Mechanical Properties of Solids

CHAPTER 9

EXAM DRILL

ANSWERS

1. (c) : The ratio of the lateral strain to the longitudinal strain in a stretched wire is called Poisson's ratio.

2. (a) : Bulk moduli of iron is 100.

3. (d) : Elastic potential energy stored per unit volume,

$$u = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{Y} = \frac{S^2}{2Y}$$

4. (b) : The ratio of change in dimension to the original dimension is known as strain.

5. (b) : N/m is not a unit of Young's modulus.

6. (c) : Gases have the highest compressibility than liquids and solids.

7. (b) : $\frac{3}{Y} = \frac{1}{\eta} + \frac{1}{3B}$

8. (c) : Tensile or compressive stress can also be terms as longitudinal stress.

9. (a) : Compressibility, $k = \frac{1}{B}$

10. (c) : Breaking stress is constant in material.

11. (i) (d) : Fluids can develop volumetric strain only.

(ii) (c) : The ratio of the lateral strain to longitudinal strain is called Poisson's ratio.

Hence, option (a) is an incorrect statement.

Its value depends only on the nature of the material.

Hence, option (b) is an incorrect statement.

It is the ratio of two like physical quantities.

Therefore, it is a unitless and dimensionless quantity.

Hence, option (c) is a correct statement.

The practical value of Poisson's ratio lies between 0 and 0.5.

Hence, option (d), is an incorrect statement.

(iii) (c) : Shearing strain = $\frac{\Delta x}{L}$

12. (b) : The stress-strain curves vary from material to material. These curves help us to understand how a given material deforms with increasing loads. Rubber is an elastomer.

13. (c) : Strain is the ratio of change in dimensions of the body to the original dimensions. Because it is a ratio, it is a dimensionless quantity.

14. All the materials undergo a change in their original state, how so ever it may be small, after the removal of deforming force. Hence, there is no such material which is perfectly elastic.

15. The reciprocal of bulk modulus of a material is called its compressibility.

i.e., compressibility = $\frac{1}{B}$

As, $B = \frac{-PV}{\Delta V}$

$\therefore$ Compressibility = $\frac{-\Delta V}{PV}$

The S.I. unit of compressibility is $N^{-1}m^2$ and dimensional formula $[M^{-1}LT^2]$.

16. It is defined as the angle θ through which a face originally perpendicular to the fixed face gets turned on applying tangential deforming force, *i.e.*,

shear strain = $\theta \simeq \tan\theta$

$$= \frac{\text{Relative displacement beween two parallel planes}}{\text{Distance between parallel planes}}$$

OR

No, it is not possible because within elastic limit strain is only of the order of 10^{-3}. Wires breaks much before it is stretched to double the length.

17. Given : $F = 200$ N

$\ell = 1$ mm $= 10^{-3}$ m

$\therefore$ Elastic potential energy stored in the wire,

$$u = \frac{1}{2} \times F \times \ell = \frac{1}{2} \times 200 \times 10^{-3} = 0.1\text{ J}$$

18. Change in length to the original length of an object is defined as a longitudinal strain.

OR

Young's modulus decreases with rise in temperature.

19. Elastic energy per unit volume, $E = \frac{1}{2} \times \text{stress} \times \text{strain}$

Using Hooke's law : Strain = $\frac{\text{Stress}}{Y}$

$$\Rightarrow E = \frac{(\text{Stress})^2}{2Y} = \frac{F^2}{2YA^2}, \text{ where } A = \frac{\pi D^2}{4}$$

$$\Rightarrow E = \frac{8F^2}{\pi^2 D^4 Y}$$

$$\therefore \frac{E_1}{E_2} = \left(\frac{D_2}{D_1}\right)^4 \qquad (\because F \text{ and } Y \text{ are same for both})$$

Given, $\frac{D_1}{D_2} = \frac{1}{2}, \frac{E_1}{E_2} = \left(\frac{2}{1}\right)^4 = \frac{16}{1} = 16:1$

20. $Y = 6 \times 10^{12}$ N/m^2

$B = 0$ [As no transverse strain]

$Y = 2\eta(1 + B) \Rightarrow Y = 2\eta$ [$\because B = 0$]

$\eta = 3 \times 10^{12}$ N/m^2

21. Energy stored in wire per unit volume

$= \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$

$= \frac{1}{2} \times 2 \times 10^{10} \times (0.01)^2 = 10^{10} \times 0.0001 = 10^6 \text{ N m}^2$

OR

Examples of normal stress are following :

(i) A load is suspended by a wire, then tensile stress is produced in the wire.

(ii) A column is supporting a vertical load, then compressive stress is produced in the wire.

22. A body is said to be perfectly plastic if it doesn't regain its original shape and size even after the removal of deforming force, *e.g.*, putty and paraffin wax.

23. Energy stored per unit volume

$= \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times \text{stress}\left(\frac{\text{stress}}{\text{Young's modulus}}\right)$

$= \frac{(\text{Stress})^2}{\text{Young's modulus}} = \frac{S^2}{2Y}$

24. Bulk modulus $(B) = \frac{\text{Hydraulic stress}}{\text{Volumetric strain}}$

or $B = \frac{\text{Hydraulic stress}}{\frac{\Delta V}{V}}$ or $\frac{\Delta V}{V} = \frac{\text{Hydraulic stress}}{B}$

$\therefore$ For contact hydraulic stress, $\frac{\Delta V}{V} \propto \frac{1}{B}$

OR

The factors affect the electricity of the material are following :

(i) Temperature : Elasticity of most of the materials decreases with increase in temperature. The elasticity of invar is not affected by temperature.

(ii) Hammering and rolling : In both of these processes, the crystal grains of the material are broken in small units due to which elasticity of the material increases.

25. Solids have the highest intermolecular force of attraction and they have least intermolecular space between molecules. Gases have the highest intermolecular space between molecules. Compressibility of gases is more than solids, because the bulk modulus is reciprocal of compressibility. Hence, elasticity of solids is more then gases.

26. The intensity of the internal force parallel or tangential to the section is called shear stress or tangential stress of a point.

$\tau = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$

SI unit of tangential stress is N/m^2 or Pa (Pascal).

27. Given : $V = 1\text{m}^3$, $h = 100$ m, $g = 9.8 \text{ m/s}^2$

ρ (water) $= 100 \text{ kg/m}^3$

$B = 22000 \text{ atm} = 22000 \times 1.013 \times 10^5 \text{ N/m}^2$

$= 22.28 \times 10^8 \text{ N/m}^2$

As $p = \rho gh = 1000 \times 9.8 \times 100 = 9.8 \times 10^5 \text{ N/m}^2$

and $B = \frac{\rho V}{\Delta V}$

or $\Delta V = \frac{\rho V}{B} = \frac{9.8 \times 10^5 \times 1}{22.28 \times 10^8}$ or $\Delta V = 4.4 \times 10^{-4} \text{ m}^3$

OR

If we consider two rods of steel and rubber, each having length ℓ and area of cross-section A. If they are subjected to the same deforming force F, then the extension Δl_s produced in the steel rod which will be less than extension Δl_r, in the rubber rod, *i.e.*, $\Delta\ell_s < \Delta\ell_r$.

As, $Y = \frac{F}{A}\frac{\ell}{\Delta\ell}$, then $Y_s = \frac{F}{A} \cdot \frac{\ell}{\Delta\ell_s}$ and $Y_r = \frac{F}{A} \cdot \frac{\ell}{\Delta\ell_r}$

$\therefore \frac{Y_s}{Y_r} = \frac{\Delta\ell_r}{\Delta\ell_r}$. So, $Y_s > Y_r$ as $\Delta\ell_s < \Delta\ell_r$

$\therefore$ We can say that Young's modules of steel is greater than that of rubber. Hence, steel is more elastic than rubber..

28. Stress $= \frac{F}{A} = \frac{\text{Mass} \times \text{acceleration}}{\text{Cross-sectional area}}$

$= \frac{550 \text{ kg} \times 9.8 \text{ m/s}^2}{0.30 \times 10^{-4} \text{ m}^2} = 1.8 \times 10^8$ Pa.

Young's modulus $(Y) = 20 \times 10^{10}$ Pa $= \frac{\text{Stress}}{\text{Strain}}$

Strain $= \frac{\text{Stress}}{Y} = \frac{1.8 \times 10^8 \text{ Pa}}{20 \times 10^{10} \text{ Pa}} = 9.0 \times 10^{-4}$.

29. Elastic potential energy in a stretched wire is the work done in stretching a wire is stored in form of potential energy of the wire.

Potential energy, U = Average force $\times$ increase in length

$= \frac{1}{2} F\Delta L = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume of the wire}$

Elastic potential energy per unit volume, $U = \frac{1}{2} \times \text{stress} \times \text{strain}$

$= \frac{1}{2} \times \text{Young's modulus} \times (\text{strain})^2$

Elastic potential energy of a stretched spring $= \frac{1}{2}kx^2$

30. From formula, increase in length

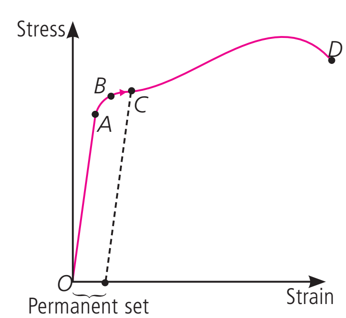
$\Delta L = \frac{FL}{AY} = \frac{4FL}{\pi D^2 Y}$; $\frac{\Delta L_s}{\Delta L_c} = \frac{F_s}{F_c}\left(\frac{D_c}{D_s}\right)^2 \frac{Y_c L_s}{Y_s L_c}$

As, $F_s = (5m + 2m)g = 7$ mg and $F_c = 5$ mg then $\frac{\Delta L_s}{\Delta L_c}$

$= \frac{7}{5} \times \left(\frac{1}{p}\right)^2 \left(\frac{1}{s}\right) q = \frac{7q}{5sp^2}$

31. Hooke's law state that strain in a solid is proportional to the applied stress within the elastic limit of that solid.

In figure *i.e.*, stress-strain curve, the region from A to B, stress and strain are not proportional. In this region body can returns to its original dimension when the stress is removed.



The point B is known as yield point or elastic limit. If the stress is further increases, strain increases rapidly even for a small change in the stress. At point C, when the load or stress is removed the body does not regain its original dimension even when stress is zero, then material is said to be permanent set.

32. Pressure (P) = 10 atm = $10 \times 1.013 \times 10^5$ Pa

($\because$ 1 atm = 1.013×10^6 Pa) = 1.013×10^6 Pa

Bulk modulus of glass (k) = 37×10^9 N/m^2

Fractional change in volume = $\frac{\Delta V}{V} = ?$

Bulk modulus (k) = $\frac{P}{\frac{\Delta V}{V}} = \frac{PV}{\Delta V}$

$$\therefore \quad \frac{\Delta V}{V} = \frac{P}{k} = \frac{1.013\times10^6}{37\times10^9} = \frac{101.3}{37}\times10^{-5} = 2.74\times10^{-5}$$

$\therefore$ Fractional change in volume $\left(\frac{\Delta V}{V}\right) = 2.74 \times 10^{-5}$.

33. Length of steel wire, $L_1 = 4.7$ m

Area of cross-section of the copper wire, $A_1 = 3.0 \times 10^{-5}$ m^2

Length of the copper wire, $L_2 = 3.5$ m

Area of cross-section of the steel wire, $A_2 = 4.0 \times 10^{-5}$ m^2

Change in length = $\Delta L_1 = \Delta L_2 = \Delta L$

Force applied in the both the cases = F

Young's modulus of the steel wire :

$$Y_1 = \frac{F_1}{A_1} \times \frac{l_1}{\Delta L_1} = \frac{F \times 4.7}{3.0\times10^{-5}\times\Delta L} \quad \text{...(i)}$$

Young's modulus of the copper wire :

$$Y_2 = \frac{F_2}{A_2} \times \frac{l_2}{\Delta L_2} = \frac{F \times 3.5}{4.0\times10^{-5}\times\Delta L} \quad \text{...(ii)}$$

Dividing (i) by (ii), we get

$$\frac{Y_1}{Y_2} = \frac{4.7\times4.0\times10^{-5}}{3.0\times10^{-5}\times3.5} = 1.79$$

The ratio of Young's modulus of steel to that cooper is 1.79.

OR

In case (a), let Y be Young's modulus of the material of the wire. If a is its area of cross-section, then we can write

$$Y = \frac{\frac{F}{a}}{\frac{\ell}{L}} = \frac{WL}{a\ell} \qquad (\because \ F = W)$$

Increase in length of wire $\ell = \frac{WL}{aY}$

In case (b), we can treat either segment of length $L/2$ in figure (b) as one of its end is fixed while other end is attached to the load W because the point M remains stationary, *i.e.*, fixed all the time due to symmetrical load. Thus, when the wire is passed over the pulley, let ℓ' be the increase in the length of each segment. Since each segment is length $L/2$, we have

$$Y = \frac{W(L/2)}{a\ell'}$$

Increase in length of wire of one side of pulley $\ell' = \frac{1}{2}\frac{WL}{aY} = \frac{\ell}{2}$.

Therefore, increase in length of both the segments of the wire = $\ell' + \ell' = \ell/2 + \ell/2 = \ell$

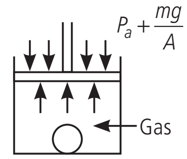
So, the increase in length remains the same.

34. (a) False, because elastic forces are not always conservative. Elastic forces are conservative as long as the loading and deloading curves are coincident, even if the curves are not linear. For example rubber.

(b) False, because when Hooke's law is obeyed, elastic forces are certainly conservative, but even when Hooke's law is not obeyed but loading and unloading curves are coincident, then too, elastic forces are conservative.

(c) False, because a part of the work done is stored as permanent energy in producing permanent set.

(d) $P_{gas} = \frac{mg}{A} + P_a = \frac{50\times10}{50\times10^{-4}} + 1\times10^5$



= 2×10^5 N/m^2

Bulk stress = $P_{gas} = 2 \times 10^5$ N/m^2

35. Let F represents some stretching force and use algebra to combine the Hooke's law account of the stretching with the Young's modulus account.

(a) According to Hooke's law, $|\vec{F}| = k\Delta L$

Young's modulus, $Y = \frac{F/A}{\Delta L/L}$

By substitution, $F = K\Delta L$ or $k = \frac{YA}{L}$

(b) The spring exerts force $-kx$. The outside agent stretching it exerts force $+kx$. We can determine the work done by integrating the force kx over the distance we stretched the wire. Then integration will reveal the work done as the wire extends.

$$W = -\int_0^{\Delta l} F\,dx = \int_0^{\Delta l} (-kx)dx = -\frac{YA}{L}\int_0^{\Delta l} x\,dx = \left[\frac{YA}{L}\left(\frac{1}{2}x^2\right)\right]_{x=0}^{x=\Delta L}$$

Therefore, $W = \frac{1}{2}\frac{YA(\Delta L)^2}{L}$

OR

Here, $P = 20{,}000$ N/cm^2 = $20{,}000 \times 10^4$ N/m^2 = 2×10^8 N/m^2

$B = 0.80 \times 10^{10}$ N/m^2

As, $B = \frac{PV}{\Delta V}$ or $\Delta V = \frac{PV}{B} = \frac{2\times10^8}{0.80\times10^{10}}\times V = \frac{V}{40}$

$\therefore$ New volume, $V' = V - \Delta V = V - \frac{V}{40} = \frac{39V}{40}$

Let ρ' be the new density of lead, under the effect of pressure applied. As mass of the lead will remain the same, so

$V\rho = V'\rho'$

or $\rho' = \frac{V\rho}{V'} = \frac{V\times11.4}{\frac{39V}{40}} = \frac{11.4\times40}{39} = 11.7$ g/cm^3