Atoms

EXAM DRILL

ANSWERS

1. **(c) :** $E_n = \frac{-13.6}{n^2}$eV, $\Delta E = E_\infty - E_2 = 0 + \frac{13.6}{2^2} = 3.4$ eV

2. **(d) :** Angular momentum, $L = mvr = \frac{nh}{2\pi}$

3. **(c) :** $r_n = r_0 n^2 \quad \therefore \quad r_3 = 9r_0$

4. **(b) :** $\frac{e^2}{2r}$. Since K.E. $= \frac{e^2}{2r}$

5. **(c) :** In Bohr's atomic model, $r_n \propto n^2$
Hence, $R_1 : R_2 : R_3 = 1 : 4 : 9$

6. **(c) :** For first Balmer line $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$

7. **(d) :** $\Delta E = E_4 - E_3 = \frac{-13.6}{4^2} - \left(\frac{-13.6}{3^2}\right) = 0.66$ eV

8. **(a) :** As $E_n = \frac{-13.6}{n^2}$, energy difference decreases between two consecutive energy states.

9. **(c) :** In transition from $n_1 = 3$ and $n_2 = 4, 5, 6, ...$. Infrared radiation of Paschen series is emitted.

10. **(c)**

11. **(c) :** In a metal every atom has certain definite energy level. In the normal state, the electron in the atom stays in lowest energy level. When the atom gets appropriate energy from outside, then this electron rises to some higher energy level *i.e.* atom is excited. Within nearly 10^{-8} sec, the electron leaves the higher energy level. Now, it can return either directly to the lowest energy level (or the ground state) or come to the ground state after passing through other lower energy levels. This gives rise to different energy levels in the metal. Since, all the free electrons lie in same energy level hence, in a metal all free electrons have same energy. But reason is false as all the electrons in a metal obey Pauli's exclusion principle.

12. **(d) :** The maximum number of photons emitted = 6 corresponding to the transitions $4 \to 3$; $3 \to 2$, $2 \to 1$, $4 \to 2$, $4 \to 1$ and $3 \to 1$.

13. **(i) (b)** **(ii) (b)** **(iii) (d)**

14. The first spectral series was discovered by Balmer.

15. Impact parameter is perpendicular distance of velocity vector of the α-particle from the centre of the nucleus.

16. As the energy of the electron is –3.4 eV in first excited state and magnitude is less for higher excited state.

17. As energy $E \propto Z^2$
For hydrogen atom $Z = 1$, For helium atom $Z = 2$
$E_{He} = 4E_n$

18. Since spectral line of wavelength 4860 Å lies in the visible region of the spectrum which is Balmer series of the spectrum.

OR

The angular momentum of the electron can only be an integral multiple to $\frac{h}{2\pi}$.

19. For longest wavelength in Balmer series, $n_1 = 2$ and $n_2 = 3$.

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right], R = 1.1 \times 10^7 \text{ m}^{-1}$$

20. In Bohr's formula,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}(e)(-e)$$

Force $\propto (-e)(e) = -e^2$

If charge on proton $\left(+\frac{4}{3}e\right)$ and charge on electron $\left(-\frac{3}{4}e\right)$ their product $\left(\frac{4}{3}e\right)\left(-\frac{3}{4}e\right) = -e^2$, remains the same.

21. As $E_n = \frac{Z^2}{n^2}E_0$

For *H*-atom : $Z = 1$ and $n = 2$,

$$E_2 = \frac{E_0}{4} = E$$

For He atom : $Z = 2$ and $n = 3$

$$\therefore \quad E_3 = \frac{4}{9}E_0. \text{ Hence } E_3 = \frac{16}{9}E$$

22. $E_{CA} = E_{CB} + E_{BA}$

$$\frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} \quad \text{or} \quad \lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$$

23. $-13.6 - (-10.2) = -3.4$ eV

$$\frac{-13.6}{n^2} = -3.4 \quad \text{or} \quad n^2 = \frac{13.6}{3.4} = 4$$

Increase in angular momentum $= \frac{2h}{2\pi} - \frac{h}{2\pi} = \frac{h}{2\pi}$

$$= \frac{6.625 \times 10^{-34}}{2 \times 3.14} \text{ Js} = 1.05 \times 10^{-34} \text{ Js}$$

OR

$$\frac{1}{\lambda} \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right), \quad \frac{\lambda_{min}}{\lambda_{max}} = \frac{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{\left(\frac{1}{2^2} - \frac{1}{\infty}\right)} = \frac{5}{9}$$

24. Rydberg's constant determines the frequencies. We have $R \propto m$. So, modified R for positronium atom is half of H atom. Hence, frequencies are reduced to half $x = \frac{1}{2}$.

25. $f = cZ^2R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\Rightarrow \quad 2.7 \times 10^{15} = cZ^2R\left[\frac{1}{1^2} - \frac{1}{2^2}\right], \quad f' = cZ^2R\left[\frac{1}{1^2} - \frac{1}{3^3}\right]$

Divide and solve to get : $f = 3.2 \times 10^{15}$ Hz

26. From n^{th} state, the atom may go to $(n-1)^{th}$ state, ..., second state or first state. So, there are $(n-1)$ possible transitions starting from the n^{th} state. The atoms reaching $(n-1)^{th}$ state may make $(n-2)$ different transitions and so on. In general, the total number of possible transition is

$(n-1) + (n-2) + (n-3) + \ldots\ldots + 2 + 1 = \frac{n(n-1)}{2}$

27. For Lyman series $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$; $n = 2, 3, 4.$

Here $R = 1.097 \times 10^7$ m

Substituting these values and arranging above equation

$$\lambda = \frac{913.4\, n^2}{n^2 - 1} \times 10^7 \text{ or } \lambda = \frac{913.4\, n^2}{n^2 - 1} \text{ Å}$$

Now, substituting values for $n = 2, 3, 4, 5$, we get the wavelengths of the first four spectral lines in the Lyman series of the hydrogen spectrum.

We get $\lambda_{21} = 1218$ Å, $\lambda_{31} = 1028$ Å, $\lambda_{41} = 974.3$ Å, $\lambda_{51} = 951.4$ Å

28. $E_{n^{th}} = \frac{-13.6}{n^2}$ eV; $n = 1, 2, 3$

$$E_1 = \frac{-13.6}{1^2} = -13.6 \text{ eV}$$

$$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV}, \; E_2 - E_1 = 10.2 \text{ eV}$$

$$E_3 = \frac{-13.6}{3^2} = -1.51 \text{ eV}, \; E_3 - E_2 = 1.89 \text{ eV}$$

$$E_4 = \frac{-13.6}{4^2} = -0.85 \text{ eV}, \; E_4 - E_3 = 0.66 \text{ eV}$$

$$E_5 = \frac{-13.6}{5^2} = -0.54 \text{ eV}, \; E_5 - E_4 = 0.31 \text{ eV}$$

(i) Clearly an electron in the hydrogen atom can not have an energy of – 6.8 eV.

(ii) We can also clearly observed that spacing between the lines (consecutive energy levels) within the given set of the observed hydrogen atom spectrum decreases as n increases.

OR

In Balmer series, H_γ line corresponds to transition from state $n_i = 5$ to state $n_f = 2$.

Energy required, $E = E_5 - E_1$

$$= \left(\frac{-13.6}{5^2}\right) - \left(\frac{-13.6}{1^2}\right) = 13.06 \text{ eV.}$$

If angular momentum of system is conserved,

Change in angular momentum of electron = Change in angular momentum of photon

$$= 5\left(\frac{h}{2\pi}\right) - 2\left(\frac{h}{2\pi}\right) = \frac{3h}{2\pi}, = \frac{3 \times 6.6 \times 10^{-34}}{2 \times 3.14} = 3.17 \times 10^{-34} \text{ Js}$$

29. $Z = 80$, K.E. = 8 MeV

At closest approach, P.E. = K.E.

$$\frac{kZe^2}{d} = 8 \times 1.6 \times 10^{-13} \text{ or } d = 128.8 \text{ fm}$$

We can see that d is inversely proportional to the K.E., so if K.E. of α-particle is doubled, distance of closest approach is halved.

30. Paschen series of hydrogen spectrum is given

$$\bar{\upsilon} = 1.1 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{n^2}\right) \text{ m}^{-1}$$

where $n = 4, 5, 6$

$$\frac{1}{\lambda} = 1.1 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{n^2}\right) \text{ m}^{-1}$$

For shortest wavelength, $n = \infty$

$$\frac{1}{\lambda} = 1.1 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{\infty}\right), \qquad \lambda = 8199 \text{ Å}$$

The series lies in infrared region of the EM spectrum.

31. (a) The longest wavelength corresponds to transition from $n_f = 5$ to $n_i = 4$: the smallest energy change.

So, $\frac{1}{\lambda_{max}} = (1.097 \times 10^7)\left(\frac{1}{4^2} - \frac{1}{5^2}\right) = 2.468 \times 10^5 \text{ m}^{-1}$

$$\lambda_{max} = \frac{1}{2.468 \times 10^5} = 4051 \text{ nm}$$

(b) The wavelength for transition $n_f = 6$ to $n_i = 4$ is

$$\frac{1}{\lambda} = (1.0947 \times 10^{-7})\left(\frac{1}{4^2} - \frac{1}{6^2}\right) = 3.801 \times 10^5$$

$$\Rightarrow \lambda = \frac{1}{3.801 \times 10^5} = 2630 \text{ nm}$$

32. Energy of an electron in n^{th} permitted energy level is given by

$$E_{n^{th}} = \frac{-13.6}{n^2} \text{ eV.}$$

(i) Energy of photon produced due to transition of an electron from second permitted energy level to the first level.

$E = E_2 - E_1$

$$E = \frac{-13.6}{(2)^2} - \frac{(-13.6)}{1^2} = 10.2 \text{ eV}$$

(ii) Energy of photon produced due to transition of an electron from highest permitted energy level to the second permitted energy level.

$E' = E_\infty - E_2$

$E' = \frac{-13.6}{(\infty)^2} - \left(\frac{-13.6}{2^2}\right) = 3.4$ eV . Ratio of $\frac{E}{E'}$ is $\frac{10.2}{3.4} = 3$

33. The energy of electron in Hydrogen like atoms in n^{th} orbit is

$$E_n = \frac{Z^2 Rhc}{n^2}$$

We have $Rhc = 1$ Rydberg.

The ionization energy $E_\infty - E_1 = Z^2 Rhc = 4$ Rydberg

$\therefore \quad Z^2 = \frac{4\,\text{Rydberg}}{Rhc} = \frac{4}{1} = 4$, $\qquad \therefore \; Z = 2$

(a) The energy required to excite the electron from $n = 1$ to $n = 2$ is given by

$$E_2 - E_1 = \frac{-Z^2 Rhc}{2^2} - \left(\frac{-Z^2 Rhc}{1^2}\right) = Z^2 Rhc\left(1 - \frac{1}{4}\right)$$

$$= \frac{3Z^2}{4} Rhc = \frac{3}{4} \times 4 \text{ Rydberg} = 3 \text{ Rydberg}$$

If λ is the wavelength of radiation emitted, then

$$\frac{hc}{\lambda} = 3 \text{ Rydberg, } i.e., \lambda = \frac{hc}{3\,\text{Rydberg}}$$

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{3 \times 2.2 \times 10^{-18}} = 301.4 \times 10^{-10} \text{ m} = 301.4 \text{ Å}$$

(b) Radius of first Bohr orbit, $r_1 = \frac{(\varepsilon_0 h^2 / \pi m e^2)}{Z}$

$$= \frac{\text{Radius of first Bohr orbit of hydrogen}}{Z} = \frac{5 \times 10^{-11}}{2}$$

$= 2.5 \times 10^{-11}$ m

OR

It is given that the energy of the electron used to bombard gaseous hydrogen at room temperatures is 12.5 eV. Also, the energy of the gaseous hydrogen in its ground state at room temperature is –13.6 eV. When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes –13.6 + 12.5 eV *i.e.*, –1.1 eV

Energy is related to orbit level (n) is given as

$$E_{n^{th}} = \frac{-13.6}{n^2} \text{ eV}$$

For $n = 3$, $E_3 = \frac{-13.6}{3^2} = -1.5$ eV

For $n = 4$, $E_4 = \frac{-13.6}{4^2} = -0.85$ eV

Since –1.1 eV lies between E_3 and E_4, so it is clear it can go upto level 3 when 12.5 eV energy is supplied. Now from $n = 3$, Hydrogen can go from

(a) $n = 3$ to $n = 1$

(b) $n = 3$ to $n = 2$, it will be Balmer series

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow R = 1.7 \times 10^7$$

Hence wavelength will be given as 656.33 nm

For $n = 3$ to $n = 1$, it will be Lyman series

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) \Rightarrow R = 1.7 \times 10^7$$

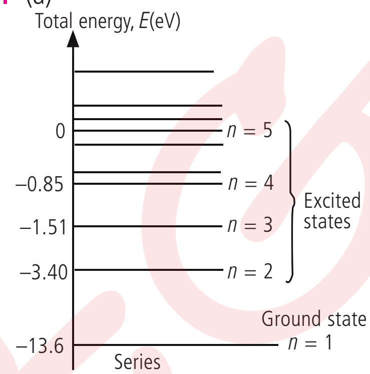
Hence wavelength will be given as 102.55 nm

For $n = 2$ to $n = 1$, it will be Lyman series

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \Rightarrow R = 1.7 \times 10^7$$

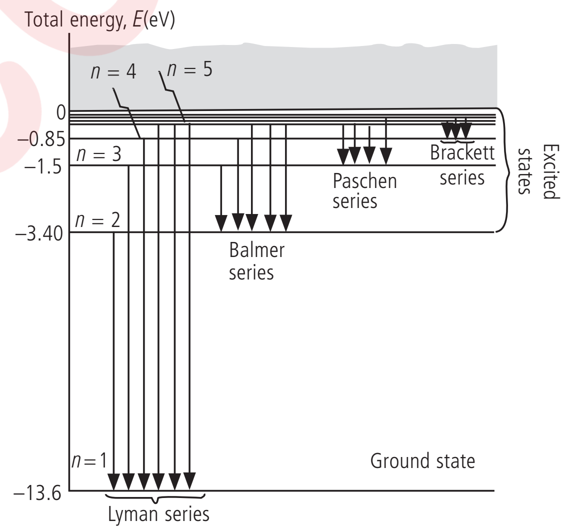
Hence, it will emit radiation of wavelength 102.55 nm and 121.5 nm in Lyman series and 656.33 nm in Balmer series.

34. (a)



(b) Lyman series, Balmer series, Paschen series, Brackett series, Pfund series.

(c)



OR

(a) In Rutherford atom model the electron is continuously moving around the nucleus in circular orbits. So the electron which is oscillating should radiate energy. Hence the radius of the circular path should be decreased and finally falls into the nucleus, *i.e.*, the atom is unstable. This problem was attempted to be solved by Bohr.

(b) (i) An electron in an atom could revolve in certain stable orbits without the emission of radiant energy. Each stable state has definite total energy and are called stationary state of atom.

(ii) The electron revolve around the nucleus only in those orbits for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$, where h is Planck's constant.

$\lambda = \frac{nh}{2\pi}$

(iii) An electron might take r transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to a energy difference between the initial and final states.

(c) $h\upsilon = E_i - E_f, r_0 = 5.3 \times 10^{-11}$ m

$r_n = r_{0n}^2$

$r_2 = r_0 \times 2^2, r^3 = r_0 \times 3^2$

$r_2 = 5.3 \times 10^{-11} \times 2^2 = 2.12 \times 10^{-10}$ m

$r_3 = 5.3 \times 10^{-11} \times 3^2 = 4.77 \times 10^{-10}$ m

35. Let μ_H and μ_D are the reduced masses of electron for hydrogen and deuterium respectively.

We know that $\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$

As n_i and n_f are fixed for by mass series for hydrogen and deuterium.

$$\lambda \propto \frac{1}{R} \quad \text{or} \quad \frac{\lambda_D}{\lambda_H} = \frac{R_H}{R_D} \qquad \ldots\text{(i)}$$

$$R_H = \frac{m_e e^4}{8\,\varepsilon_0 ch^3} = \frac{\mu_H e^4}{8\varepsilon_0 ch^3}, R_D = \frac{m_e e^4}{8\,\varepsilon_0\, ch^3} = \frac{\mu_D\, e^4}{8\,\varepsilon_0 ch^3}$$

$$\therefore \quad \frac{R_H}{R_D} = \frac{\mu_H}{\mu_D} \qquad \ldots\text{(ii)}$$

From equation (i) and (ii)

$$\frac{\lambda_D}{\lambda_H} = \frac{\mu_H}{\mu_D} \qquad \ldots\text{(iii)}$$

Reduced mass of hydrogen, $\mu_H = \frac{m_e}{1 + m_e / M}$; $m_e\left(1 - \frac{m_e}{M}\right)$

Reduced mass of deuterium, $\mu_D = \frac{2M \cdot m_e}{2M\left(1 + \frac{m_e}{2M}\right)} \simeq m_e\left(1 - \frac{m_e}{2M}\right)$

where M is mass of proton

$$\frac{\mu_H}{\mu_D} = \frac{m_e\left(1 - \frac{m_e}{M}\right)}{m_e\left(1 - \frac{m_e}{2M}\right)} = \left(1 - \frac{m_e}{M}\right)\left(1 - \frac{m_e}{2M}\right)^{-1}$$

$$= \left(1 - \frac{m_e}{M}\right)\left(1 + \frac{m_e}{2M}\right), \frac{\mu_H}{\mu_D} = \left(1 - \frac{m_e}{2M}\right)$$

$$\frac{\mu_H}{\mu_D} = \left(1 - \frac{1}{2 \times 1840}\right) = 0.99973 \qquad \ldots\text{(iv)}$$

$(\because \quad M = 1840\, m_e)$

From (iii) and (iv)

$\frac{\lambda_D}{\lambda_H} = 0.99973, \quad \lambda_D = 0.99973\, \lambda_H.$

Percentage difference in wavelength of lyman series in ^{1}H (hydrogen) and ^{2}H (deuterium) $= \left(\frac{\lambda_H - \lambda_D}{\lambda_H}\right) \times 100$

$= (1 - 0.99973) \times 100 = 0.027\% = 2.7 \times 10^{-2}\,\%$.

OR

In a H-atom in ground state, electron revolves round the point-size proton in a circular orbit of radius r_B (Bohr's radius).

As $mvr_B = \hbar$ and $\frac{mv^2}{r_B} = \frac{-1 \times e \times e}{4\pi\,\varepsilon_0 r_B^2}, \frac{m}{r_B}\left(\frac{-\hbar^2}{m^2 r_B^2}\right) = \frac{e^2}{4\pi\varepsilon_0 r_B^2},$

$r_B = \frac{4\pi\varepsilon_0}{e^2}\frac{\hbar^2}{m} = 0.53$ Å

K.E. $= \frac{1}{2}mv^2 = \left(\frac{m}{2}\right)\left(\frac{\hbar}{mr_B}\right)^2 = \frac{\hbar^2}{2mr_B^2} = 13.6$ eV

PE of the electron and proton,

$U = \frac{1}{4\pi\varepsilon_0}\frac{e(-e)}{r_B} = -\frac{e^2}{4\pi\varepsilon_0 r_B} = -27.2$ eV

Total energy of the electron, *i.e.*,

$E = K + U = +13.6$ eV $- 27.2$ eV $= -13.6$ eV

(i) When, $R = 0.1$ Å : $R < r_B$ (as $r_B = 0.51$ Å) and the ground state energy is the same as obtained earlier for point-size proton, *i.e.*, -13.6 eV

(ii) When $R = 10$ Å : $R >> r_B$, the electron moves inside the proton (assumed to be a sphere of radius R) with new Bohr's radius r'_B.

Clearly, $r'_B = \frac{4\pi\,\epsilon_0\,\hbar^2}{m(e)(e')}$

[replacing e^2 by $(e)(e')$, where e' is the charge on the sphere of radius r'_B]

Since $e' = \left[\frac{e}{(4\pi/3)R^3}\right]\left[\left(\frac{4\pi}{3}\right)r_B'^3\right] = \frac{er_B'^3}{R^3},$

$r'_B = \frac{4\pi\varepsilon_0\hbar^2}{me(er_B'^3 / R^3)} = \left(\frac{4\pi\varepsilon_0\hbar^2}{me^2}\right)\left(\frac{R^3}{r_B'^3}\right)$

or $\quad r_B'^4 = \left(\frac{4\pi\varepsilon_0\hbar^2}{me^2}\right)R^3 = (0.51\,\text{Å})(10\,\text{Å})^3 = (510\,\text{Å}^4)$

$\therefore \quad r'_B = 4.8$ Å, which is less than R (=10 Å)

KE of the electron,

$K' = \frac{1}{2}mv'^2 = \left(\frac{m}{2}\right)\left(\frac{\hbar^2}{m^2 r_B'^2}\right) = \frac{\hbar^2}{2mr_B'^2}$

$= \left(\frac{\hbar^2}{2mr_B^2}\right)\left(\frac{r_B}{r'_B}\right)^2 = (13.6\text{ eV})\left(\frac{0.51\,\text{Å}}{4.8\,\text{Å}}\right)^2 = 0.16$ eV

Potential at a point inside the charged proton, *i.e.*,

$V = \frac{k_e e}{R}\left(3 - \frac{r_B'^2}{R^2}\right) = k_e e\left(\frac{3R^2 - r_B'^2}{R^3}\right)$

Potential energy of electron and proton, $U = -eV$

$= -e(k_e e)\left[\frac{3R^2 - r_B'^2}{R^3}\right], \quad = -\left(\frac{e^2}{4\pi\varepsilon_0\, r_B}\right)\left[\frac{r_B(3R^2 - r_B'^2)}{R^3}\right]$

$= -(27.2\text{ eV})\left[\frac{(0.51\,\text{Å})(300\,\text{Å} - 23.03\,\text{Å})}{(1000\text{Å})}\right]$

$= -3.83$ eV

Total energy of the electron,

$E = K + U = 0.16$ eV $- 3.83$ eV $= -3.67$ eV