.g.s. system, the absolute unit of force is dyne.

one dyne = 1 gram × 1cm s^{-2}

1 dyne = 1 g × 1 cm s^{-2} = 1 g cm s^{-2}

One dyne of force is that much force which produces an acceleration of 1 cm s^{-2} in a body of mass 1 g.

Relation between newton and dyne

1 N = 1 kg × 1 m s^{-2} = 10^3 g × 10^2 cm s^{-2}

∴ 1 N = 10^5 dyne

(ii) Gravitational or practical units of force

(a) In S.I., the gravitational unit of force is kilogram weight (kg-wt) or kilogram force (kgf).

1 kg-wt or 1 kgf = 1 kg × 9.8 m s^{-2}

= 9.8 kg m s^{-2} = 9.8 N

(b) In c.g.s. system, gravitational unit of force is gram weight (g-wt) or gram force (gf).

1 g-wt (or 1 gf) = 1 g × 980 cm s^{-2} = 980 g cms^{-2} = 980 dyne

One kilogram weight of force is that much force which produces an acceleration of 980 cm s^{-2} in a body of mass of 1 kilogram.

Gravitational unit of force is used to express the weight of a body. For example, weight of a body of mass 4 kg is 4 kg-wt or 4 kgf.

Dimensional Formula :

[Force] = [Mass] [Acceleration] = $[M^1L^0T^0][M^0L^1T^{-2}] = [M^1L^1T^{-2}]$

33. (iii) Rocket propulsion : Motion of rockets and jet engines can also be explained with the concept of conservation of momentum. Here the hot gases produced due to the combustion of fuels inside a rocket, escapes out of it with large momentum. Thus, the machine gains an equal and opposite momentum, enabling it to move with very high velocities.

Expression for the velocity gained by a rocket :

Suppose at time $t = 0$, we have

m_0 = initial mass of the rocket including that of the fuel

v_0 = initial velocity of the rocket

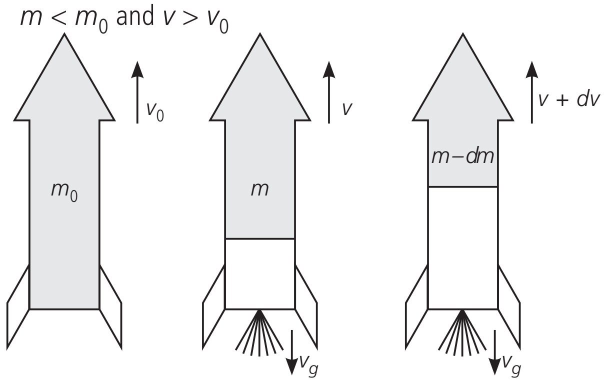
Suppose at time $t = t$, we have

m = mass of the rocket and fuel left

v = velocity acquired by the rocket

As the gases are escaping from rocket, we must have

$m < m_0$ and $v > v_0$



At $t = 0$ At $t = t$ At $t = t + dt$

Suppose in a small interval of time dt,

dm = a small decrease in mass of the rocket = mass of the exhaust gases that escape

dv = corresponding small increase in velocity of the rocket.

v_g = velocity of exhaust gases w.r.t. the earth

Using the principle of conservation of linear momentum, we get

$$mv = (m - dm)(v + dv) + dm(-v_g) \quad \text{...(i)}$$

Here v_g is taken negative because it acts downwards as the rocket moves upward.

From (i), we have

$$mv = mv + m(dv) - (dm)v - dm\,dv - dmv_g$$

As dm and dv both are small, their product term can be neglected.

$$\therefore \quad mdv = dm(v + v_g) \quad \text{...(ii)}$$

If u is relative velocity of burnt gases w.r.t. rocket, then

u = Velocity of burnt gases w.r.t. the earth – Velocity of the rocket w.r.t. the earth

$= -v_g - v = -(v_g + v)$

or $v + v_g = -u$

The negative sign shows the downward direction of motion of the gases.

$$\therefore \quad mdv = -udm \quad \text{...(iii)}$$

$$dv = -u\frac{dm}{m}$$

Integrating both sides, we get

$$\int_{v_0}^{v} dv = -u\int_{m_0}^{m}\frac{dm}{m}$$

$$[v]_{v_0}^{v} = -u[\log_e m]_{m_0}^{m}$$

or $v - v_0 = -u[\log_e m - \log_e m_0]$

$$= -u\log_e\left(\frac{m}{m_0}\right) = +u\log_e\left(\frac{m_0}{m}\right)$$

$$\text{or} \quad V = v_0 + u\log_e\left(\frac{m_0}{m}\right) \quad \text{...(iv)}$$

This gives the velocity of the rocket at any time t, when its mass is m.

At time t =0, v_0 = 0, so $v = u\log_e\left(\frac{m_0}{m}\right)$

Thus in the absence of any external force, the instantaneous velocity of the rocket is proportional to

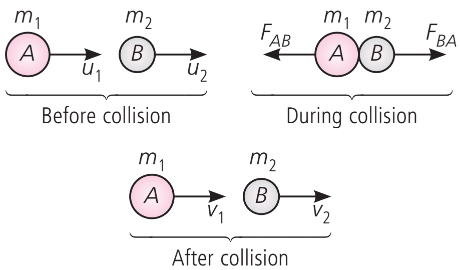
(i) the exhaust speed of the burnt gases.

(ii) natural logarithm of the ratio of initial mass of the rocket to its mass at the instant.

OR

(a) Law of conservation of momentum states that "if no external force acts on the system of particles, their total linear momentum remains conserved".

(b) The law of conservation of momentum states that "in the absence of external forces, the total momentum of the system is conserved".



Consider an isolated system consisting of two bodies A and B of masses m_1 and m_2. Let the two bodies, m_1 and m_2, be moving along a straight line in the same path with velocities $\vec{u}_1$ and $\vec{u}_2$ respectively (Assume $\vec{u}_1 > \vec{u}_2$). After some time they will collide. Let Δt be the time of contact. Let $\vec{v}_1$ and $\vec{v}_2$ be the velocities of the bodies A and B after collision.

Before collision

Initial momentum of body $A = m_1\vec{u}_1$

Initial momentum of body $B = m_2\vec{u}_2$

Total initial momentum of the system $= m_1\vec{u}_1 + m_2\vec{u}_2$ ...(i)

After collision

Final momentum of body $A = m_1\vec{v}_1$

Final momentum of body $B = m_2\vec{v}_2$

Total final momentum of the system $= m_1\vec{v}_1 + m_2\vec{v}_2$ ...(ii)

Force exerted by A on B, $\vec{F}_{AB} = \dfrac{m_2(\vec{v}_2 - \vec{u}_2)}{\Delta t}$ ...(iii)

Force exerted by B on A, $\vec{F}_{BA} = \dfrac{m_1(\vec{v}_1 - \vec{u}_1)}{\Delta t}$ ...(iv)

If no force is acting on them, then according to Newton's third law,

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

$$m_2\frac{(\vec{v}_2 - \vec{u}_2)}{\Delta t} = -m_1\frac{(\vec{v}_1 - \vec{u}_1)}{\Delta t}$$

$$m_2\vec{v}_2 - m_2\vec{u}_2 = -m_1\vec{v}_1 + m_1\vec{u}_1$$

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2 \quad \text{...(v)}$$

Eqn. (v) shows that total momentum of a system before collision is equal to the total momentum after collision.

(c) Here , mass of bullet, $m = 10\text{ g} = \dfrac{10}{1000}$ kg

Mass of ice, $M = 5$ kg

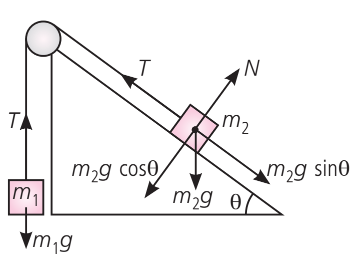
According to the conservation of linear momentum, we get

$$m \times 300 + M \times 0 = m \times 0 + Mv$$

or $\dfrac{10}{1000} \times 300 + M \times 0 = 5v$

or $v = \dfrac{3}{5} = 0.6$ m/s $= 60$ cm/s

34. (a) From the figure, the block m_2 goes down with an acceleration, a which is equal to the acceleration with which the block m_1 goes up



∴ For block m_1,

$$T - m_1g = m_1a$$

or $T = m_1(a + g)$ ...(i)

∴ For block m_2,

$$N = mg\cos\theta$$

and $m_2g\sin\theta - T = m_2a$ ...(ii)

Substituting (i) in (ii)

$$m_2(g\sin\theta - a) = m_1(a + g)$$

$$m_2g\sin\theta - m_1g = (m_1 + m_2)a$$

or $a = \dfrac{g(m_2\sin\theta - m_1)}{m_1 + m_2}$

$$\therefore\ T = m_1\left(\frac{g(m_2\sin\theta - m_1 + m_1 + m_2)}{m_1 + m_2}\right)$$

$$= \frac{m_1m_2g}{m_1 + m_2}(\sin\theta + 1)$$

(b) Here $4g - T_2 = 4a$, $T_1 - 2g = 2a$ and $T_2 - T_1 = 8a$

On solving, $a = 1.4\text{ m s}^{-2}$, $T_1 = 22.4$ N, $T_2 = 33.6$ N.

OR

(a) In mechanics, equilibrium of particles refer to a situation where the net external force on the particle is zero or in other words, the vector sum of all the forces acting on a particle is zero. This, according to the first law means that either the particle is at rest or the particle is under uniform motion.

In general, particle is in equilibrium under the action of n forces, if these forces can be represented by the sides of an n-sided polygon taken in the same order *i.e.*,

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n = 0$$

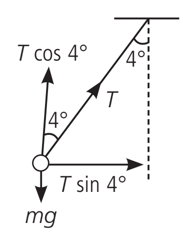
or if we take the components of these forces along x, y and z directions respectively, then

$$\vec{F}_{1x} + \vec{F}_{2x} + \vec{F}_{3x} + \ldots + \vec{F}_{nx} = 0$$

$$\vec{F}_{1y} + \vec{F}_{2y} + \vec{F}_{3y} + \ldots + \vec{F}_{ny} = 0$$

$$\vec{F}_{1z} + \vec{F}_{2z} + \vec{F}_{3z} + \ldots + \vec{F}_{nz} = 0$$

(b) Figure shows the equilibrium position of a pendulum suspended in a train which is moving towards right with acceleration a along horizontal track.



Two forces acting on the bob are :

(i) Weight mg acting vertically downwards,

(ii) Tension T acting along the string.

Resolving T along horizontal and vertical directions, we find that

$T\cos 4° = mg$ ...(i)

As the acceleration a of the train and hence that of the pendulum is responsible for tension $T\sin 4°$, so

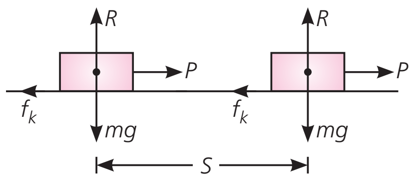
$T\sin 4° = ma$ ...(ii)

Dividing (ii) by, (i), we get

$\tan 4° = a/g$

or $a = g\tan 4° = 10 \times 0.07 = 0.7\text{ m s}^{-2}$.

35. Work done in sliding a body over a horizontal surface : Consider a body of weight mg resting on a rough horizontal surface, as shown in figure. The weight mg is balanced by the normal reaction R of the horizontal surface.



Suppose a force P is applied horizontally so that the body just begins to slide. Let f_k be the kinetic friction.

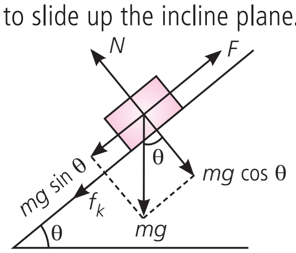
Work done against friction in moving the body through distance S will be

$$W = f_k \times S$$

But $f_k = \mu_k R = \mu_k \cdot mg$

$\therefore \quad W = \mu_k \cdot mg\, S$

Work done in moving a body up an incline plane : Suppose a body of weight mg is placed on an incline plane, as shown in figure. Let θ be the angle of inclination. A force P is applied on the body so that it just begins to slide up the incline plane.



The weight mg of the body has two components :

(i) $mg \cos\theta$ perpendicular to the inclined plane. It balances the normal reaction R. Thus

$$R = mg \cos\theta$$

(ii) $mg \sin\theta$ down the incline plane.

If f_k is the kinetic friction, then the force needed to be applied upwards to just move the body up the inclined plane must be

$$P = mg \sin\theta + f_k$$

But $f_k = \mu_k R = \mu_k\, mg \cos\theta$

$\therefore \quad P = mg \sin\theta + \mu_k\, mg \cos\theta = mg(\sin\theta + \mu_k \cos\theta)$

Work done in pulling the body through distance S up the inclined plane is

$$W = P \times S = mg(\sin\theta + \mu_k \cos\theta)S$$

OR

Banking of the curved road : The large amount of friction between the tyres and the road produces considerable wear and tear of the tyres. To avoid this, the curved road is given an inclination sloping upwards towards the outer circumference. This reduces wearing out of the tyres because the horizontal component of the normal reaction provides the necessary centripetal force.

The system of raising the outer edge of a curved road above its inner edge is called banking of the curved road. The angle through which the outer edge of the curved road is raised above the inner edge is called angle of banking.

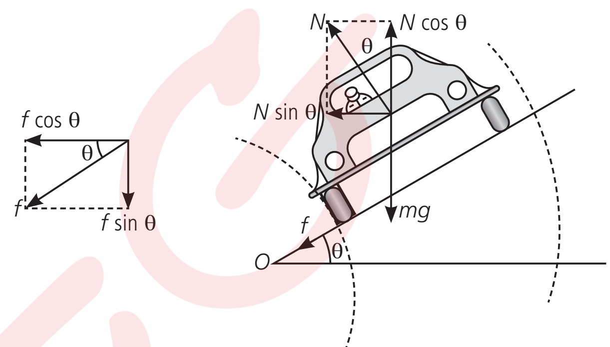
Circular motion of a car on a banked road

As shown in figure, consider a car of weight mg going along a curved path of radius r with speed v on a road banked at an angle θ. The forces acting on the vehicle are

(i) Weight mg acting vertically downwards

(ii) Normal reaction N of the road acting at an angle θ with the vertical.

(iii) Force of friction f acting downwards along the inclined plane.



Equating the forces along horizontal and vertical directions respectively, we get

$$N \sin\theta + f \cos\theta = \frac{mv^2}{r} \qquad \text{...(i)}$$

$mg + f \sin\theta = N \cos\theta$, where $f = \mu N$

or $\quad N \cos\theta - f \sin\theta = mg \qquad$ ...(ii)

Dividing equation (i) by equation (ii), we get

$$\frac{N \sin\theta + f \cos\theta}{N \cos\theta - f \sin\theta} = \frac{v^2}{rg}$$

Dividing numerator and denominator of L.H.S. by $N \cos\theta$, we get

$$\frac{\tan\theta + \frac{f}{N}}{1 - \frac{f}{N}\tan\theta} = \frac{v^2}{rg} \text{ or } \frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{v^2}{rg} \qquad \left[\because \mu = \frac{f}{N}\right]$$

$$\text{or } v^2 = rg\left[\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right] \text{ or } v = \sqrt{rg \cdot \frac{\mu + \tan\theta}{1 - \mu\tan\theta}}$$

Special case : When there is no friction between the road and the tyres, $\mu = 0$, so that the safe limit for maximum speed is

$v = \sqrt{rg \tan\theta}$.

The angle of banking θ for minimum wear and tear of tyres is given by

$$\text{or} \quad \tan\theta = \frac{v^2}{rg} \text{ or } \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$