CHAPTER 12

Atoms

NCERT FOCUS **ANSWERS**

Topic 1

1. (a) not different from
(b) Thomson's model, Rutherford's model
(c) Rutherford's model
(d) Thomson's model, Rutherford's model
(e) both the models

2. (a) Nearly the same. This is because we are considering the average angle of deflection.
(b) Much less, because there is no such massive core (nucleus) in Thomson's model as in Rutherford's model.
(c) This suggests that scattering is mainly due to a single collision, because the chance of a single collision increases linearly with the number of the target atoms, and hence linearly with the thickness of the foil.
(d) In Thomson model, positive charge is distributed uniformly in the atom. So single collision causes very little deflection. The observed average scattering angle can be explained only by considering multiple scattering. Hence, it is wrong to ignore multiple scattering in Thomson's model.

3. The nucleus of a hydrogen atom is a proton. The mass of a proton is 1.67×10^{-27} kg, whereas the mass of an incident α-particle is 6.64×10^{-27} kg. Because the incident α-particles are more massive than the target nuclei (protons), the α-particle won't bounce back even in a head on collision. It is similar to a football colliding with a tennis ball at rest. Thus, there would be no appreciable scattering.

Topic 2

1. For shortest wavelength of Paschen series, $n_1 = 3$, $n_2 = \infty$

$$\therefore \quad \frac{1}{\lambda_s} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right] = \frac{R}{9}$$

$$\text{or} \quad \lambda_s = \frac{9}{R} = \frac{9}{1.097 \times 10^7} = 8.2041 \times 10^{-7} \text{ m} = 8204.1 \text{ Å}.$$

2. Here $E = 2.3$ eV $= 2.3 \times 1.6 \times 10^{-19}$ J

As $E = h\upsilon$

$\therefore$ Frequency,

$$\upsilon = \frac{E}{h} = \frac{2.3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 5.6 \times 10^{14} \text{ Hz.}$$

3. Total energy, $E = -13.6$ eV

K.E. $= -E = -(-13.6) = 13.6$ eV

P.E. $= -2$K.E. $= -2 \times 13.6 = -27.2$ eV.

4. Energy of an electron in the n^{th} orbit of H-atom,

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

Energy in the ground ($n = 1$) level,

$$E_1 = -\frac{13.6}{1^2} = -13.6 \text{ eV}$$

Energy in the fourth ($n = 4$) level,

$$E_4 = -\frac{13.6}{4^2} = -0.85 \text{ eV}$$

Energy radiated during emission

$\Delta E = E_4 - E_1 = -0.85 - (-13.6) = 12.75$ eV

As $h\upsilon = \Delta E$

$$\therefore \text{ Frequency, } \upsilon = \frac{\Delta E}{h} = \frac{12.75 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$= 3.078 \times 10^{15}$ Hz

$$\text{Wavelength, } \lambda = \frac{c}{\upsilon} = \frac{3 \times 10^8}{3.078 \times 10^{15}}$$

$= 0.9744 \times 10^{-7}$ m $= 974.4$ Å

5. (a) Speed of the electron in Bohr's n^{th} orbit is

$$v = \frac{e^2}{2nh\varepsilon_0} = \alpha\frac{c}{n}$$

Speed of the electron in Bohr's first ($n = 1$) orbit is

$$v_1 = \frac{1}{137} \times \frac{3 \times 10^8}{1} = 2.186 \times 10^6 \text{ m s}^{-1}$$

$= 2.186 \times 10^6$ m s^{-1}

$$v_2 = \frac{v_1}{2} = 1.093 \times 10^6 \text{ m s}^{-1},$$

$$v_3 = \frac{v_1}{3} = 0.729 \times 10^6 \text{ m s}^{-1}.$$

(b) Orbital period of electron in Bohr's first orbit is

$$T_1 = \frac{2\pi r_1}{v_1} = \frac{2 \times 3.14 \times 0.53 \times 10^{-10}}{2.186 \times 10^6} \text{ s}$$

$= 1.52 \times 10^{-16}$ s

As $T_n = n^3 T_1$

$\therefore \quad T_2 = (2)^3 \times 1.52 \times 10^{-16} = 12.16 \times 10^{-16}$
$= 1.22 \times 10^{-15}$ s

$T_3 = (3)^3 \times 1.52 \times 10^{-16} = 41.04 \times 10^{-16}$
$= 4.10 \times 10^{-15}$ s.

6. Radius of innermost electron

$$r = \frac{n^2h^2\varepsilon_0}{\pi me^2}$$

For $n = 1$, $r_1 = \frac{h^2\varepsilon_0}{\pi me^2} = 5.3\times10^{-11}$ m

For $n = 2$, $r_2 = (2)^2 r_1 = 2.12 \times 10^{-10}$ m

For $n = 3$, $r_3 = (3)^2 r_1 = 4.77 \times 10^{-10}$ m.

7. In ground state, energy of gaseous hydrogen at room temperature = – 13.6 eV. When it is bombarded with 12.5 eV electron beam, the energy becomes – 13.6 + 12.5 = – 1.1 eV. The electron would jump from n = 1 to n = 3, where $E_3 = -\frac{13.6}{3^2} = -1.5\,\text{eV}$. On de-excitation the electron may jump from n = 3 to n = 2 giving rise to Balmer series. It may also jump from n = 3 to n = 1, giving rise to Lyman series.

8. According to Bohr's quantization condition of angular momentum,

Angular momentum of the earth around the sun,

$$mvr = \frac{nh}{2\pi}$$

$$\therefore \quad n = \frac{2\pi mvr}{h}$$

$$= \frac{2\times3.14\times6.0\times10^{24}\times1.5\times10^{11}\times3\times10^4}{6.6\times10^{-34}}$$

$= 2.57 \times 10^{74}$

9. The radius of the first orbit in Bohr's model is calculated by considering the electrostatic force between electrons and proton in nucleus.

$$r_0 = \frac{n^2h^2}{4\pi^2 m_e ke^2} \quad \text{where } k = \frac{1}{4\pi\varepsilon_0}$$

If we consider the atom bound by gravitational force $\frac{Gm_pm_e}{r^2}$, then the term ke^2 should be replaced by Gm_pm_e. The radius of the first Bohr orbit in a gravitational bound hydrogen atom will be

$$r' = \frac{n^2h^2}{4\pi^2 Gm_pm_e^2}$$

$$= \frac{(1)^2\,(6.6\times10^{-34})^2}{4\times9.87\times6.67\times10^{-11}\times1.625\times10^{-27}\times(9.1\times10^{-31})^2}$$

$r' = 1.194 \times 10^{29}$ m $\approx 1.2 \times 10^{29}$ m.

This radius is much greater than the estimated size of the whole universe.

10. Let us first find the frequency of revolution of electron in the orbit classically.

In Bohr's model velocity of electron in n^{th} orbit, $v = \frac{nh}{2\pi mr}$

where radius $r = \frac{n^2h^2\varepsilon_0}{\pi me^2}$

Thus orbital frequency of electron in n^{th} orbit is

$$\upsilon = \frac{v}{2\pi r} = \frac{nh/2\pi mr}{2\pi r}$$

$$\upsilon = \frac{nh}{4\pi^2 mr^2} = \frac{nh}{4\pi^2 m}\left[\frac{\pi me^2}{n^2h^2\varepsilon_0}\right]^2$$

$$\text{or} \quad \upsilon = \frac{me^4}{4n^3h^3\varepsilon_0^{\,2}} \qquad \text{...(i)}$$

Now, let us find the frequency of radiation emitted when a hydrogen atom de-excites from level n to level (n – 1).

$$\upsilon = \frac{me^4}{8\varepsilon_0^2h^3}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\upsilon = \frac{me^4}{8\varepsilon_0^2h^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$\text{or} \quad \upsilon = \frac{me^4}{8\varepsilon_0^2h^3}\left[\frac{2n-1}{n^2\,(n-1)^2}\right]$$

For large n, $2n - 1 \approx 2n$ and $n - 1 \approx n$,

frequency,

$$\upsilon = \frac{me^4}{8\varepsilon_0^2h^3}\left[\frac{2n}{n^4}\right] \quad \text{or} \quad \upsilon = \frac{me^4}{4n^3h^3\varepsilon_0^{\,2}} \qquad \text{...(ii)}$$

Equation (i) and (ii) are equal, hence for large value of n, the classical frequency of revolution of electron in n^{th} orbit is same as frequency of radiation when electron de-excite from level n to (n – 1).

11. (a) The quantity is $\frac{ke^2}{r^2}$ or $\frac{e^2}{4\pi\varepsilon_0 mc^2}$ which has the dimension of length.

$$\left[\frac{e^2}{4\pi\varepsilon_0 mc^2}\right] = \frac{[Q^2]}{[M^{-1}L^{-3}T^2Q^2M(LT^{-1})^2]} = L$$

$$\text{Also } \frac{e^2}{4\pi\varepsilon_0 mc^2} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{9.1\times10^{-31}\times(3\times10^8)^2}\text{ m}$$

$= 2.8 \times 10^{-15}$ m.

This length is much smaller than the typical atomic size ($\approx 10^{-10}$ m).

(b) A quantity with the dimension of length from h, m_e and e

$$= \frac{\varepsilon_0 h^2}{\pi me^2} = \frac{(6.6\times10^{-34})^2\times8.86\times10^{-12}}{3.14\times9.1\times10^{-31}\times(1.6\times10^{-19})^2}$$

$= 0.53 \times 10^{-10}$ m = 0.53 Å

The length is of the order of atomic size (10^{-10} m).

12. Kinetic energy of an electron in an orbit,

$$= \frac{1}{4\pi\varepsilon_0}\frac{e(Ze)}{2r} \quad ...(i)$$

Potential energy of electron in the orbit

$$U = -\frac{1}{4\pi\varepsilon_0}\left(\frac{Ze^2}{r}\right) \quad ...(ii)$$

Total energy $E = K + U$

$$= -\frac{1}{4\pi\varepsilon_0}\left(\frac{Ze^2}{2r}\right) = -K = \frac{U}{2}$$

It is given, total energy $E = (-3.4 \text{ eV})$

(a) Kinetic energy of electron in this state $E = -K$

So, $K = E = -(-3.4 \text{ eV}) = 3.4$ eV

(b) Potential energy $E = U/2$

$U = 2E = 2(-3.4) = -6.8$ eV

(c) If the zero of the potential energy is chosen differently, the kinetic energy remain the same. Although potential energy and hence total energy changes.

13. Angular momentum $mvr = n\frac{h}{2\pi}$ associated with planetary motion are incomparably large relative to h.

For example angular momentum of earth in its orbital motion is of the order of $10^{70}\ \frac{h}{2\pi}$.

For such large value of n, the difference in successive energies and angular momenta of the quantised levels of the Bohr model are so small that one can predict the energy level continuous.

14. In Bohr's model, the radius of n^{th} orbit,

$$r = \frac{n^2h^2\varepsilon_0}{\pi Zme^2}$$

In the given muonic hydrogen atom, a negatively charged muon (μ^-) of mass 207 m_e revolve around a proton.

Therefore radius of electron and muon can be written as

$$\frac{r_\mu}{r_e} = \frac{m_e}{m_\mu} = \frac{m_e}{207m_e}$$

$$r_\mu = \frac{r_e}{207} = \frac{0.53 \times 10^{-10}}{207} \text{ m} = 2.5 \times 10^{-13} \text{ m}$$

Energy of electron in the orbit, $E = -\frac{me^4}{8\varepsilon_0 n^2h^2}$

$$\frac{E_e}{E_\mu} = \frac{m_e}{m_\mu} = \frac{m_e}{207m_e}$$

$E_\mu = 207\ E_e = 207\ [-13.6 \text{ eV}] = -2.8$ keV.