Systems of Particles and Rotational Motion

CHAPTER 7

TRY YOURSELF

ANSWERS

1. Centre of mass of a body is defined as a point where whole mass of the body is supposed to be concentrated.

2. A body with uniformly distributed mass is said to be homogeneous.

3. The centre of mass of homogeneous bodies of regular shapes like rings, discs, spheres, rods etc., lie at their geometric centres.

4. Not necessarily. For example, the centre of mass of a ring lies at its hollow portion, at the centre

5. (i) Motion of Earth-Moon system around the sun.

(ii) Motion of CM of a fire cracker or a projectile exploding in air.

6. When two balls are thrown simultaneously in air, the force acting on the system is the gravitational pull.

$\therefore$ The force on centre of mass is the force due to gravity. This implies that the acceleration of the centre of mass is equal to the acceleration due to gravity.

7. Two stars which move in their elliptical orbits around a common centre of mass are said to form binary stars. For example, Sirius is an example of binary star system.

8. Angular velocity of a particle is defined as the time rate of change of its angular displacement.

$$\omega = \frac{\Delta\theta}{dt}$$

Its SI unit is rad s^{-1}.

9. Right-handed thumb rule : If we have two vectors $\vec{a}$ and $\vec{b}$, then their cross product is $\vec{c} = \vec{a} \times \vec{b}$. To find out the direction of this cross product we use right handed screw rule, according to which "if we curls up the fingers of right hand around a line perpendicular to the plane of the vector $\vec{a}$ and $\vec{b}$ and if the fingers are curled up in the direction from $\vec{a}$ to $\vec{b}$, then the stretched thumb points in the direction of $\vec{c}$.

10. The relation between linear velocity and angular velocity is given by

$$v = r\omega$$

where, r is the radius.

In vector form,

$$\vec{v} = \vec{\omega} \times \vec{r}$$

11. Here, $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k},\ \vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k},\ \vec{v} = ?$

As $\vec{v} = \vec{\omega} \times \vec{r}$

$$\therefore \quad \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$$

$$= \hat{i}(-24+6) - \hat{j}(18-5) + \hat{k}(-18+20) = -18\hat{i} - 13\hat{j} + 2\hat{k}$$

12. Since $\dfrac{d\vec{L}}{dt} = \vec{\tau}_{ext}$

If $\quad \vec{\tau}_{ext} = 0$, then $\dfrac{d\vec{L}}{dt} = 0$ or $\vec{L} = \text{constant}$

i.e. if total external torque on a system is zero, its angular momentum remains constant. This is known as principle of conservation of angular momentum.

13. (i) Angular momentum

(ii) Torque

14. $\tau = \dfrac{dL}{dt} = \dfrac{4A - A}{4} = \dfrac{3}{4}A$

15. Torque or moment of force is the turning effect of the force about the axis of rotation. It is measured as the product of magnitude of force and perpendicular distance of the line of action of force from the axis of rotation.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Torque is a vector quantity.

16. A pair of equal and opposite forces with different lines of action are said to form a couple. A couple produces rotation without translation.

For example, when we open the lid of a bottle by turning it, our fingers apply a couple on the lid.

17. Yes, a body in motion can be in equilibrium if it has no linear and angular acceleration. Hence a body moving with uniform velocity along a straight line will be in equilibrium.

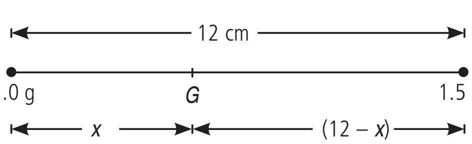
18. Translational equilibrium : The resultant of all the external forces acting on the body must be zero.

$$\Sigma\vec{F}_{ext} = 0$$

Rotational equilibrium : The resultant of torques due to all the forces acting on the body about any point must be zero.

$$\Sigma\vec{F}_{ext} = \Sigma\vec{r}_i \times \vec{F}_i^{\,ext} = 0$$

19.



Taking moment of forces about centre of gravity G,

$(1.0)\,g\,x = 1.5\,g\,(12 - x)$

or $x = 7.2$ cm

20. The property of a body by virtue of which it opposes the torque tending to change its state of rest or of uniform rotation about an axis is called rotational inertia or moment of inertia. Unit of moment of inertia is kg m^2 or g cm^2 and its dimensions are $[ML^2 T^0]$.

21. Here, $M = 4$ kg, $k = 25$ cm $= 0.25$ m

As, $I = Mk^2 = 4 \times (0.25)^2 = 0.25$ kg m^2

22. $I = \frac{2}{5}MR^2$, where M is the mass and R is the radius of the solid sphere.

23. Kinetic energy of rotation $= \frac{1}{2}I\omega^2$ where I is the moment of inertia of the body rotating about an axis of rotation.

24. As $I = MR^2$

Given that, $I = 200$ g cm^2 and $R = 5$ cm

So, $200 = M(5)^2$

$\therefore \quad M = \frac{200}{25} = 8$ g

25.

Equations of translational motion	Equations of rotational motion
(i) $v = u + at$	(i) $\omega = \omega_0 + \alpha t$
(ii) $x = x_0 + v_0 t + \frac{1}{2}at^2$	(ii) $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
(iii) $v^2 = v_0^2 + 2ax$ where the symbols have their usual meanings.	(iii) $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ where the symbols have their usual meanings.

26. Newton's second law of rotational motion about a fixed axis states that the angular acceleration during rotational motion of a rigid body is directly proportional to the applied torque and inversely proportional to the moment of inertia of that body.

$\tau = I\alpha$ or $\alpha = \frac{\tau}{I}$

27. Here $\omega = 200\ s^{-1}$, $\tau = 120$ N m, $P = ?$

As, $P = \tau\omega$

$\therefore$ Required power of the engine,

$P = 120 \times 200 = 24000$ W $= 24$ kW

28. Comparison of translational and rotational motion

Linear motion	Rotational motion about a fixed axis
Displacement x	Angular displacement θ
Velocity $v = dx/dt$	Angular velocity $\omega = d\theta/dt$
Acceleration $a = dv/dt$	Angular acceleration $\alpha = d\omega/dt$
Mass M	Moment of inertia I
Force $F = Ma$	Torque $\tau = I\alpha$
Work $dW = F\,ds$	Work $W = \tau d\theta$
Kinetic energy $K = Mv^2/2$	Kinetic energy $K = I\omega^2/2$
Power $P = F\,v$	Power $P = \tau\omega$
Linear momentum $p = Mv$	Angular momentum $L = I\omega$

29. $E_{Tot} = E_T + E_R = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{1}{2}mr^2 \times \frac{v^2}{r^2}$

$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$

$\therefore \quad \frac{E_R}{E_{Tot}} = \frac{\frac{1}{4}mv^2}{\frac{3}{4}mv^2} = 1:3$

30. When a ballet dancer stretches her hands and a leg outward, her moment of inertia increases and hence angular speed decreases to conserve the angular momentum. On the other hand, when she folds her hands and legs near her body, the moment of inertia decreases and she is able to increase the angular speed.