Wave Optics

CHAPTER 10

NCERT FOCUS

ANSWERS

Topic 1

1. (a) In the process of reflection wavelength, frequency and speed of incident light remain unchanged.

So, speed of reflected light = speed of incident light

$c = 3 \times 10^8$ m s^{-1}

Wavelength of reflected light = wavelength of incident light

$\lambda = 589 \times 10^{-9}$ m

frequency of reflected light = frequency of incident light

$$f = \frac{c}{\lambda} = \frac{3\times10^8}{589\times10^{-9}} = 5.09\times10^{14}\ \text{Hz}$$

(b) In the process of refraction wavelength and speed changes but the frequency remain the same.

Speed of light in water

$$v = \frac{c}{{}^a\mu_w} = \frac{3\times10^8}{1.33} = 2.26\times10^8\ \text{m s}^{-1}$$

Wavelength of light in water

$$\lambda = \frac{v}{f} = \frac{2.26\times10^8}{5.09\times10^{14}} = 444\times10^{-9}\ \text{m}$$

or $\lambda = 444$ nm.

2. (a) Spherical wavefront : All particles vibrating in same phase will lie on a sphere.

(b) Plane wavefront : Light will be a parallel beam after passing through the convex lens.

(c) Plane wavefront : Light rays from a distant star are nearly parallel as a small portion of a huge spherical wavefront is nearly plane.

3. (a) Speed of light in glass,

$$v = \frac{c}{{}^a\mu_g} = \frac{3\times10^8}{1.5} = 2\times10^8\ \text{m s}^{-1}$$

(b) Yes, speed of light in glass depends upon the colour of light (*i.e.*, λ). Thus, speed of light is different for red and violet colours.

As $\mu_V > \mu_R$, so, $\lambda_V < \lambda_R$, hence, $v_V < v_R$

Speed of red colour is more than violet colour light in glass.

4. By Brewster's law, $\tan i_p = {}^a\mu_g = 1.5$

$\therefore$ Brewster's angle, $i_p = \tan^{-1}(1.5) = 56.3°$.

5. In the reflected light the wavelength and frequency remain the same as that of incident light.

Wavelength of reflected light = 5000 Å, frequency of reflected light

$c = f\lambda$

$$f = \frac{c}{\lambda} = \frac{3\times10^8}{5\times10^{-7}} = 6\times10^{14}\ \text{Hz}$$

When reflected ray is normal to incident ray,

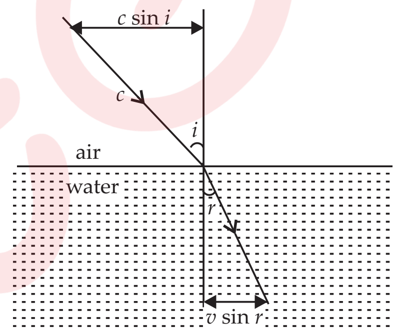
$\angle i + \angle r = 90°$

$2\angle i = 90°$ $\quad (\because \angle i = \angle r)$

$\angle i = 45°$

For an angle of incidence, 45° the reflected ray is normal to incident ray.

6. In Newton's corpuscular (particle) picture of refraction, particles of light incident from a rarer to a denser medium experience a force of attraction normal to the surface. This results in an increase in the normal component of velocity but the component along the surface remains unchanged.



Considering a ray of light going from a rarer medium (air) to a denser medium (water).

Let c = speed of light in vacuum (or air),

v = speed of light in water,

i = angle of incidence, and r = angle of refraction

Then according to Newton's corpuscular theory,

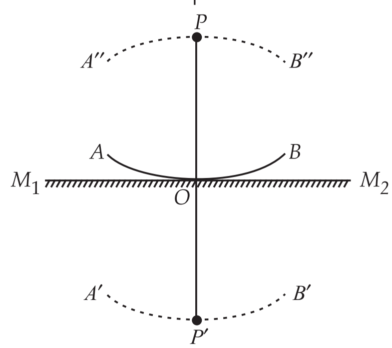
Component of velocity c along surface of separation = Component of velocity v along the surface of separation

$$\therefore \quad c\sin i = v\sin r \quad \text{or} \quad \frac{v}{c} = \frac{\sin i}{\sin r} = {}^a\mu_w$$

As ${}^a\mu_w > 1$, therefore, $v > c$.

So, according to Newton's corpuscular theory the speed of light in medium is larger than speed of light in air. $v > c$ but in fact the experimental observation shows that speed of light is smaller in denser medium as compared to rare medium $v < c$.

7.



In figure, P is a point object placed at a distance r from a plane mirror M_1M_2. With P as centre and $PO = r$ as radius, draw a spherical arc; AB. This is the spherical wavefront from the object, incident on M_1M_2.

If mirrors were not present, the position of wavefront AB would be $A'B'$ where $PP' = 2r$. In the presence of the mirror, wave front AB would appear as $A''PB''$, according to Huygen's construction. As it is clear from the figure $A'B'$ and $A''B''$ are two spherical arcs located symmetrically on either side of M_1M_2. Therefore, $A'P'B'$ can be treated as reflected image of $A''PB''$. From simple geometry, we find $OP = OP'$, which was to be proved.

8. (a) Speed of light in vacuum is independent of all the factors listed above. It is also independent of relative motion between source and observer.

(b) Dependence of speed of light in a medium:

(i) The speed of light in a medium does not depend on the nature of the source. Although speed is determined by the properties of the medium of propagation.

(ii) The speed of light in a medium is independent of the direction of propagation for an isotropic media.

(iii) The speed of light is independent of the motion of the source relative to the medium but it depends upon the motion of the observer relative to the medium.

(iv) The speed of light in a medium depends on wavelength of light *i.e.*, $v \propto \lambda$.

(v) The speed of light in a medium is independent of intensity.

Topic 2

1. Here $d = 0.28$ mm, $D = 1.4$ m

Distance of fourth bright fringe from center

$= 1.2 \text{ cm} = 1.2 \times 10^{-2}$ m

Linear position of n^{th} bright fringe, $y_n = \dfrac{nD\lambda}{d}$

Linear position of 4^{th} bright fringe, $y_4 = \dfrac{4D\lambda}{d}$

$$1.2 \times 10^{-2} = \frac{4\,(1.4)\,\lambda}{0.28 \times 10^{-3}}, \quad \lambda = 600 \text{ nm}$$

2. In Young's double-slit experiment net intensity of light at a point on screen is

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

For $I_1 = I_2 = I$, $I_{net} = 2I + 2I\cos\phi$

Relation between path difference and phase difference is

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

For path difference λ, phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}\lambda = 2\pi$$

$I_{net} = K = 2I + 2I\cos 2\pi$ or $K = 4I$ ...(i)

For a path difference $\lambda/3$, phase difference

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x \;;\; \Delta\phi = \frac{2\pi}{\lambda}\left(\frac{\lambda}{3}\right) = \frac{2\pi}{3}$$

Now the intensity, $I_{net} = 2I + 2I\cos\dfrac{2\pi}{3}$

$I_{net} = 2I - 2I\sin 30°$

$I_{net} = I = \dfrac{K}{4}$

3. Here, $d = 2$ mm, $D = 1.2$ m,

$\lambda_1 = 650 \text{ nm} = 650 \times 10^{-9}$ m,

$\lambda_2 = 520 \text{ nm} = 520 \times 10^{-9}$ m

(a) Distance of third bright fringe from the central maximum for the wavelength 650 nm.

$$y_3 = \frac{3\lambda D}{d} = \frac{3\,(650 \times 10^{-9})\,1{\cdot}2}{2 \times 10^{-3}} = 1{\cdot}17 \text{ mm.}$$

(b) Let at linear distance 'y' from center of screen the bright fringes due to both wavelength coincides. Let n_1 number of bright fringe with wavelength λ_1 coincides with n_2 number of bright fringe with wavelength λ_2.

We can write

$y = n_1\beta_1 = n_2\beta_2$

$n_1\dfrac{\lambda_1 D}{d} = n_2\dfrac{D\lambda_2}{d}$ or $n_1\lambda_1 = n_2\lambda_2$ ...(i)

Also at first position of coincide, the n^{th} bright fringe of one will coincide with $(n + 1)^{th}$ bright fringe of other.

If $\lambda_2 < \lambda_1$,

So, then $n_2 > n_1$

then $n_2 = n_1 + 1$ ...(ii)

Using equation (ii) in equation (i)

$n_1\lambda_1 = (n_1 + 1)\,\lambda_2$

$n_1\,(650) \times 10^{-9} = (n_1 + 1)\,520 \times 10^{-9}$

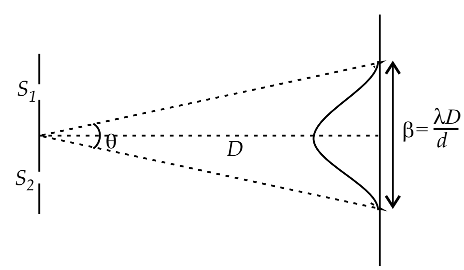
$65\,n_1 = 52\,n_1 + 52$ or $12\,n_1 = 52$ or $n_1 = 4$

Thus, $y = n_1\beta_1 = 4\left[\dfrac{(6.5 \times 10^{-7})(1.2)}{2 \times 10^{-3}}\right]$

$= 1.56 \times 10^{-3}$ m $= 1.56$ mm

So, the fourth bright fringe of wavelength 520 nm coincides with 5^{th} bright fringe of wavelength 650 nm.

4.



Angular width $\theta = \dfrac{\text{arc}}{\text{radius}}$

$\theta = \dfrac{\lambda D/d}{D} = \lambda/d$ ...(i)

With air between slit and screen.

$$\theta = 0.2 = \frac{600 \times 10^{-9}}{d}$$

With water as the medium between slit and screen.

$\lambda' = \lambda/\mu$

Angular width $\theta' = \lambda'/d$ ...(ii)

Dividing $\dfrac{\theta'}{\theta} = \dfrac{\lambda'}{\lambda}$

$$\theta' = \frac{\lambda'}{\lambda}\theta = \frac{\lambda}{\mu\lambda}\theta = \frac{\theta}{\mu} \quad \text{or} \quad \theta' = \frac{0.2°}{(4/3)} = 0.15°$$

5. Angular width, $\theta = \dfrac{\lambda}{d}$

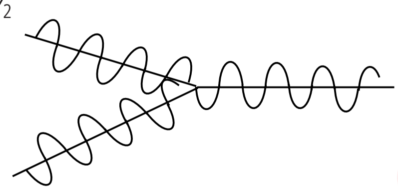
$$0.1° = \frac{0.1}{180}\pi \text{ rad} = \frac{6\times10^{-7}}{d}$$

$$\therefore \quad d = \frac{6\times10^{-7}\times180}{0.1\times\pi} = 3.44\times10^{-4}\text{ m}$$

6. (a) The low flying aircraft reflects the TV signals. Due to superposition between the direct signal received by the antenna and the reflected signals from aircraft. We sometimes notice slight shaking of the picture on the TV screen.

(b) Superposition principle states how to explain the formation of resultant wave by combination of two or more waves. Let y_1 and y_2 represent instantaneous displacement of two superimposing waves, then resultant waves instantaneous displacement is given by

$y = y_1 + y_2$



Topic 3

1. Fresnel distance required for a sufficient spreading of central bright fringe, so that diffraction is appreciable

$$D_F = \frac{a^2}{\lambda} = \frac{(4\times10^{-3})^2}{400\times10^{-9}} = 40\text{ m}$$

So, for distance less than 40 m between slit and screen, ray optics is a good approximation as within this distance, the spreading is negligible.

2. (a) Linear width of central maximum

$$\beta = \frac{2\lambda D}{d}$$

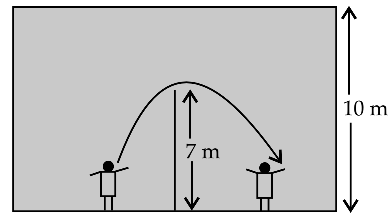
On doubling the slit width 'd', the size of central diffraction band is halved.

Because the width of central maximum is halved. Its area become 1/4 times and hence the intensity become 4 times the initial intensity.

(b) In double slit experiment, an interference pattern is observed by waves from two slits but as each slit provide a diffraction pattern of its own, thus the intensity of interference pattern in Young's double slit experiment is modified by diffraction pattern of each slit.

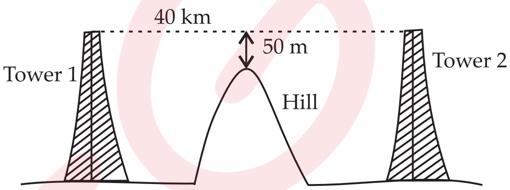
(c) Waves from the distant source are diffracted by the edge of the circular obstacle and these waves superimpose constructively at the centre of obstacle's shadow producing a bright spot.

(d) We know for diffraction to take place, size of the obstacle/aperture should be of the order of wavelength. Wavelength of sound waves is of the order of few meters that is why sound waves can bend through the aperture in partition wall but wavelength of light waves is of the order of micrometer, hence light waves can not bend through same big aperture. That is why the two students can hear each other but cannot see each other.



(e) In optical instruments, the sizes of apertures are much larger as compared to wavelength of light. So the diffraction effects are negligibly small. Hence, the assumption that light travels in straight lines is used in the optical instruments.

3.



For diffraction of radiowaves not to occur the distance of middle hill should be less than fresnel distance for a slit width 'a' of 50 m.

$$Z_F = \frac{a^2}{\lambda}$$

Distance between one of the towers and the hill halfway in between

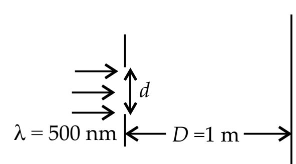
$$= \frac{40}{2}\text{ km} = 20000\text{ m} = 20\times10^3\text{ m}$$

Longest wavelength of radiowave which can be sent without appreciable diffraction effect

$$\lambda = \frac{a^2}{Z_F} = \frac{50\times50}{20\times10^3} = 12.5\text{ cm}$$

Thus wavelength of radio waves longer than 12.5 cm will bend due to the hill in the middle of towers.

4.



First minimum is observed at a distance 2.5 mm from centre of the screen.

So, $x = 2.5\text{ mm} = 2.5\times10^{-3}\text{ m}$

$\lambda = 500\text{ nm} = 500\times10^{-9}\text{ m} = 5\times10^{-7}\text{ m}$

$D = 1\text{ m}, n = 1$

So, condition for minima is $\dfrac{dx}{D} = n\lambda$

$$\therefore \quad \text{Slit width, } d = \frac{n\lambda D}{x}$$

$$\therefore \quad d = \frac{5\times10^{-7}\times1\times1}{2.5\times10^{-3}} = 0.2\text{ mm}$$

5. Let the single-slit of width a be divided into n smaller slits. If a' is the width of each one of the smaller slits, $a' = a/n$. For the single-slit to produce zero intensity, each one of the smaller slits should also produce zero intensity. This is possible if

$$\theta = \lambda/a' \text{ or } \theta = \frac{\lambda}{a/n} \quad \text{or } \theta = \frac{n\lambda}{a}$$

6. Here, $\lambda = 6563$ Å, $\Delta\lambda = 15$ Å,

$$c = 3 \times 10^8 \text{ m s}^{-1}$$

Since the star is receding away, hence its velocity v is negative (*i.e.* if $\Delta\lambda$ is positive, v is negative)

$$\therefore \quad \Delta\lambda = -\frac{v\lambda}{c}$$

$$\text{or } v = -\frac{c\Delta\lambda}{\lambda} = -\frac{3\times10^8\times15}{6563} = -6.86\times10^5 \text{ m s}^{-1}$$

Here, negative sign shows recession of star.

7. Sound waves require a medium for propagation.

The Doppler formula for frequency shift differs slightly in two situations

(i) Source at rest, observer moving

$$\upsilon' = \upsilon\left[\frac{v \pm v_0}{v}\right]$$

(ii) Observer at rest, source moving

$$\upsilon' = \upsilon\left[\frac{v}{v \pm v_0}\right]$$

The two formulas are different because motion of the observer relative to the medium is different in the two situations for light waves in vacuum. No such relative relation of observer and medium exist.

Hence only the relative motion between the source and the observer counts and the frequency shift is same, $\upsilon' = \upsilon\left[1 \pm \frac{v}{c}\right]$.