

# Wave Optics



## ANSWERS

### Topic 1

1. (a) In the process of reflection wavelength, frequency and speed of incident light remain unchanged.

So, speed of reflected light = speed of incident light

$$c = 3 \times 10^8 \text{ m s}^{-1}$$

Wavelength of reflected light = wavelength of incident light

$$\lambda = 589 \times 10^{-9} \text{ m}$$

frequency of reflected light = frequency of incident light

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{589 \times 10^{-9}} = 5.09 \times 10^{14} \text{ Hz}$$

(b) In the process of refraction wavelength and speed changes but the frequency remain the same.

Speed of light in water

$$v = \frac{c}{\mu_w} = \frac{3 \times 10^8}{1.33} = 2.26 \times 10^8 \text{ m s}^{-1}$$

Wavelength of light in water

$$\lambda = \frac{v}{f} = \frac{2.26 \times 10^8}{5.09 \times 10^{14}} = 444 \times 10^{-9} \text{ m}$$

or  $\lambda = 444 \text{ nm}$ .

2. (a) Spherical wavefront : All particles vibrating in same phase will lie on a sphere.

(b) Plane wavefront : Light will be a parallel beam after passing through the convex lens.

(c) Plane wavefront : Light rays from a distant star are nearly parallel as a small portion of a huge spherical wavefront is nearly plane.

3. (a) Speed of light in glass,

$$v = \frac{c}{\mu_g} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m s}^{-1}$$

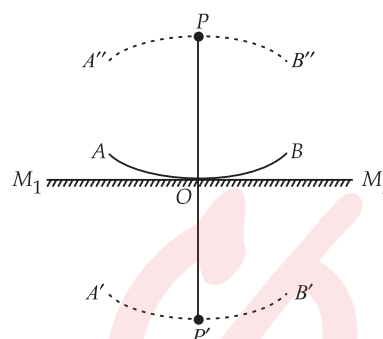
(b) Yes, speed of light in glass depends upon the colour of light (i.e.,  $\lambda$ ).

Thus, speed of light is different for red and violet colours.

As  $\mu_v > \mu_r$ , so,  $\lambda_v < \lambda_r$ , hence,  $v_v < v_r$

Speed of red colour is more than violet colour light in glass.

4.



In figure,  $P$  is a point object placed at a distance  $r$  from a plane mirror  $M_1M_2$ . With  $P$  as centre and  $PO = r$  as radius, draw a spherical arc  $AB$ . This is the spherical wavefront from the object, incident on  $M_1M_2$ .

If mirrors were not present, the position of wavefront  $AB$  would be  $A'B'$  where  $PP' = 2r$ . In the presence of the mirror, wave front  $AB$  would appear as  $A''PB''$ , according to Huygen's construction. As it is clear from the figure  $A'B'$  and  $A''PB''$  are two spherical arcs located symmetrically on either side of  $M_1M_2$ . Therefore,  $A'PB'$  can be treated as reflected image of  $A''PB''$ . From simple geometry, we find  $OP = OP'$ , which was to be proved.

5. (a) Speed of light in vacuum is independent of all the factors listed above. It is also independent of relative motion between source and observer.

(b) Dependence of speed of light in a medium:

(i) The speed of light in a medium does not depend on the nature of the source. Although speed is determined by the properties of the medium of propagation.

(ii) The speed of light in a medium is independent of the direction of propagation for an isotropic media.

(iii) The speed of light is independent of the motion of the source relative to the medium but it depends upon the motion of the observer relative to the medium.

(iv) The speed of light in a medium depends on wavelength of light i.e.,  $v \propto \lambda$ .

(v) The speed of light in a medium is independent of intensity.

### Topic 2

1. Here  $d = 0.28 \text{ mm}$ ,  $D = 1.4 \text{ m}$

Distance of fourth bright fringe from center

$$= 1.2 \text{ cm} = 1.2 \times 10^{-2} \text{ m}$$

Linear position of  $n^{\text{th}}$  bright fringe,  $y_n = \frac{nD\lambda}{d}$

Linear position of 4<sup>th</sup> bright fringe,  $y_4 = \frac{4D\lambda}{d}$

$$1.2 \times 10^{-2} = \frac{4(1.4)\lambda}{0.28 \times 10^{-3}}, \lambda = 600 \text{ nm}$$

2. In Young's double-slit experiment net intensity of light at a point on screen is

$$I_{\text{net}} = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

$$\text{For } I_1 = I_2 = I, I_{\text{net}} = 2I + 2I \cos \phi$$

Relation between path difference and phase difference is

$$\Delta \phi = \frac{2\pi}{\lambda} \Delta x$$

For path difference  $\lambda$ , phase difference,

$$\Delta \phi = \frac{2\pi}{\lambda} \lambda = 2\pi$$

$$I_{\text{net}} = K = 2I + 2I \cos 2\pi \quad \text{or} \quad K = 4I$$

For a path difference  $\lambda/3$ , phase difference

$$\Delta \phi = \frac{2\pi}{\lambda} \Delta x; \quad \Delta \phi = \frac{2\pi}{\lambda} \left( \frac{\lambda}{3} \right) = \frac{2\pi}{3}$$

Now the intensity,  $I_{\text{net}} = 2I + 2I \cos \frac{2\pi}{3}$

$$I_{\text{net}} = 2I - 2I \sin 30^\circ$$

$$I_{\text{net}} = I = \frac{K}{4}$$

3. Here,  $d = 2 \text{ mm}$ ,  $D = 1.2 \text{ m}$ ,

$$\lambda_1 = 650 \text{ nm} = 650 \times 10^{-9} \text{ m},$$

$$\lambda_2 = 520 \text{ nm} = 520 \times 10^{-9} \text{ m}$$

(a) Distance of third bright fringe from the central maximum for the wavelength 650 nm.

$$y_3 = \frac{3\lambda D}{d} = \frac{3(650 \times 10^{-9})1.2}{2 \times 10^{-3}} = 1.17 \text{ mm}.$$

(b) Let at linear distance 'y' from center of screen the bright fringes due to both wavelength coincides. Let  $n_1$  number of bright fringe with wavelength  $\lambda_1$  coincides with  $n_2$  number of bright fringe with wavelength  $\lambda_2$ .

We can write

$$y = n_1 \beta_1 = n_2 \beta_2$$

$$n_1 \frac{\lambda_1 D}{d} = n_2 \frac{D \lambda_2}{d} \quad \text{or} \quad n_1 \lambda_1 = n_2 \lambda_2 \quad \dots(i)$$

Also at first position of coincide, the  $n^{\text{th}}$  bright fringe of one will coincide with  $(n+1)^{\text{th}}$  bright fringe of other.

...(i) If  $\lambda_2 < \lambda_1$ ,

So, then  $n_2 > n_1$

then  $n_2 = n_1 + 1$

...(ii)

Using equation (ii) in equation (i)

$$n_1 \lambda_1 = (n_1 + 1) \lambda_2$$

$$n_1 (650) \times 10^{-9} = (n_1 + 1) 520 \times 10^{-9}$$

$$65 n_1 = 52 n_1 + 52 \quad \text{or} \quad 12 n_1 = 52 \quad \text{or} \quad n_1 = 4$$

$$\text{Thus, } y = n_1 \beta_1 = 4 \left[ \frac{(6.5 \times 10^{-7})(1.2)}{2 \times 10^{-3}} \right]$$

$$= 1.56 \times 10^{-3} \text{ m} = 1.56 \text{ mm}$$

So, the fourth bright fringe of wavelength 520 nm coincides with 5<sup>th</sup> bright fringe of wavelength 650 nm.

