Work, Energy and Power

EXAM DRILL

ANSWERS

1. (c) : Let the ball is projected with velocity u.

Then, $E=\frac{1}{2}mu^2$

At the highest point, the velocity of the ball will be equal to the horizontal component of the velocity of projection.

$\therefore\quad v=u\cos30°=\frac{\sqrt{3}}{2}u$

Now, the kinetic energy of the ball at the highest point,

$$E'=\frac{1}{2}mv^2=\frac{1}{2}m\left(\frac{\sqrt{3}u}{2}\right)^2=\frac{3}{4}\times\frac{1}{2}mu^2=\frac{3}{4}E$$

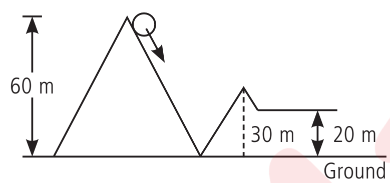
2. (b) : Total energy at 60 m height = Total energy at 20 m height

$mgh_1=mgh_2+\frac{1}{2}mv^2$

$v=\sqrt{2g(h_1-h_2)}$

$=\sqrt{2\times10(60-20)}$

$=\sqrt{800}=20\sqrt{2}\ \text{m s}^{-1}$



3. (c) : As the road does not move at all, therefore, work done by the cycle on the road must be zero.

4. (d) : The kinetic energy K and momentum p of a body are related as

$K=\frac{p^2}{2m}$ where m is the mass of the body.

Here, p = 6 N s, m = 4 kg $\quad\therefore\ K=\frac{(6\ \text{N s})^2}{2(4\ \text{kg})}=4.5$ J

5. (a) : $P=\vec{F}\cdot\vec{v}$

6. (b) : Loss in P·E = Gain in K·E

$mgh=\frac{1}{2}mv^2$

or $v=\sqrt{2gh}=\sqrt{2\times10\times20}=20\ \text{m s}^{-1}$

7. (b) : When two bodies of equal masses suffers perfectly elastic collision, their velocities gets interchanged. Thus, the velocity of B is 10 m/s.

8. (d) : As the earth moves once around the sun in its elliptical orbit, its kinetic energy is maximum when it is closest to the sun and minimum when it is farthest from the sun. As kinetic energy is never zero during its motion, hence option (d) is correct.

9. (a) : $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$

or $\quad|\vec{A}+\vec{B}|^2=|\vec{A}-\vec{B}|^2$

or $\quad(\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B})=(\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B})$

or $\quad\vec{A}\cdot\vec{A}+\vec{B}\cdot\vec{B}+2\vec{A}\cdot\vec{B}=\vec{A}\cdot\vec{A}+\vec{B}\cdot\vec{B}-2\vec{A}\cdot\vec{B}$

or $\quad4\vec{A}\cdot\vec{B}=0$

or $\quad4AB\cos\theta=0$

or $\quad\theta=90°$ $\qquad$ ($\because A$ and B are non-zero)

10. (c) : Force, $F=(5+3x)$ N

Work done, $W=\int_{x_1}^{x_2}F\,dx=\int_2^6(5+3x)\,dx$

$$=\left[5x+\frac{3}{2}x^2\right]_2^6=68\ \text{J}$$

11. (d) : A body may not have momentum but may have potential energy buy virtue of its position (*e.g.*, compressed or stretched spring). But if the body has no energy, then its kinetic energy is zero and therefore its momentum is also zero. Also, dimensions of momentum = $[MLT^{-1}]$ and dimensions of energy = $[ML^2T^{-2}]$ *i.e.*, dimensions of momentum is not equal to dimensions of energy.

12. (d) : Kinetic energy is conserved in elastic collision, but in a perfectly inelastic collision kinetic energy is not conserved, as some of the total energy may be converted into heat energy etc.

13. (i) (b) : Work done, $W=\vec{F}\cdot\vec{d}=Fd\cos\theta$, where, θ is the angle between the force $\vec{F}$ and displacement $\vec{d}$.

As the body moves along the direction of applied force.

$\therefore\quad\theta=0°$, $W=Fd\cos0°=Fd$. It is positive.

As the body moves in a direction opposite to the gravitational force which acts vertically downwards.

$\therefore\quad\theta=180°, W=Fd\cos180°=-Fd$. It is negative.

(ii) (c) : When a weightlifter lifts a weight, work done by the lifting force,

$W_1=Fd\cos0°=Fd$

Work done in holding it up

$W_2=0$ (because displacement is zero)

(iii) (b) : Here, $\vec{F}=3\hat{i}+4\hat{j}-5\hat{k}$ unit

$\vec{d}=5\hat{i}+4\hat{j}+3\hat{k}$ unit

$\therefore\ |\vec{F}|=\sqrt{(3)^2+(4)^2+(-5)^2}=\sqrt{50}$ unit

$|\vec{d}|=\sqrt{(5)^2+(4)^2+(3)^2}=\sqrt{50}$ unit

Let θ be angle between $\vec{F}$ and $\vec{d}$.

$$\therefore\quad\cos\theta=\frac{\vec{F}\cdot\vec{d}}{|\vec{F}||\vec{d}|}=\frac{(3\hat{i}+4\hat{j}-5\hat{k})\cdot(5\hat{i}+4\hat{j}+3\hat{k})}{\sqrt{50}\sqrt{50}}$$

$$=\frac{15+16-15}{50}=\frac{16}{50}=0.32\ \text{ or }\ \theta=\cos^{-1}(0.32)$$

14. $v = \sqrt{gr} = \sqrt{10 \times 90} = \sqrt{900} = 30 \text{ m s}^{-1}$

15. By doing work through the internal forces, the kinetic energy of a system can be increased or decreased without applying any external force on the system. For example, in explosion of a bomb.

16. Work done by a conservative force like gravity in taking an object from initial position (x_i) to final position (x_f) is equal to the difference between initial and final potential energy of the object.

$$F(x) = -\frac{dU}{dx}$$

or $$\int_{x_i}^{x_f} F(x)dx = \int_{U_i}^{U_f} dU = U_i - U_f$$

OR

Work done in stretching a spring of force constant k through a distance x is

$$W = \frac{1}{2}kx^2$$

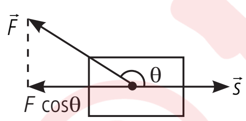
$$\therefore \quad \frac{W_A}{W_B} = \frac{\frac{1}{2}k_A x^2}{\frac{1}{2}k_B x^2} = \frac{k_A}{k_B}$$

As $k_A > k_B$, thus $W_A > W_B$

17. (i) The applied force must act on the object.
(ii) The applied force must displace the object.

18. The type of collision involved in the given process is inelastic collision because the objects suffer loss of kinetic energy which appears in the form of heat energy thereby increasing the temperature of the object.

19. If a force acting on the body has a component in the opposite direction of displacement, the work done is negative.



Negative work (θ > 90°)

Examples : (i) When a body slides against a rough horizontal surface, its displacement is opposite to the force of friction. The work done by friction is negative.
(ii) If we lift an object up from the ground, then the work done due to gravitational force is negative.

20. (i) $W_s = \frac{1}{2}K(x_1^2 - x_2^2)$

$x_1 = 0$ and $x_2 = 10$ cm $= 0.1$ m, $K = 100\,\frac{\text{N}}{\text{m}}$

$$\Rightarrow \quad W_s = \frac{1}{2}\left(100\frac{\text{N}}{\text{m}}\right)(0 - (0.1\text{ m})^2) = -0.5 \text{ J}$$

(ii) $\Delta U_s = -W_s = \frac{1}{2}K(x_2^2 - x_1^2)$

$$= \frac{1}{2}\left(100\frac{\text{N}}{\text{m}}\right)[(0.1\text{ m})^2 - 0^2] = +0.5 \text{ J}$$

21. Work energy theorem for a constant force states that the work done by the net force acting on a body is equal to the change produced in the kinetic energy of the body.

Suppose a constant force F is acting on an object of mass m, producing an acceleration a in it. If the velocity of the body changes from the u to v, after covering a distance s.

Then, from the equation of motion,

$$v^2 - u^2 = 2as$$

Multiplying both sides by $\frac{1}{2}m$, we get $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$

Using Newton's second law, the applied force, $F = ma$

$$\therefore \quad \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs \quad \text{or} \quad \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = W$$

or $\quad K_f - K_i = W$

Change in kinetic energy of the object = Work done on the object by the net force

This proves the work energy theorem for a constant force.

OR

Initial kinetic energy of the body,

$$K_i = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$$

Increase in momentum = 10% of p

$$= \frac{10}{100} \times p = \frac{p}{10}$$

Final momentum $= p + \frac{p}{10} = \frac{11p}{10}$

Final kinetic energy of the body,

$$K_f = \frac{(11p/10)^2}{2m} = \frac{121}{100}\frac{p^2}{2m}$$

or $$K_f = \frac{121}{100}K_i$$

Increase in kinetic energy $= K_f - K_i$

$$= \frac{121}{100}K_i - K_i = \frac{21}{100}K_i$$

% increase in $K.E = \frac{K_f - K_i}{K_i} \times 100$

$$= \frac{\frac{21}{100}K_i}{K_i} \times 100 = 21\%$$

22. (a) Properties of potential energy :

(i) Potential energy is defined only for conservative forces. It does not exists for non-conservative forces.

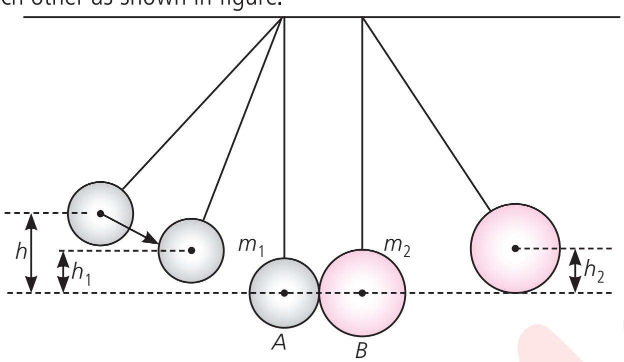
(ii) Potential energy depends upon the frame of reference. It may be positive or negative. In a conservative field, potential energy is equal to negative.

(iii) A body in motion may or may not have potential energy.

(b) When two bodies travelling initially along the same straight line collide without loss of kinetic energy and move along different directions in a plane after collision, the collision is said to be elastic collision in two dimensions or oblique collision.

23. Mass of two deuterons = 2 × 2.0141 = 4.0282 u
Mass of ${}_2He^3$ and neutron = 3.0160 + 1.0087 = 4.0282 u
Loss of mass = Δm = 4.0282 – 4.0247 = 0.0035 u
$= 0.0035 \times 1.661 \times 10^{-27}$ kg
Energy released, $E = \Delta mc^2$
$= 0.0035 \times 1.661 \times 10^{-27} \times (3 \times 10^8)^2$
$= 52.2 \times 10^{-14}$ J
$\because$ 1 MeV = 1.602×10^{-13} J
$$\therefore \quad E = \frac{52.2\times10^{-14}}{1.602\times10^{-13}} = 3.26 \text{ MeV}$$

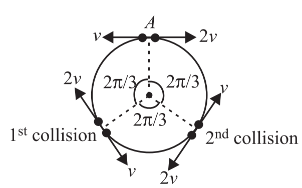
24. Let us consider two balls *A* and *B* of masses m_1 and m_2 respectively, suspended as simple pendulums from a fixed common support.
In the normal position of rest, the two balls are in contact with each other as shown in figure.



Let the ball *A* is displaced to a height *h* and released. The velocity acquired by the ball *A* just before striking the ball *B* is,
$$u_1 = \sqrt{2gh}$$
As the ball *B* is initially at rest, thus $u_2 = 0$.
After collision, let the ball *B* rise to a height h_2 and the ball *A* recoil back to a height h_1.
$$\therefore \quad v_1 = \sqrt{2gh_1} \text{ and } v_2 = \sqrt{2gh_2}$$
Coefficient of restitution, $e = \dfrac{v_2 - v_1}{u_1 - u_2}$
$$\therefore \quad e = \frac{\sqrt{2gh_2} - \sqrt{2gh_1}}{\sqrt{2gh} - 0} = \frac{\sqrt{h_2} - \sqrt{h_1}}{\sqrt{h}}$$
If we know the values of *h*, h_1 and h_2, we can calculate the coefficient of restitution of the collision.

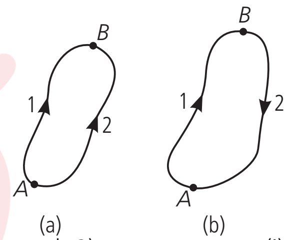
OR

At first collision, particle having speed 2*v* will rotate by 240° $\left(\text{or } \dfrac{4\pi}{3}\right)$ while other particle having speed *v* will rotate by 120° $\left(\text{or } \dfrac{2\pi}{3}\right)$. At first collision, they will exchange their velocities. Now as shown in figure, after two collisions they will again reach at point *A*.



25. Lost potential energy = *mgh*
$= 7.2 \times 10^4 \times 10 \times 100$
$= 7.2 \times 10^7$ J
Electric energy produced = $\dfrac{1}{2} \times 7.2 \times 10^7 = 3.6 \times 10^7$ J
Power produced, $P = \dfrac{3.6\times10^7}{t} = \dfrac{3.6\times10^7}{3600} = 10^4$ W

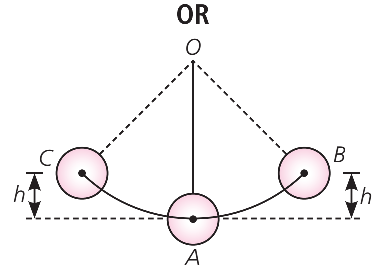
26. If the work done by a force in displacing a particle from one point to another is independent of the path followed by the particle and depends only on the end points, then the force is called conservative force.
Suppose a body moves from point *A* to point *B* along either of the path 1 or path 2 as shown in figure (a). If a conservative force *F* acts on the body, then the work done on the body is same along the two paths.



Mathematically,
W_{AB} (along path 1) = W_{AB} (along path 2) ...(i)
Now, suppose the body moves in a round trip from point *A* to *B* along path 1 and then back to point *A* along path 2 as shown in figure (b). Now, for a conservative force,
Work done on the body along path 1 from *A* to *B* = –Work done on the body along path 2 from *B* to *A*
W_{AB} (along path 1) = $-W_{BA}$ (along path 2) ...(ii)
From eqn. (i) and (ii),
W_{AB} (along path 1) = $-W_{BA}$ (along path 2)
or W_{AB} (along path 1) + W_{BA} (along path 2) = 0
or $W_{\text{closed path}} = 0$
Hence, a force is conservative if the work done by the force in moving a body around any closed path is zero.

27. (a) When the two objects are dropped from the same height, the two objects will hit the ground with the same velocity which is given by,
$$v = \sqrt{2gh}$$
Therefore, the ratio of momentum of the two objects is,
$$\frac{p_2}{p_1} = \frac{m_2}{m_1} = n$$
(b) Kinetic energy, $K = \dfrac{p^2}{2m}$
$$\therefore \quad p = \sqrt{2mK}$$
Since, the two objects have same kinetic energies,
$$\therefore \quad \frac{p_2}{p_1} = \sqrt{\frac{m_2}{m_1}} = \sqrt{n}$$

OR



Consider a simple pendulum as shown in figure, in which OA is normal position of rest of the pendulum. When the bob of the pendulum is displaced to B, through a height h, the potential energy is given by $U = mgh$ where m is the mass of the bob. On releasing the bob from B, it moves towards A. At A, potential energy of the bob is converted into kinetic energy , therefore, the bob do not stop at A. Due to inertia, it overshoots the position A and reaches C at the same height h above A.

The entire kinetic energy of the bob at A is converted into potential energy at C.

The whole process is repeated and the pendulum vibrates about equilibrium position OA.

At extreme positions B and C, the bob is momentarily at rest. Therefore, kinetic energy at extreme positions is zero. The entire energy at B and C is potential energy.

At A, there is no height and hence no potential energy. The entire energy at A is kinetic energy.

Therefore, in a simple pendulum, there is a constant exchange between kinetic energy and gravitational potential energy but the total mechanical energy remains constant.

28. (a) Here, mass of each ball is m and collision between two bobs is elastic. Hence, after collision, the two bobs will exchange their velocities. Therefore, bob A would come to rest and does not rise at all. Bob B will have the velocity of bob A.

(b) Speed of bob A just before collision,

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1} = 4.42\text{ m s}^{-1}$$

$\therefore$ Speed of bob B, $v = 4.42$ m s^{-1}

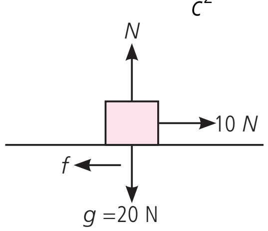
29. Mass-Energy relation or mass-energy equivalence, unified the law of conservation of mass and the law of conservation of energy into a single law of conservation of mass-energy.

Einstein discovered that mass can be transformed into energy and energy can be transformed into mass. He showed that mass and energy are equivalent and are related as,

$$E = mc^2$$

where c is the velocity of light in vacuum *i.e.*, 3×10^8 m s^{-1}.

According to Einstein's mass-energy relation, if mass m disappears, an energy E (= mc^2) appears in some form. This is when mass is being converted into energy. Conversely, when energy E is converted into mass, energy E disappears and a mass m appears which is equal to $m = \dfrac{E}{c^2}$

30. 

Work done by applied force, $W = 10 \times 5 = 50$ J

Frictional force, $f = \mu N = 0.4 \times 20 = 8$ N

Therefore, work done by frictional force,

$$W = -fd = -8 \times 5$$
$$= -40 \text{ J (force and displacement are in opposite direction)}$$

Work done by net force,

$$W = (10 - f) \times d = (10 - 8) \times 5 = 10 \text{ J}$$

31. Given, $m = 3 \times 10^{-5}$ kg, $v = 9$ m s^{-1}

Rain falls in a year, $h = 100$ cm = 1 m

$\therefore$ $A = 1\ m^2$

Volume of rain falling, $V = Ah = 1\ m^3$

Mass of the rain, $M = V\rho = 1 \times 10^3 = 10^3$ kg

($\because \rho = 10^3$ kg m^{-3})

Net energy transferred by the rain is given by

$$E = \frac{1}{2}Mv^2$$
$$= \frac{1}{2}(10^3)(9)^2 = 40.5 \times 10^3 \text{ J} = 4.05 \times 10^4 \text{ J}$$

32. Let the mass of the two spheres be m.

Before collision After collision

From conservation of linear momentum,

$$mu = mv_1 + mv_2$$

or $u = v_1 + v_2$ ...(i)

From definition of e,

$v_1 - v_2 = eu$ ...(ii)

From eq's (i) and (ii), we get

$$v_1 = \left(\frac{1+e}{2}\right)u \text{ and } v_2 = \left(\frac{1-e}{2}\right)u$$

$$\therefore \frac{v_1}{v_2} = \left(\frac{1+e}{1-e}\right)$$

33. (a) When a body tied to one end of a string is rotated uniformly in a circle, work done by the centripetal force applied along the string is zero. As the angle between radius and tangent is 90°

$\therefore$ $W = \vec{F} \cdot \vec{s}$

$= Fs \cos 90° = 0$

(b) For horizontal motion, the angle between force and displacement is 90°.

Here, $F = 20$ N, $s = 10$ m and $\theta = 90°$

$\therefore$ Work done, $W_1 = Fs\cos\theta$

$= 20 \times 10 \times \cos 90° = 0$

For vertical motion, the angle between force and displacement is 0°.

Here, $F = 20$ N, $s = 20$ m and $\theta = 0°$

$\therefore$ Work done, $W_2 = Fs\cos\theta$

$= 20 \times 20 \times \cos 0° = 400$ J

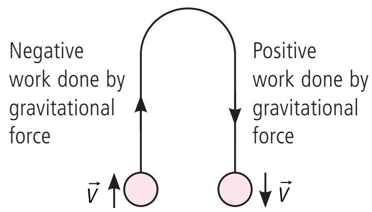
(c) Work done, $W = Fs\cos\theta$ or $\cos\theta = \dfrac{W}{Fs}$

Here, $F = 50$ N, $s = 6$ m, $W = 150$ J

$$\therefore \cos\theta = \frac{150}{50 \times 6} = \frac{150}{300} = \frac{1}{2} \text{ or } \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

OR

(a) Suppose a ball is thrown vertically upwards. The gravitational force opposes its upward motion and does negative work on it, thereby decreasing its kinetic energy. When the ball descends, the gravitational force does positive work on it and increasing its kinetic energy. The ball falls back to the point of projection with the same velocity and kinetic energy with which it was thrown up. The net work done by the gravitational force on the ball during the round trip is zero. This shows that the gravitational force is a conservative force.



(b) If an object is moved from a position *A* to another position *B* on a rough surface, work done against frictional force (*W*), shall depend on the length of the path between *A* and *B* and also on the positions *A* and *B*.

Further, if the body is brought back to its initial position *A*, on the same path, the same work (*W*) has to be done against the frictional forces, which always opposes the motion. Hence, net work done against the frictional forces in moving a body over a round trip is not zero. It is numerically equal to 2*W*.

If, E_i = total value of initial energy and

E_f = total value of final energy, then

$E_i - E_f = 2W$

Hence, force of friction is a non- conservative force.

34. (a) : Let the original speeds of the heavier and lighter particles be v_h and v_l respectively. Then,

The original kinetic energy of the heavier particle is

$$K_h = \frac{1}{2}mv_h^2$$

and that of the lighter particle is

$$K_l = \frac{1}{2}\frac{m}{2}v_l^2$$

But, $K_h = \frac{1}{2}K_l \quad \therefore \quad \frac{1}{2}mv_h^2 = \frac{1}{2}\left(\frac{1}{2}\frac{m}{2}v_l^2\right)$ or $v_h^2 = \frac{v_l^2}{4}$

or $\quad v_h = \frac{v_l}{2}$ ...(i)

When the speed of the heavier particle is increased by 3 m s^{-1}, its kinetic energy becomes

$$K_h' = \frac{1}{2}m(v_h + 3)^2$$

But $K_h' = K_l$ (given)

$$\therefore \quad \frac{1}{2}m(v_h + 3)^2 = \frac{1}{2}\frac{m}{2}v_l^2$$

or $(v_h + 3)^2 = \frac{v_l^2}{2}$ or $v_h + 3 = \frac{v_l}{\sqrt{2}}$

Using (i), we get

$v_h + 3 = \frac{2v_h}{\sqrt{2}} = \sqrt{2}v_h$ or $(\sqrt{2} - 1)v_h = 3$

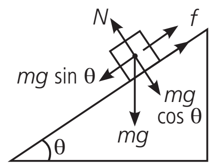
or $\quad v_h = \frac{3}{(\sqrt{2}-1)} = \frac{3(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$$= 3\sqrt{2} + 3 = 7.2 \text{ m s}^{-1}$$

Substituting this value of v_h in eqn. (i), we get

$7.2 \text{ m s}^{-1} = \frac{v_l}{2}$ or $v_l = 14.4 \text{ m s}^{-1}$

Thus, v_h = 7.2 m s^{-1} and v_l = 14.4 m s^{-1}



(b) Given : Angle, θ = 30°

F = *mg* = 2(9.8) = 19.6 N

μ = 0.1, S = 1 m

(i) Work done by gravity, $W_{gravity}$

= (Component of weight along displacement) × (displacement)

= (*mg* sin θ) (*S*) = (19.6 N) × (1 m) sin 30°

$= + 19.6 \times \frac{1}{2} = + 9.8$ J

(ii) Work done by frictional force,

$W_{friction} = FS \cos$ (Angle between $\vec{F}$ and $\vec{S}$)

= (*m*N) *S* cos 180° = (μ *mg* cos θ) (*S*) (–1)

= –μ *mg* cos θ *S* = –(0.1) (19.6 N) (cos 30°) (1 m)

$= -(0.1)(19.6 \text{ N}) \left(\frac{1.732}{2}\text{m}\right) = -1.7$ J.

OR

(a) Let, mass of the rocket (+ gas) at time *t* = *M*

Speed of the rocket at time *t* = *v*

Mass of ejected gas at time (*t* +Δ*t*) = Δ*m*

Speed of the rocket at time (*t* + Δ*t*) = (*v* + Δ*v*)

Speed of gas at (*t* + Δ*t*) = *v* – *u*

KE of the entire system at time *t*, is given by

$$K_t = \frac{1}{2}Mv^2 \quad \text{...(i)}$$

KE of the entire system at time (*t* + Δ*t*), is given by

$$K_{t+\Delta t} = \left[\frac{1}{2}(M - \Delta m)(v + \Delta v)^2\right] + \frac{1}{2}\Delta m\,(v - u)^2$$

$$\Rightarrow \quad K_{t+\Delta t} = \frac{1}{2}[Mv^2 + 2Mv\Delta v + M(\Delta v)^2 - v^2\Delta m - \Delta m(\Delta v)^2 - 2v\Delta m\Delta v] + \frac{1}{2}\Delta m\,(u^2 + v^2 - 2uv)$$

Neglecting the terms containing $(\Delta v)^2$ and Δ*m* Δ*v*,

$$\therefore \quad K_{t+\Delta t} = \frac{1}{2}Mv^2 + Mv\Delta v + \frac{1}{2}\Delta mu^2 - uv\Delta m \quad \text{...(ii)}$$

According to conservation of linear momentum,

momentum of rocket = – momentum of escaping gas

$\Rightarrow \quad M\Delta v = -(\Delta m)(-u)$

$\Rightarrow \quad M\Delta v - u\Delta m = 0$ ...(iii)

From equation (ii) and (iii),

$$K_{t+\Delta t} = \frac{1}{2}Mv^2 + \frac{1}{2}\Delta mu^2 \quad \text{...(iv)}$$

Applying work-energy theorem,
work done by the ejecting gas

$W = K_{t+\Delta t} - K_t$

$\Rightarrow \quad W = \frac{1}{2}(\Delta m)u^2$ [using (i) and (iv)]

(b) Given, length of shock absorber, l = 1.5 m
Total mass of the system, $m = 5 \times 10^4$ kg
Speed of the system $v = 36$ km h^{-1}

$$= 36 \times \frac{5}{18}\text{ m s}^{-1} = 10\text{ m s}^{-1}$$

Kinetic energy of the system, $K = \frac{1}{2}mv^2$

$$\Rightarrow K = \frac{1}{2}(5\times10^4)(10)^2 = 2.5\times10^6 \text{ J}$$

Since, 90% of the KE is lost due to friction.
So, elastic potential of the spring,
U = 10% of $K = 2.5 \times 10^5$ J
We know,

$$U = \frac{1}{2}kx^2 \Rightarrow k = \frac{2U}{x^2} = \frac{2\times2.5\times10^5}{(1)^2}$$

$\Rightarrow \quad k = 5 \times 10^5$ N m^{-1}

35. (a) At maximum compression the kinetic energy of the car is converted entirely into the potential energy of the spring.

Here, m = 1000 kg, v = 18 km h^{-1} = 18 × $\frac{5}{18}$ m s^{-1} = 5 m s^{-1}

Kinetic energy of the moving car is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\times10^3\times5\times5 = 1.25\times10^4 \text{ J}$$

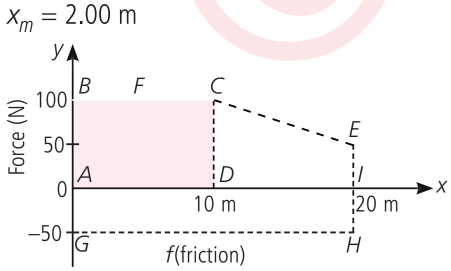
Let the maximum compression be x_m, where the potential energy U of the spring is equal to the kinetic energy K of the moving car from the principle of conservation of mechanical energy.

$$U = \frac{1}{2}kx_m^2 = 1.25\times10^4 \text{ J}$$

$$\Rightarrow x_m^2 = \frac{1.25\times10^4\times2}{6.25\times10^3}$$

$\Rightarrow \quad x_m$ = 2.00 m

(b)



The plot of the applied force is shown in figure.
At x = 20 m, F = 50 N
We are given that the frictional force f is $|f|$= 50 N. It opposes motion and acts in a direction opposite to F. It is therefore, shown on the negative side of the force axis.
Work done by the woman is
W_F = area of the rectangle $ABCD$ + area of the trapezium $CEID$

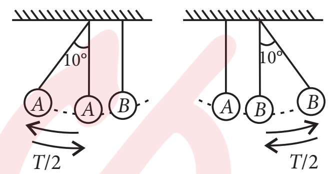
or $\quad W_F = 100 \times 10 + \frac{1}{2}(100 + 50) \times 10$

$\quad = 1000 + 750 = 1750$ J

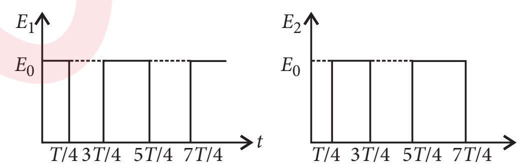
Work done by the frictional force
W_f = area of the rectangle $AGHI$
or $\quad W_f = (-50) \times 20 = -1000$ J

OR

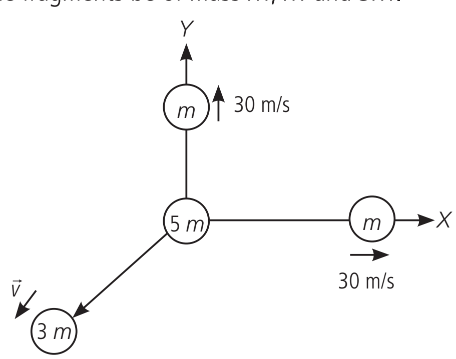
(a) (i) We know that, both the pendulums are identical and their bobs are in contact. If one of the bobs is displaced by 10° and released, it collides elastically head on with the other bob. Then, motion of two bobs is shown below



Each pendulum will swing only for $T/2$ time.
(T is the time period of each pendulum).
KE of the two bobs is conserved.
(ii) Let, energy of pendulum $A = E_1$
Energy of pendulum $B = E_2$
$E_1 = E_2 = E_0$ (∵ pendulums are identical and collision is elastic)



(b) Let the fragments be of mass m, m and $3m$.



Let us choose the X-Y axes along the flying directions of the lighter fragments.

$\vec{p}_{\text{initial}} = 0$

$\vec{p}_{\text{final}} = (m)(30\hat{i}) + m(30)\hat{j} + 3m\vec{v}$

$\Rightarrow m(30\hat{i} + 30\hat{j} + 3\vec{v}) = 0$

or $\quad \vec{v} = -(10\hat{i} + 10\hat{j})$

$\Rightarrow v = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ m/s

The speed of the heavier fragment is $10\sqrt{2}$ m/s
or $\quad$ 14.14 m/s.