

Dual Nature of Radiation and Matter

CHAPTER 11



ANSWERS

Topic 1

1. (a) Maximum energy of X-ray photon = Maximum energy of an accelerated electron

$$h\nu_{\max} = eV$$

$$\therefore \nu_{\max} = \frac{eV}{h} = \frac{1.6 \times 10^{-19} \times 30 \times 10^3}{6.63 \times 10^{-34}} \\ = 7.24 \times 10^{18} \text{ Hz}$$

$$(b) \lambda_{\min} = \frac{c}{\nu_{\max}} = \frac{3 \times 10^8}{7.24 \times 10^{18}} = 0.0414 \times 10^{-9} \\ = 0.0414 \text{ nm.}$$

2. Here $W_0 = 2.14 \text{ eV}$, $\nu = 6 \times 10^{14} \text{ Hz}$

$$(a) K_{\max} = h\nu - W_0 \\ = 6.63 \times 10^{-34} \times 6 \times 10^{14} \text{ J} - 2.14 \text{ eV} \\ = \frac{6.63 \times 6 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} - 2.14 \text{ eV} \\ = 2.48 - 2.14 = 0.34 \text{ eV.}$$

$$(b) \text{ As } eV_0 = K_{\max} = 0.34 \text{ eV} \\ \therefore \text{ Stopping potential, } V_0 = 0.34 \text{ V.}$$

$$(c) K_{\max} = \frac{1}{2} m v_{\max}^2 = 0.34 \text{ eV} \\ = 0.34 \times 1.6 \times 10^{-19} \text{ J} \\ \text{or } v_{\max}^2 = \frac{2 \times 0.34 \times 1.6 \times 10^{-19}}{m} \\ = \frac{2 \times 0.34 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}} = 119560.4 \times 10^6$$

$$\text{or } v_{\max} = 345.8 \times 10^3 \text{ m s}^{-1} = 345.8 \text{ km s}^{-1}.$$

$$3. \text{ Here, } V_0 = 1.5 \text{ V} \\ K_{\max} = eV_0 = 1.5 \text{ eV} = 1.5 \times 1.6 \times 10^{-19} \text{ J} \\ = 2.4 \times 10^{-19} \text{ J}$$

$$4. \text{ Here } \lambda = 632.8 \text{ nm} = 632.8 \times 10^{-9} \text{ m}, \\ P = 9.42 \text{ mW} = 9.42 \times 10^{-3} \text{ W}$$

$$(a) \text{ Energy of each photon,} \\ E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{632.8 \times 10^{-9}} = 3.14 \times 10^{-19} \text{ J}$$

Momentum of each photon,

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{632.8 \times 10^{-9}} = 1.05 \times 10^{-27} \text{ kg m s}^{-1}.$$

- (b) Number of photons arriving per second at the target,

$$N = \frac{P}{E} = \frac{9.42 \times 10^{-3}}{3.14 \times 10^{-19}} \\ = 3 \times 10^{16} \text{ photons per second.}$$

- (c) As $mv = p$

$$\therefore \text{ Velocity, } v = \frac{p}{m} = \frac{1.05 \times 10^{-27} \text{ kg m s}^{-1}}{1.67 \times 10^{-27} \text{ kg}} \\ = 0.63 \text{ m s}^{-1}.$$

5. Energy of each photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{550 \times 10^{-9}} = 3.62 \times 10^{-19} \text{ J}$$

Number of photons incident on earth's surface per second per square metre

$$= \frac{\text{total energy per square metre} \\ \text{per second reaching on earth surface}}{\text{energy of each photon}}$$

$$= \frac{1.388 \times 10^3}{3.62 \times 10^{-19}} = 3.8 \times 10^{21}$$

6. Given, slope of graph = $4.12 \times 10^{-15} \text{ V s}$

$$\text{slope} = \frac{h}{e} \\ 4.12 \times 10^{-15} = \frac{h}{1.6 \times 10^{-19}}$$

$$\text{or } h = 6.592 \times 10^{-34} \text{ J s}$$

7. Here $P = 100 \text{ W}$, $\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$

(a) Energy of each photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{589 \times 10^{-9}} \text{ J} = 3.38 \times 10^{-19} \text{ J} \\ = \frac{3.38 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 2.11 \text{ eV}$$

(b) Rate at which photons are delivered to sphere,
 $N = \text{Total energy/energy of each photon}$

$$N = \frac{P}{E} = \frac{100 \text{ J s}^{-1}}{3.38 \times 10^{-19} \text{ J}} \\ = 3.0 \times 10^{20} \text{ photons/second.}$$

8. According to Einstein's relation

$$h\nu = h\nu_0 + \frac{1}{2}mv_{\max}^2$$

Maximum kinetic energy of emitted electron

$$\frac{1}{2}mv_{\max}^2 = h(\nu - \nu_0)$$

$$= 6.63 \times 10^{-34} (8.2 \times 10^{14} - 3.3 \times 10^{14})$$

$$= 32.49 \times 10^{-20} \text{ joule} \approx 2 \text{ eV}$$

∴ Cut off potential for emitted electron will be 2 volt.

9. Let us calculate the energy associated with photons incident

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}} = 6.027 \times 10^{-19} \text{ J}$$

$$E = \frac{6.027 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 3.77 \text{ eV}$$

Since, energy of incident photons *i.e.*, 3.77 eV is less than work function, hence no emission will take place.

10. According to Einstein's equation

$$h\nu = h\nu_0 + \frac{1}{2}mv_{\max}^2$$

So, threshold frequency

$$\nu_0 = \frac{h\nu - \frac{1}{2}mv_{\max}^2}{h} \quad \text{or} \quad \nu_0 = \nu - \frac{mv_{\max}^2}{2h}$$

$$\nu_0 = 7.21 \times 10^{14} - \frac{9.1 \times 10^{-31} \times (6 \times 10^5)^2}{2 \times (6.63 \times 10^{-34})}$$

$$\nu_0 = 4.74 \times 10^{14} \text{ Hz.}$$

11. Energy of incident radiation

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{488 \times 10^{-9} \times 1.6 \times 10^{-19}} \text{ eV} = 2.55 \text{ eV}$$

Now using Einstein's relation

$$E = W_0 + eV_0$$

where V_0 is stopping potential

$$2.55 \text{ eV} = W_0 + e \times 0.38 \text{ V}$$

$$\Rightarrow W_0 = 2.55 - 0.38$$

$$\Rightarrow W_0 = 2.17 \text{ eV.}$$

12. (a) Here, power of transmitter,

$$P = 10 \text{ kW} = 10^4 \text{ W}$$

Total energy emitted per second = $P \times t = 10^4 \times 1 = 10^4 \text{ J}$

Energy of each photon,

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{500}$$

If n is the number of photons emitted, then $nE = 10^4$

$$\text{or } n = \frac{10^4}{E} = \frac{10^4 \times 500}{6.63 \times 3 \times 10^{-26}} = 2.51 \times 10^{31}$$

We see that the energy of a radio photon is exceedingly small and the number of photons emitted per second in a radio beam is enormously large. Therefore, negligible error involved in ignoring the existence of a minimum quantum of energy (photon) and treating the total energy of a radio wave as continuous.

(b) Here, area of the pupil, $A = 0.4 \text{ cm}^2$

$$= 0.4 \times 10^{-4} \text{ m}^2, \nu = 6 \times 10^{14} \text{ Hz}$$

$$\text{Intensity} = 10^{-10} \text{ W m}^{-2}$$

Energy of a photon is given by, $E = h\nu$

$$= 6.63 \times 10^{-34} \times 6 \times 10^{14} \text{ J} \approx 4 \times 10^{-19} \text{ J.}$$

If n = number of photons falling per sec per unit area, the energy per unit area per sec due to these photons = total energy of n photons = $n \times 4 \times 10^{-19} \text{ J m}^{-2}$

Since, intensity = energy per unit area per second

$$\therefore 10^{-10} = n \times 4 \times 10^{-19}$$

$$\text{or } n = \frac{10^{-10}}{4 \times 10^{-19}} = 2.5 \times 10^8 \text{ m}^{-2} \text{ s}^{-1}$$

∴ Number of photons entering the pupil per second = $n \times \text{area of the pupil}$

$$= 2.5 \times 10^8 \times 0.4 \times 10^{-4} \text{ s}^{-1} = 10^4 \text{ s}^{-1}.$$

Though this number is not large as in part (a) above, it is large enough for us to 'sense' or 'count' the individual photons by our eye.

13. Wave picture of radiation state that incident energy is uniformly distributed among all the electrons continuously. Let us first calculate the total number of recipient electrons in 5 layers of sodium.

Consider each sodium atom has one electron free as conduction electron.

$$\text{Effective atomic area} = 10^{-20} \text{ m}^2$$

Number of conduction electrons in 5 layers

$$n = \frac{5 \times \text{area of each layer}}{\text{effective area of each atom}}$$

$$\text{or } n = \frac{5 \times 2 \times 10^{-4}}{10^{-20}} = 10^{17}$$

Incident power = incident intensity \times area

$$= 10^{-5} \times 2 \times 10^{-4} = 2 \times 10^{-9} \text{ W}$$

As incident energy is equally distributed among all conduction electrons.

$$\text{Energy to each conduction electron per second} = \frac{2 \times 10^{-9}}{10^{17}} = 2 \times 10^{-26} \text{ W}$$

Time required for emission by each electron

$$t = \frac{\text{total work function energy}}{\text{energy received per second}}$$

$$t = \frac{2 \text{ eV}}{2 \times 10^{-26} \text{ W}} = \frac{2 \times 1.6 \times 10^{-19}}{2 \times 10^{-26}} = 1.6 \times 10^7 \text{ s} \\ = 0.5 \text{ year}$$

where experimental observation shows that emission of photoelectrons is instantaneous $\approx 10^{-9}$ sec

Thus, wave picture fails to explain photoelectric effect.

14. From the first data, work function of given photosensitive material can be calculated.

$$E = h\nu = W_0 + \frac{1}{2}mv_{\max}^2$$

$$\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{640.2 \times 10^{-9} \times 1.6 \times 10^{-19}} = W_0 + 0.54 \text{ eV}$$

$$1.94 \text{ eV} = W_0 + 0.54 \text{ eV} \quad \text{or} \quad W_0 = 1.4 \text{ eV}$$

Now the source is replaced by iron source which produce 427.2 nm wavelength.

$$E = W_0 + \frac{1}{2}mv_{\max}^2$$

$$\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{427.2 \times 10^{-9} \times 1.6 \times 10^{-19}} = 1.4 \text{ eV} + (K.E.)_{\max}$$

$$2.9 \text{ eV} = 1.4 \text{ eV} + (K.E.)_{\max}$$

$$\text{or } (K.E.)_{\max} = 1.5 \text{ eV}$$

Stopping potential required is 1.5 volt.

15. The distance between laser source and receiver does not affect the energy of each photon incident, hence does not affect the energy of emitted photoelectrons.

But the reduction in distance will increase the intensity of incident light and hence number of photons. This will increase the photoelectric current.

where, wavelength of incident radiation is $\lambda = 3300 \text{ \AA}$

So, energy of incident radiation is

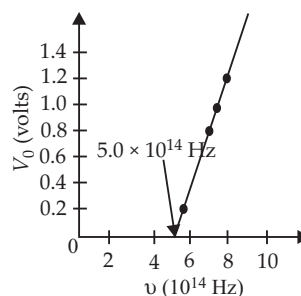
$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{3300 \times 10^{-10} \times 1.6 \times 10^{-19}} = 3.75 \text{ eV}$$

Now, work function of Mo : 4.17 eV, Ni : 5.15 eV is more than energy of incident photon, hence these two metals will not give photoelectric emission.

16. In order to calculate Planck's constant 'h' we need slope of the graph between cut off voltage and frequency.

So, let us first calculate the frequency ($\nu = c/\lambda$) in each case and the table shows corresponding stopping potential.

λ	ν	V_0
3650 \AA	$8.2 \times 10^{14} \text{ Hz}$	1.28 V
4047 \AA	$7.4 \times 10^{14} \text{ Hz}$	0.95 V
4358 \AA	$6.9 \times 10^{14} \text{ Hz}$	0.74 V
5461 \AA	$5.49 \times 10^{14} \text{ Hz}$	0.16 V
6907 \AA	$4.3 \times 10^{14} \text{ Hz}$	0.0 V



V_0 versus plot shows that the first four points lie nearly on a straight line which intercepts the x-axis at threshold frequency, $\nu_0 = 5.0 \times 10^{14} \text{ Hz}$. The fifth point $\nu (= 4.3 \times 10^{14} \text{ Hz})$ corresponds to $\nu < \nu_0$, so there is no photoelectric emission and no stopping voltage is required to stop the current.

Slope of, V_0 versus ν graph is

$$\begin{aligned} \tan \theta &= \frac{\Delta V_0}{\Delta \nu} = \frac{(1.28 - 0) \text{ V}}{(8.2 - 5.0) \times 10^{14} \text{ s}^{-1}} = \frac{h}{e} \\ &= 4.0 \times 10^{-15} \text{ Vs} = \frac{h}{e} \end{aligned}$$

Planck's constant,

$$\begin{aligned} h &= e \times 4.0 \times 10^{-15} \text{ Js} = 1.6 \times 10^{-19} \times 4.0 \times 10^{-15} \text{ Js} \\ &= 6.4 \times 10^{-34} \text{ J s.} \end{aligned}$$

(b) Threshold frequency, $\nu_0 = 5.0 \times 10^{14} \text{ Hz}$

\therefore Work function,

$$\begin{aligned} W_0 &= h\nu_0 = 6.4 \times 10^{-34} \times 5.0 \times 10^{14} \text{ J} \\ &= \frac{6.4 \times 5.0 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} = 2.00 \text{ eV.} \end{aligned}$$

Topic 2

1. An electron which is accelerated through a potential difference of 56 V will have kinetic energy $K = 56 \text{ eV}$

(a) Momentum associated with accelerated electron

$$\begin{aligned} p &= \sqrt{2Km} = \sqrt{2 \times 56 \times 1.6 \times 10^{-19} \times 9.1 \times 10^{-31}} \\ &= 4.04 \times 10^{-24} \text{ kg m s}^{-1} \end{aligned}$$

(b) Wavelength of electron accelerated

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{4.04 \times 10^{-24}} = 0.164 \text{ nm}$$

2. Kinetic energy of electron

$$= 120 \times 1.6 \times 10^{-19} \text{ J} = 1.92 \times 10^{-17} \text{ J}$$

(a) Momentum of electron,

$$\begin{aligned} p &= \sqrt{2Km} = \sqrt{2 \times 1.92 \times 10^{-17} \times 9.1 \times 10^{-31}} \\ p &= 5.91 \times 10^{-24} \text{ kg m s}^{-1} \end{aligned}$$

(b) Speed of electron

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 1.92 \times 10^{-17}}{9.1 \times 10^{-31}}} = 6.5 \times 10^6 \text{ m s}^{-1}$$

- (c) de-Broglie wavelength associated with electron

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{5.91 \times 10^{-24}} = 1.122 \text{ Å}.$$

3. (a) Kinetic energy required by electron to have de-Broglie wavelength of 589 nm

$$K = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (589 \times 10^{-9})^2}$$

$$K = 6.95 \times 10^{-25} \text{ J} = 4.34 \text{ µeV}$$

- (b) Kinetic energy of neutron to have de-Broglie wavelength of 589 nm

$$K = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (589 \times 10^{-9})^2}$$

$$K = 3.78 \times 10^{-28} \text{ J} = 2.36 \text{ neV}$$

4. de Broglie wavelength $\lambda = \frac{h}{mv}$

- (a) Wavelength associated with bullet

$$\lambda_{\text{bullet}} = \frac{6.63 \times 10^{-34}}{0.04 \times 10^3} = 1.7 \times 10^{-35} \text{ m}$$

- (b) Wavelength associated with ball

$$\lambda_{\text{ball}} = \frac{6.63 \times 10^{-34}}{0.06 \times 1} = 1.1 \times 10^{-32} \text{ m}$$

- (c) Wavelength associated with dust particle

$$\lambda_{\text{particle}} = \frac{6.63 \times 10^{-34}}{10^{-9} \times 2.2} = 3 \times 10^{-25} \text{ m}$$

5. (a) de Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mK}}$

∴ Kinetic energy (K) of neutron

$$K = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.677 \times 10^{-27} \times (1.40 \times 10^{-10})^2}$$

$$= 6.686 \times 10^{-21} \text{ J}$$

$$= \frac{6.686 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV} = 4.178 \times 10^{-2} \text{ eV}.$$

- (b) $K = \frac{3}{2}kT$, where k = Boltzmann constant

$$\therefore \lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.677 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} \text{ m}$$

$$= \frac{6.63 \times 10^{-10}}{\sqrt{20.8}} = \frac{6.63 \times 10^{-10}}{4.56} \text{ m}$$

$$= 1.45 \times 10^{-10} \text{ m} = 0.145 \text{ nm}.$$

6. For a photon, de Broglie wavelength,

$$\lambda = \frac{h}{p}.$$

For an electromagnetic radiation of frequency ν and wavelength λ' ($= c/\nu$),

Momentum,

$$p = \frac{E}{c} = \frac{h\nu}{c} \quad \text{or} \quad p = \frac{h}{c} \cdot \frac{c}{\lambda'} = \frac{h}{\lambda'}$$

Then, $\lambda' \cdot \frac{h}{c} = \frac{h}{p} = \lambda$

Thus the wavelength of the electromagnetic radiation is the same as the de Broglie wavelength of the photon.

7. Let us first calculate mass of each N_2 molecule.

$$m = 2 \times 14.0076 \times 1.66 \times 10^{-27} \text{ kg} = 46.5 \times 10^{-27} \text{ kg}$$

$$T = 300 \text{ K}$$

Average kinetic energy of N_2 molecules at temperature T

$$\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT \quad \text{or} \quad v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

$$\therefore \lambda = \frac{h}{mv_{\text{rms}}} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 46.5 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} \text{ m}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{577.53 \times 10^{-24}}} = \frac{6.63 \times 10^{-10}}{24.03} \text{ m}$$

$$= 0.0276 \times 10^{-9} \text{ m} \approx 0.028 \text{ nm}.$$

8. Order of interatomic spacing is 1 Å in the crystal lattice. So, for diffraction to take place the wavelength should be of the same order.

For X-ray photon (energy for wavelength of 1 Å)

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-10} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= 12.4 \times 10^3 \text{ eV} = 12.4 \text{ keV}$$

For electron (energy to provide wavelength of 1 Å)

$$\text{momentum required, } p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-10}}$$

$$= 6.63 \times 10^{-24} \text{ kg m s}^{-1}$$

$$\text{Kinetic energy, } K = \frac{p^2}{2m} = \frac{(6.63 \times 10^{-24})^2}{2 \times 9.11 \times 10^{-31}} \text{ J}$$

$$K = \frac{(6.63 \times 10^{-24})^2}{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19}} \text{ eV} = 150.6 \text{ eV}$$

Thus in order to produce same wavelength X-ray photon should have higher energy than electron.

9. Let us first find mass ' m ' of each helium atom.

$$m = \frac{\text{Atomic weight of Helium}}{\text{Avogadro's number}}$$

$$m = \frac{4}{6 \times 10^{23}} \text{ g} = \frac{2}{3} \times 10^{-23} \text{ g} = \frac{2}{3} \times 10^{-26} \text{ kg}$$

$$\text{Absolute temperature, } T = 273 + 27 = 300 \text{ K}$$

Average K.E. of a He atom at absolute temperature T

$$K = \frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$m^2v^2 = p^2 = 3mkT$$

$$\text{Momentum, } p = \sqrt{3mkT}$$

Wavelength of the wave associated with He atom at room temperature

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}} = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times \frac{2}{3} \times 10^{-26} \times 1.38 \times 10^{-23} \times 300}} \\ = 0.73 \times 10^{-10} \text{ m} \quad \dots(i)$$

Now let us find mean separation between He atoms.

$$\text{Mean separation, } r = \left[\frac{\text{Molar volume}}{\text{Avogadro's number}} \right]^{1/3}$$

Here for 1 mole, $PV = RT$

$$PV = NkT \text{ or } \frac{V}{N} = \frac{kT}{P}$$

$$\text{So, mean separation } r = \left[\frac{kT}{P} \right]^{1/3}$$

Here, k = Boltzmann's constant = $1.38 \times 10^{-23} \text{ J K}^{-1}$

T = Absolute temperature = 300 K

P = Atmospheric pressure = $1.01 \times 10^5 \text{ Pa}$

$$r = \left[\frac{1.38 \times 10^{-23} \times 300}{1.01 \times 10^5} \right]^{1/3} \text{ m} = 3.4 \times 10^{-9} \text{ m} \quad \dots(ii)$$

Comparing the wavelength ' λ ' with mean separation ' r ', it can be observed that separation is larger than wavelength, i.e. ($r \gg \lambda$)

10. K.E. of an electron, accelerated by voltage of 50 kV.

$$K = 50 \text{ keV} = 1.6 \times 10^{-19} \times 5 \times 10^4 \text{ J} = 8 \times 10^{-15} \text{ J}$$

\therefore de Broglie wavelength associated with electron

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 8 \times 10^{-15}}}$$

$$\lambda = \frac{6.63 \times 10^{-11}}{12.07} \text{ m} = 5.5 \times 10^{-12} \text{ m}$$

Wavelength of yellow light, $\lambda_y = 5.9 \times 10^{-7} \text{ m}$.

Resolving power of a microscope $\propto \frac{1}{\lambda}$

$$\therefore \frac{\text{Resolving power of electron microscope}}{\text{Resolving power of optical microscope}} = \frac{\lambda_y}{\lambda} \\ = \frac{5.9 \times 10^{-7}}{5.5 \times 10^{-12}} \approx 10^5$$

Thus, the resolving power of an electron microscope is about 10^5 times greater than that of an optical microscope.

11. (a) Let us first calculate the wavelength of matter wave associated with neutron of kinetic energy 150 eV.

$$K = \frac{p^2}{2m}$$

So, momentum, $p = \sqrt{2mK}$

$$\text{Wavelength, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 150 \times 1.6 \times 10^{-19}}} \text{ m} \\ = 2.33 \times 10^{-12} \text{ m}$$

As the interatomic spacing ($1 \text{ \AA} = 10^{-10} \text{ m}$) is about hundred times greater than this wavelength, so a neutron beam of 150 eV energy is not suitable for diffraction experiments.

(b) Average kinetic energy of a neutron at absolute temperature T is

$$\frac{1}{2}mv^2 = \frac{3}{2}kT \text{ or } \frac{p^2}{2m} = \frac{3}{2}kT$$

or $p = \sqrt{3mkT}$

$$\therefore \text{ de Broglie wavelength, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}$$

Given $m_n = 1.675 \times 10^{-27} \text{ kg}$, $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$

$T = 27 + 273 = 300 \text{ K}$, $h = 6.63 \times 10^{-34} \text{ J s}$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.675 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} \\ = \frac{6.63 \times 10^{-10}}{4.56} \text{ m} = 1.45 \times 10^{-10} \text{ m} = 1.45 \text{ \AA}$$

As this wavelength is comparable to interatomic spacing ($\approx 1 \text{ \AA}$) in a crystal, so thermal neutrons can be used for diffraction experiments. So high energy neutron beam should be first thermalised before using it for diffraction.

12. Considering free electrons as gas. Kinetic energy at temperature T

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$m^2v^2 = 3mkT$$

Wavelength associated with moving electron

$$\lambda = \frac{h}{\sqrt{3mkT}} \\ = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}} \text{ m} \\ = 6.2 \times 10^{-9} \text{ m}$$

Given that mean separation between two electrons is about $2 \times 10^{-10} \text{ m}$.

$$\therefore \frac{\lambda}{r} = \frac{6.2 \times 10^{-9}}{2 \times 10^{-10}} = 31$$

So, de-Broglie wavelength is much greater than the electron separation.

