CHAPTER 8

Gravitation

EXAM DRILL

ANSWERS

1. **(a) :** A heavenly body revolving around a planet in an orbit is called natural satellite.

2. **(a) :** Gravitational constant is a scalar quantity.

3. **(b) :** The value of G in SI unit is 6.67×10^{11} N m^2 kg^{-2}.

4. **(b) :** The space around a material body in which its gravitational pull can be experienced is called gravitational field.

5. **(d) :** Zero.

6. **(c) :** Zero.

7. **(d) :** Newton formulated the universal law of gravitation.

8. **(d) :** It remains same on the moon.

9. **(d) :** The radius and mass of Jupiter is larger than that of earth. Therefore, the escape velocity $\left(=\frac{\sqrt{2GM}}{R}\right)$ is more at the surface of Jupiter than at earth's surface.

10. **(b) :** A heavenly body revolving round the sun is called a planet and there are nine planets in our solar system. A heavenly body made of gaseous material and luminous due to its own energy, is called a star.

11. **(a) :** For downward accelerated motion of the lift, apparent weight $R = m(g - a)$.
For free fall $a = g$, then $R = m(g - g) = 0$, *i.e.* the man will feel weightlessness.

12. **(i) (a) :** From Kepler's second law of planetary motion, the linear speed of a planet is maximum when its distance from the sun is least.

(ii) (c) : Kepler's second law is a consequence of conservation of angular momentum.

(iii) (d) : Since radius vector sweeps equal areas in equal interval of time, area swept in one week

$$= \frac{1}{4} \times \text{area swept in one month} = \frac{A}{4}$$

13. No, because both the body and the artificial satellite are in a state of free fall.

14. Gravitation is the force of attraction between any two bodies in the universe while gravity refers to the force of attraction between any body and the earth.

15. It states that every body in this universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

16. The motion of a body under the influence of gravity alone is called a free fall.

17. $\rho = \frac{3g}{4\pi GR}$

18. The value of g depends on (i) shape of earth (ii) latitude (iii) altitude (iv) depth.

OR

The mass of the body will remain 5 kg at the centre of earth because it remains same at all places. At cenre of the earth $g = 0$ thus weight of (the body (mg) is zero at the cenre of the earth.

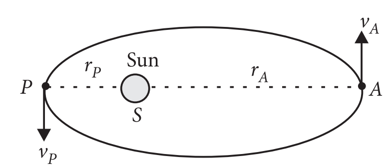
19. The maximum value of potential energy of a heavenly body is zero. The gravitational P.E. of a body of mass m near the surface of the earth is

$$U = -\frac{GMm}{R}$$

20. If a body is projected vertically upwards with a speed 11200 m/s, it will escape earth's gravitational field because escape speed on earth surface is 11.2 km/s. The body will escape the gravitational field of the earth.

$$\frac{v_e}{v_e'} = \sqrt{\frac{2GM}{2R}} \times \sqrt{\frac{R}{2G \times 4M}} = \frac{1}{\sqrt{8}} = 1 : \sqrt{8}$$

21. In the figure, point A represents aphelion and point P perihelion.



According to Kepler's second law, areal velocity is constant. So, $r_P \times v_P = r_A \times v_A$

$$\Rightarrow \frac{r_A}{r_P} = \frac{v_P}{v_A}$$

$\because$ $r_A > r_P$. So, $v_P > v_A$.
Hence, speed of the earth at the perihelion is more than that at the aphelion.

(b) According to Kepler's third law of periods, the square of the period of revolution of a planet around the Sun is proportional to the cube of the semi-major axis of its elliptical orbit.

22. (a) Orbital velocity, $v_0 = \sqrt{\frac{GM}{R+h}}$

Clearly, the satellite revolving at smaller height h from the surface of the earth will have a greater velocity.

(b) $v_e = \sqrt{2gR}$

OR

Here, $R = 6 \times 10^6$ m, $g = 10$ m/s^2,
$h = 400 \times 10^3$ m $= 0.4 \times 10^6$ m

(a) Orbital speed

$$v = R\sqrt{\frac{g}{R+h}} \Rightarrow 6\times10^6\sqrt{\frac{10}{6\times10^6+0.4\times10^6}} = 7.5\times10^3 \text{ m/s}$$

(b) Period of revolution, $T = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$

$$= \frac{2\times(22/7)}{6\times10^6}\sqrt{\frac{(6\times10^6+0.4\times10^6)^3}{10}} = 5368.5\text{ s}$$

23. Here $\rho = 5.5\times10^3$ kg m^{-3}, $G = 6.67\times10^{-11}$ N m^2 kg^{-2}

A satellite is orbiting close to the surface of a planet so $R + h \approx R$. Orbital velocity of planet is

$$v_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{GM}{R}} = \sqrt{\frac{G\times4/3\,\pi R^3\rho}{R}} = 2R\sqrt{\frac{\pi G\rho}{3}}$$

Time period of revolution of satellite

$$T = \frac{2\pi R}{v_0} = \frac{2\pi R}{2R\sqrt{\pi G\rho/3}}$$

$$= \sqrt{\frac{3\pi}{G\rho}} = \sqrt{\frac{3\times3.14}{(6.67\times10^{-11})\times5.5\times10^3}} = 5067\text{ s}$$

24. Given, $\frac{R_E}{R_P} = \frac{2}{3}, \frac{\rho_E}{\rho_P} = \frac{4}{5}, h_E = 1.5\text{ m}$

Mass of the earth, $M_E = \frac{4}{3}\pi R_E^3\rho_E$

Mass of the planet, $M_P = \frac{4}{3}\pi R_E^3\rho_P$

Gravitational acceleration on earth,

$$g_E = \frac{GM_E}{R_E^2} = \frac{G\times\frac{4}{3}\pi R_E^3\rho_E}{R_E^2} = \frac{4}{3}\pi R_E\rho_E\cdot G$$

Similarly, $g_P = \frac{4}{3}\pi R_P\rho_P\cdot G$

Energy would be common in both planets and at maximum height energy will be converted into potential energy.

$\therefore\quad mg_E h_E = mg_p h_p$

$$\text{or}\quad h_P = h_E\times\frac{g_E}{g_P} = h_E\times\frac{\frac{4}{3}\pi R_E\rho_E\cdot G}{\frac{4}{3}\pi R_P\rho_P\cdot G}$$

$$= h_E\times\frac{R_E}{R_P}\times\frac{\rho_E}{\rho_P} = 1.5\times\frac{2}{3}\times\frac{4}{5} = 0.8\text{ m}$$

25. Let m be the mass of a body.

$\therefore$ Weight of the body on the surface of the earth is

$W = mg = 250$ N

Acceleration due to gravity at a depth d below the surface of the earth is

$$g' = g\left(1-\frac{d}{R_E}\right)$$

Weight of the body at depth d is $W' = mg' = mg\left(1-\frac{d}{R_E}\right)$

Here, $d = \frac{R_E}{2}$

$$\therefore\quad W' = mg\left(1-\frac{R_E/2}{R_E}\right) = \frac{mg}{2} = \frac{W}{2} = \frac{250\text{ N}}{2} = 125\text{ N}$$

26. Here $h = 32$ km, $R = 6400$ km

As $h << R$, so

$$g_h = g\left(1-\frac{2h}{R}\right) = g - \frac{2gh}{R} \quad\text{or}\quad g - g_h = \frac{2gh}{R}$$

Percent decrease in weight

$$= \frac{mg-mg_h}{mg}\times100 = \frac{g-g_h}{g}\times100$$

$$= \frac{2gh}{g\times R}\times100 = \frac{2h}{R}\times100 = \frac{2\times32}{6400}\times100 = 1$$

27. At a height h ($<< R$), we have

$$g_h = g\left(1-\frac{2h}{R}\right) = g - \frac{2gh}{R} \quad\therefore\quad g - g_h = \frac{2gh}{R}$$

Change in weight = Weight of the body at the bottom of tower – Weight of the body at the top of the tower

$$= mg - mg_h = m(g-g_h) = \frac{2mgh}{R}$$

But $mg = 0.5$ kg, $h = 20$ m

$R = 6400$ km $= 6.4\times10^6$ m

$$\therefore\ \text{Change in weight} = \frac{2\times0.5\times20}{6.4\times10^6} = 3.125\times10^{-6}\text{ kg}$$

28. In equilibrium, weight of the suspended body = stretching force

$\therefore$ At the earth's surface, $mg = k\times x$

At a height h, $mg' = k\times x'$

$$\frac{g'}{g} = \frac{x'}{x} = \frac{R^2}{(R+h)^2} = \frac{(6400)^2}{(6400+1600)^2} = \left(\frac{6400}{8000}\right)^2 = \frac{16}{25}$$

$$\text{or}\quad x' = \frac{16}{25}\times x = \frac{16}{25}\times1\text{ cm} = 0.64\text{ cm}$$

29. Here, $h = 20$ m

Gravitational potential difference = 16 J kg^{-1}

The vertical distance through which the body need to be raised is $8\sin60° = 8\times(\sqrt{3}/2) = 4\sqrt{3}$

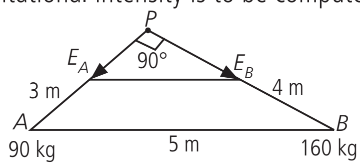
Since gravitational potential difference for a distance of 20 m is 16 J/kg, hence the potential difference for a distance of $4\sqrt{3}$ m is $\left(\frac{16}{20}\times4\sqrt{3}\right)$ J kg^{-1}.

Work done in lifting a 4 kg body through a vertical height

$$\left(\frac{16}{20}\times4\sqrt{3}\right)\times4 = \frac{64\times1.732}{5} = 22.16\text{ J}$$

OR

Let A and B be the positions of the masses and P be the point at which the gravitational intensity is to be computed.



Gravitational intensity at P due to mass at A will be

$$E_A = G\frac{90}{3^2} = 10G, \text{ along } PA.$$

Gravitational intensity at P due to mass at B will be

$$E_B = G\frac{160}{4^2} = 10G, \text{ along } PB.$$

In ΔAPB, $AB^2 = AP^2 + PB^2$ $\quad (\because \angle APB = 90°)$

Therefore, the magnitude of resultant gravitational intensity at P will be

$$E = \sqrt{E_A^2 + E_B^2} = \sqrt{(10G) + (10G)^2} = 10\sqrt{2}G$$

$$= 10\sqrt{2} \times 6.61 \times 10^{-11}\ \text{N/kg} = 9.43 \times 10^{-10}\ \text{N kg}^{-1}$$

30. Let r be the distance of the given point from the centre of earth. Then,

Gravitational potential, $V = -\dfrac{GM}{r} = -5.12 \times 10^7\ \text{J kg}^{-1}$ ...(i)

Acceleration due to gravity $g = \dfrac{GM}{r^2} = 6.4\ \text{ms}^{-2}$ ...(ii)

Divide (i) by (ii), $r = \dfrac{5.12 \times 10^7}{6.4} = 8 \times 10^6\ \text{m} = 8000\ \text{km}$

Height of the point from earth's surface

= 8000 – 6400 = 1600 km

31. (i) According to the law of conservation of angular momentum, the angular momentum of planet remains constant.

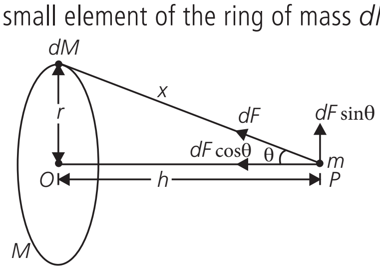
(ii) Since linear speed of the planet around the sun changes, therefore, its kinetic energy also changes continuously.

(iii) P.E. depends upon the distance between Sun and planet, hence P.E. changes as the distance between the Sun and planet changes continuously.

(iv) Total energy of planet remains constant.

32. The upward and downward sense is due to gravitational force of attraction between the body and earth. In spaceship gravitational force is counter balanced by the centripetal force needed by satellite to move around the earth in circular orbit. Hence in absence of force, the astronaut will not able to distinguish between up and down.

33. Consider a small element of the ring of mass dM.



Distance between dM and $m = x$

Also $x^2 = r^2 + h^2$

$\therefore$ Gravitational force between dM and m

$$dF = \frac{G(dM)m}{x^2}$$

Now, dF has two components as shown in figure.

$dF\cos\theta$ along PO and $dF\sin\theta$ perpendicular to PO.

Due to symmetry of ring, $\int dF\sin\theta = 0$

So, net force on mass m due to ring is given by

$$F = \int dF\cos\theta = \int \frac{G(dM)m}{x^2}\cdot\frac{h}{x}$$

$$\Rightarrow \quad F = \frac{Gm}{x^3}h\int dM = \frac{GMm \times h}{x^3} \Rightarrow \quad F = \frac{GMmh}{(r^2+h^2)^{3/2}}$$

Now, when $h = r$, $F = \dfrac{GMmr}{(r^2+r^2)^{3/2}} = \dfrac{GMm}{2\sqrt{2}r^2}$

When, $h = 2r$, $F' = \dfrac{GMm(2r)}{(r^2+4r^2)^{3/2}} = \dfrac{2GMm}{5\sqrt{5}r^2}$

$$\therefore \quad \frac{F'}{F} = \frac{\frac{2}{5\sqrt{5}}}{\frac{1}{2\sqrt{2}}} = \frac{4\sqrt{2}}{5\sqrt{5}}$$

OR

The gravitational potential energy of a body of mass m on earth's surface is

$$U(R) = -\frac{GMm}{R}$$

where M is the mass of the earth (supposed to be concentrated at its centre) and R is the radius of earth (distance of the particle from the centre of earth). The gravitational energy of the same body at a height $10R$ from earth's surface *i.e.*, at a distance $11R$ from earth's centre is

$$U(11R) = -\frac{GMm}{11R}$$

Therefore, change in potential energy,

$$U(11R) - U(R) = -\frac{GMm}{11R}\left(-\frac{GMm}{R}\right) = \frac{10}{11}\frac{GMm}{R}$$

The difference must come from the initial kinetic energy given to the body in sending it to that height.

Now, suppose the body is moving with a vertical speed v, so that the initial kinetic energy is $\frac{1}{2}mv^2$.

$$\frac{1}{2}mv^2 = \frac{10}{11}\frac{GMm}{R} \text{ or } v = \sqrt{\frac{20}{11}\frac{GM}{R}}$$

Putting the given values,

$$v = \sqrt{\frac{(20 \times 6.7 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}) \times (6 \times 10^{24}\ \text{kg})}{11 \times 6.4 \times 10^8}}$$

$= 1.07 \times 10^4$ m/s.

34. Radial acceleration

$$= \frac{v^2}{r} = \frac{(\sqrt{GM/r})^2}{r} = \frac{GM}{r^2}$$

According to the given condition, $\frac{GM}{r^2} = \frac{g}{2}$

or $r = \sqrt{\frac{2GM}{g}} = \sqrt{\frac{2GM}{GM/R^2}} = \sqrt{2}R$

or $(R+h) = \sqrt{2}R$ (as $r = R + h$)

or $h = (\sqrt{2}-1)R = (1.414-1)(6.4\times10^6\,\text{m})$

$= 2.6 \times 10^6$ m

$$v = \sqrt{\frac{rg}{2}} = \sqrt{\frac{\sqrt{2}Rg}{2}} = \sqrt{\frac{Rg}{\sqrt{2}}}$$

$$= \sqrt{\frac{(6.4\times10^6)9.8}{1.414}}\ \text{m/s} = 6.7\times10^3\,\text{m/s}$$

$= 6.7$ km/s

$$T = \frac{2\pi r}{v} = \frac{2\pi(\sqrt{2}R)}{v}$$

$$= \frac{2\times3.14\times1.414(6.4\times10^6\,\text{m})}{6.67\times10^3\,\text{m/s}} = 8.5\times10^3\,\text{s}$$

OR

$r_a = a(1+e) = 6R$; $r_p = a(1-e) = 2R \Rightarrow e = \frac{1}{2}$

Conservation of angular momentum :

angular momentum at perigee = angular momentum at apogee

$mv_pr_p = mv_ar_a$; $v_a/v_p = 1/3$.

Conservation of Energy :

Energy at perigee = Energy at apogee

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a}$$

$$\therefore\ v_p^2\left(1-\frac{1}{9}\right) = -2GM\left[\frac{1}{r_a}-\frac{1}{r_p}\right] = 2GM\left[\frac{1}{r_a}-\frac{1}{r_p}\right]$$

$$v_p = \frac{2GM\left[\frac{1}{r_p}-\frac{1}{r_a}\right]^{1/2}}{\left[1-\left(\frac{v_a}{v_p}\right)\right]^2} = \left[\frac{\frac{2GM}{R}\left[\frac{1}{2}-\frac{1}{6}\right]}{\left(1-\frac{1}{9}\right)}\right]^{1/2}$$

$$= \left(\frac{2/3}{8/9}\frac{GM}{R}\right)^{1/2} = \sqrt{\frac{3}{4}\frac{GM}{R}} = 6.85\,\text{km/s}$$

$v_p = 6.85$ km/s, $v_a = 2.28$ km/s

For $r = 6R$, $v_c = \sqrt{\frac{GM}{6R}} = 3.23\,\text{km/s}$.

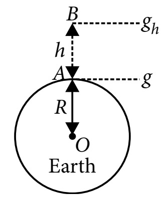
Hence to transfer to a circular orbit of apogee, we have to boost the velocity by $\Delta = (3.23 - 2.28) = 0.95$ km/s. This can be done by suitably firing rocket from the satellite.

35. (a) Acceleration due to gravity on the surface of earth at point A is given by $g = \frac{GM}{R^2}$ ...(i)

($M \to$ Mass of earth ; $R \to$ Radius of earth)

Let g_h be the acceleration due to gravity at a point B at a height 'h' above the surface of earth. Then

$$g_h = \frac{GM}{(R+h)^2} \quad \text{...(ii)}$$



Dividing (ii) by (i), we get

$$\frac{g_h}{g} = \frac{GM}{(R+h)^2}\times\frac{R^2}{GM} = \frac{R^2}{(R+h)^2}$$

$$\frac{g_h}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2} \quad \text{or} \quad g_h = g\left(1+\frac{h}{R}\right)^{-2}$$

Using binomial theorem and neglecting the higher powers of h/R $\because (R >> h)$, we get

$$g_h = g\left(1-\frac{2h}{R}\right)$$

(b) (i) Acceleration due to gravity g at height h is given by

$$g' = \frac{gR^2}{(R+h)^2} = \frac{gR^2}{(R+R/2)^2} = \frac{4}{9}g$$

Gravitational force on body at height h is

$F = mg' = m\frac{4}{9}g = mg\frac{4}{9} = 63\times\frac{4}{9} = 28$ N $[\because mg = 63]$

(ii) Variation of acceleration due to gravity g with depth d is

$$g' = g\left(1-\frac{d}{R}\right)$$

Weight of body at a depth d

$$mg' = mg\left(1-\frac{d}{R}\right) = 250\left(1-\frac{R/2}{R}\right) = 125\ \text{N}.$$

OR

Radial acceleration

$$= \frac{v^2}{r} = \frac{(\sqrt{GM/r})^2}{r} = \frac{GM}{r^2}$$

According to the given condition, $\frac{GM}{r^2} = \frac{g}{2}$

or $r = \sqrt{\frac{2GM}{g}} = \sqrt{\frac{2GM}{GM/R^2}} = \sqrt{2}R$

or $(R+h) = \sqrt{2}R$ (as $r = R + h$)

or $h = (\sqrt{2}-1)R = (1.414-1)(6.4\times10^6\,\text{m}) = 2.6 \times 10^6$ m

$$v = \sqrt{\frac{rg}{2}} = \sqrt{\frac{\sqrt{2}Rg}{2}} = \sqrt{\frac{Rg}{\sqrt{2}}}$$

$$= \sqrt{\frac{(6.4\times10^6)9.8}{1.414}}\ \text{m/s} = 6.7\times10^3\,\text{m/s}$$

$= 6.7$ km/s

$$T = \frac{2\pi r}{v} = \frac{2\pi(\sqrt{2}R)}{v}$$

$$= \frac{2\times3.14\times1.414(6.4\times10^6\,\text{m})}{6.67\times10^3\,\text{m/s}} = 8.5\times10^3\,\text{s}$$