CHAPTER 10

Wave Optics

EXAM DRILL

ANSWERS

1. **(a) :** Speed and wavelength of light remains unchanged in the same medium.

2. **(b) :** When source of light is linear, all the points equidistant from the linear source lie on the surface of cylinder.

3. **(c) :** If plane wavefront is incident on a convex lens then emergent wavefront will be spherical with focus.

4. **(a) :** Coherent sources have light having same frequency, wavelength and in same phase or having a constant phase difference.

5. **(b) :** Soap bubbles appearing coloured is phenomenon of interference by division of wavefront.

6. **(a) :** Constructive interference gives maximum amplitude as it is sum of crest of two combining waves.

7. **(b) :** Using relation, $Z_F = \frac{a^2}{\lambda}$

where Z_F = fresnel distance

α = aperture width

λ = wavelength

$$\therefore Z_f = \frac{(2\times10^{-3})^2}{500\times10^{-9}} = 8\text{m}$$

8. **(a) :** Width of central maximum is proportional to wavelength of light source.

9. **(c) :** Assertion is true but reason is false.

For diffraction of a wave, size of an obstacle or aperture should be comparable to the size of wavelength of the wave. As wavelength of light is of the order of 10^{-6} m and obstacle / aperture of this size are rare, therefore, diffraction is not common in light waves. On the contrary, wavelength of sound is of the order of 1 m and obstacle / aperture of this size are readily available, therefore, diffraction is common in sound.

10. **(c) :** Assertion is true but reason is false.

When source in Young's double slit experiment is of white light, the central fringe is white as all colours meet there in phase.

11. **(c) :** Assertion is true but reason is false.

When one of slits is covered with cellophane paper, the intensity of light emerging from the slit is decreased (because this medium is translucent). Now the two interfering beam have different intensities or amplitudes. Hence intensity at minima will not be zero and fringes will become indistinct.

12. **(i)** **(c) :** The optical path difference at P is

$\Delta x = S_1P - S_2P = d\cos\theta$

$\because \quad \cos\theta = 1 - \frac{\theta^2}{2}$ for small θ

$$\therefore \quad \Delta x = d\left(1-\frac{\theta^2}{2}\right) = d\left[1-\frac{y^2}{2D^2}\right], \text{ where } D + d = D$$

(ii) **(b) :** For n^{th} maxima,

$$\Rightarrow \quad \Delta x = n\lambda \;\; d\left[1-\frac{y^2}{2D^2}\right] = n\lambda$$

y = radius of the n^{th} bright ring $= D\sqrt{2\left(1-\frac{n\lambda}{d}\right)}$

(iii) **(d) :** At the central maxima, $\theta = 0$.

$\Delta x = d = n\lambda$

$$\Rightarrow \quad n = \frac{d}{\lambda} = \frac{0.5}{0.5\times10^{-3}} = 1000$$

Hence, for the closet second bright fringe, $n = 998$.

(iv) **(c) :** Light waves from two coherent sources must have a constant phase difference.

(v) **(d) :** Interference is shown by transverse as well as mechanical waves.

13. The width of slits should be of order of wavelength of light.

14. Here, $2\mu t = \frac{\lambda}{2}$

$$\therefore \quad t_{min} = \frac{\lambda}{4\mu} = \frac{750}{4\times1.33} = 141\text{nm}$$

OR

Red shift is the spectrum of star indicates increase in apparent wavelength or decrease in apparent frequency of light reaching the observer. So, the star is receding away from earth.

15. Resultant intensity, $I_r = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$

Here, $I_1 = I$, $I_2 = 9I$, $\phi_P = \frac{\pi}{2}$, $\phi_Q = \pi$

$$\therefore \quad I_P = I + 9I + 2\sqrt{I\times9I}\cos\frac{\pi}{2} = 10I + 2\times3I\times0 = 10I$$

$$I_Q = I + 9I + 2\sqrt{I\times9I}\cos\pi = 10I + 2\times3I(-1) = 10I - 6I = 4I$$

Hence, $I_P - I_Q = 10I - 4I = 6I$

16. Here, $\Delta\lambda = 5896$ Å $- 5892$ Å $= 4$ Å, $\lambda = 5892$ Å

As $\frac{\Delta\lambda}{\lambda}=\frac{v}{c}$

$\therefore\quad v=\frac{\Delta\lambda}{\lambda}c=\frac{4}{5892}\times3\times10^{8}\ \text{m/s}$

$= 2.04\times10^{5}\ \text{m/s} = 204\ \text{km/s}$

Hence, the speed of galaxy is 204 km/s.

17. Here, $\lambda = 600$ nm $= 6\times10^{-7}$ m,
$D = 1.2$ m, $n = 1$, $x = 4$ mm $= 4\times10^{-3}$ m, $a = ?$

For first minimum, $a\sin\theta = n\lambda$

$a\cdot\left(\frac{x}{D}\right)=1\lambda$

$a=\frac{\lambda D}{x}=\frac{6\times10^{-9}\times1.2}{4\times10^{-3}}=1.8\times10^{-6}\ \text{m}$

18. As Intensity $\propto$ width of slit (w)

Also, Intensity $\propto$ square of amplitude

$\therefore\quad \frac{w_1}{w_2}=\frac{I_1}{I_2}=\frac{a^2}{b^2}=\frac{1}{4}\quad\text{or}\quad \frac{a}{b}=\sqrt{\frac{1}{4}}=\frac{1}{2}\ \text{or}\ b=2a$

$\therefore\quad \frac{I_{max}}{I_{min}}=\frac{(a+b)^2}{(a-b)^2}=\frac{(a+2a)^2}{(a-2a)^2}=\frac{9}{1}$

19. When light waves from two coherent sources superimpose such that at any particular point, crest of one wave falls on trough of the other and trough falls on crest, the amplitude of the resultant wave is zero. Hence the resultant intensity is zero. It is the phenomenon of destructive interference.

OR

Yes, electromagnetic waves follow superposition principle according to which two light wave of equal frequency having same phase or constant phase difference will superimpose constructively. The crest of one wave will meet crest of another wave and similarly for trough. If superimposition of crest and trough takes place then there is destructive interference.

20. It is the minimum distance a beam of light has to travel before its deviation from straight line path becomes significant.

Fresnel's distance $z_f=\frac{a^2}{\lambda}$

21. $u=-\frac{\Delta\lambda}{\lambda}\times c=-\frac{0.048}{100}\times3\times10^{8}\ \text{m/s} = -1.44\times10^{5}\ \text{m/s}$

22. According to Huygens's principle,

(i) Every point on given wave front (primary wave front) acts as fresh source of new disturbance, called secondary wavelets.

(ii) The secondary wavelet spread out in all the directions with speed of light in medium.

(iii) Surface touching these secondary wavelets tangentially in the forward direction at any instant gives the new (secondary) wavefront at that instant.

23.

	Interference		Diffraction
1.	It is the phenomenon of superposition of two different coherent sources	1.	It is the phenomenon of two waves coming from two parts of the same wave front.
2.	In interference pattern, all bright lines are equally bright and equally spaced.	2.	All bright lines are not equally bright and equally spaced. Brightness and width goes on decreasing with the angle of diffraction.
3.	All dark lines are perfectly dark.	3.	Dark lines are not perfectly dark. Their contrast with bright lines and width goes on decreasing with angle of diffraction.
4.	In interference bands are large in number.	4.	In diffraction bands are a few in number.

OR

Total intensity at any point is given by

$I = kR^2 = k(a^2 + b^2 + 2ab\cos\phi)$

when $\quad b = a,\ I = k(a^2 + a^2 + 2a^2\cos\phi)$

$= 2ka^2(1+\cos\phi)$

At the centre of a bright fringe, $\phi = 0°$

$\therefore\quad I_1 = 2ka^2(1+\cos 0°) = 4ka^2$

Distance between two fringes $= \beta$, which is proportional to wavelength (λ)

Now $\frac{\lambda}{4}$ corresponds to a phase difference of $\frac{2\pi}{4}$

$=\frac{2\pi}{4}=\frac{\pi}{2}\qquad\therefore\ I_2=2ka^2\left(1+\cos\frac{\pi}{2}\right)=2ka^2$

$\therefore\quad \frac{I_1}{I_2}=\frac{4ka^2}{2ka^2}=\frac{2}{1}$

24. The position of the 30th bright fringe is given by

$x_n=n\frac{\lambda D}{d}\ \Rightarrow\ x_{30}=30\frac{\lambda D}{d}$

Hence the shift of the central fringe is

$x_{30}=30\frac{\lambda D}{d}$

But $x_{30}=\frac{D}{d}(\mu-1)t\ \ \therefore\ x_{30}=\frac{30\lambda D}{d}=\frac{D}{d}(\mu-1)t$

$\Rightarrow\quad (\mu-1)=\frac{30\lambda}{t}=\frac{30\times(6000\times10^{-10})}{(3.6\times10^{-5})}=0.5$

or $\quad \mu = 1.5$

25. Given, $y_1 = a\cos\omega t$

$y_2 = a\cos(\omega t+\phi)$

The resultant displacement is given by

$y = y_1 + y_2 = a \cos \omega t + a \cos (\omega t + \phi)$

$= a \cos \omega t + a \cos \omega t \cos \phi - a \sin \omega t \sin\phi$

$= a \cos \omega t(1 + \cos \phi) - a \sin \omega t \sin \phi$

Put $R \cos \phi = a (1 + \cos \phi)$ ....(i)

$R \sin \phi = a \sin \phi$ ...(ii)

By squaring and adding eqs. (i) and (ii), we get

$R^2 = a^2 (1 + \cos^2 \phi + 2 \cos\phi) + a^2\sin^2\phi$

$= 2a^2(1 + \cos \phi) = 4a^2 \cos^2 \frac{\phi}{2}$

$\therefore \quad I = R^2 = 4a^2 \cos^2 \frac{\phi}{2} \quad \therefore \quad I = 4 I_0 \cos^2 \frac{\phi}{2}$

26. Here, $\lambda_1 = 5890$ Å $= 5890 \times 10^{-10}$ m,

$\lambda_2 = 5896$ Å $= 5896 \times 10^{-10}$ m, $D = 2$ m

$a = 2\ \mu$ m $= 2 \times 10^{-6}$ m

For the first secondary maxima,

$$\sin\theta = \frac{3\lambda_1}{2a} = \frac{x_1}{D} \text{ or } x_1 = \frac{3\lambda_1 D}{2a} \quad \text{and} \quad x_2 = \frac{3\lambda_2 D}{2a}$$

$\therefore$ Spacing between the first secondary maxima of two sodium lines

$$x_2 - x_1 = \frac{3D}{2a}(\lambda_2 - \lambda_1)$$

$$\frac{3 \times 2(5896 - 5890) \times 10^{-10}}{2 \times 2 \times 10^{-6}} = 9 \times 10^{-4} \text{ m}$$

27. (i) Reflection and refraction arise through interaction of incident light with atomic constituents of matter which vibrate with the same frequency as that of the incident light. Hence frequency remains unchanged.

(ii) Energy carried by a wave depends on the frequency of the wave, not on the speed of wave propagation.

(iii) For a given frequency, intensity of light in the photon picture is determined by

$$I = \frac{\text{Energy of photons}}{\text{area} \times \text{time}} = \frac{n \times h\upsilon}{A \times t}$$

where n is the number of photons incident normally on crossing area A in time t.

28. Conditions for sustained interference pattern

(i) Sources of light must be coherent. Two sources are said to be coherent if phase difference between them does not change with time. If phase difference keeps changing with time, then position of maxima and minima keeps on changing and a sustained interference pattern is not obtained on screen.

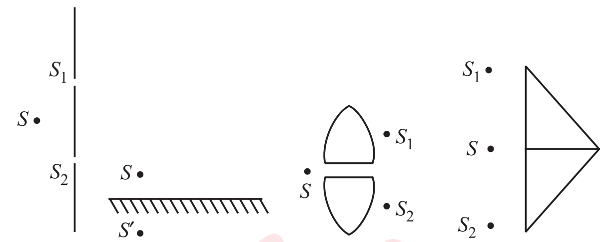
Since two independent sources of light can never be coherent so a single source is split in two coherent sources. Some ways of splitting a single source into two coherent source are

(a) Light from source S falls on two small openings which acts two sources. The openings are called slits.

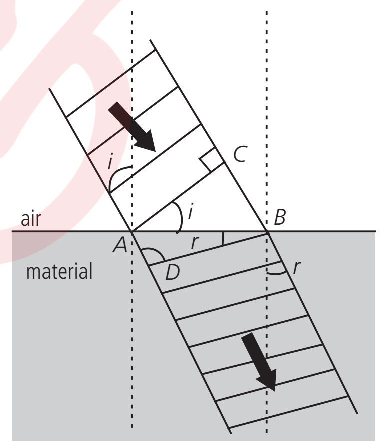
(b) A source S is kept in front of plane mirror. The source S and its reflection S' acts as two coherent source of light.

(c) A point source is kept infront of a lens split into two parts and separated by a small distance. The images of the source by the two lens acts as two coherent sources.

(d) Sources S kept infront of two thin prism. The prism deviates the light and it appears to come from S_1 and S_2 which act as two coherent sources.



29. Refraction: Consider a plane monochromatic wave hitting the surface of a transparent material of refractive index n. The velocity of light in the material is c_m and that in air c_a. Now in Figure.



$CB = AB \sin i$

$AD = AB \sin r$

The same applies about the new envelope as in the case of reflection:

time to travel $CB = CB/c_a = AB \sin i / c_a$

time to travel $AD = AD/c_m = AB \sin i / c_m$

But these times are equal and therefore

$c_a/c_m = \sin i / \sin r = {}^a n_m$.

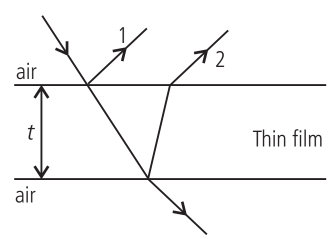
This is Snell's law, and it was verified later by Foucault and others.

Notice that since the refractive index of a transparent material is greater than 1, Huygen's theory requires that the velocity of light in air should be greater than that in the material.

OR

Thin Film Interference :

If light rays are incident normally on a thin film of uniform thickness t, then waves reflected from the upper surface interfere with waves reflected from the lower surface. If oil is floating on surface of water and white light falls on it, various colours are observed due to interference of waves reflected from upper surface and lower surface.



Wave 1 suffers a phase change of π due to reflection against denser medium (A phase change of π is equivalent to path length of $\lambda/2$)

Wave 2 travels an extra path of $2t$(for normal incidence) in medium of refractive index μ.

Equivalent path in air = $\mu(2t)$, Path difference in the two reflected rays 1 and 2

$$\Delta x = x_2 - x_1 = 2\mu t - \lambda/2$$

For constructive interference, $\Delta x = n\lambda$

$$2\mu t - \frac{\lambda}{2} = n\lambda \quad \Rightarrow \quad 2\mu t = n\lambda + \frac{\lambda}{2} \quad \text{or} \quad 2\mu t = (2n+1)\frac{\lambda}{2}$$

For destructive interference

$$2\mu t - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2} \quad \Rightarrow \quad 2\mu t = n\lambda$$

30. The diffraction pattern produced by a single slit of width a when a monochromatic light or wavelength λ is incident normally. The diffraction pattern consists of a central maximum bright band with alternating bright and dark bands of decreasing intensity on both sides. of the central maximum.

(i) Condition you nth secondary maximum is

$\Delta x = a \sin\theta_n = (2n + 1)\frac{\lambda}{2}$

where n = 1, 2, 3...

(ii) Condition for nth secondary minimum is

$\Delta x = a\sin\theta_n = n\lambda$

where n = 1,2,3...

Further width of secondary maxima or minima is $\beta = \frac{\lambda D}{a}$

whereas, for central maximum = $\frac{\alpha\lambda D}{a}$

Angular width of central maximum is, and for secondary maxima is $\frac{\lambda}{a}$. Hence, central maximum is twice the angular width of secondary maximum.