EXAM DRILL

Nuclei

CHAPTER 13

ANSWERS

1. **(a) :** $\frac{R_e}{r_n} = 10^5$

2. **(b)**

3. **(c) :** $R \propto A^{\frac{1}{3}}$

$$\frac{R_1}{R_2} = \left(\frac{1}{8}\right)^{\frac{1}{3}} = 1:2$$

4. **(b)**

5. **(c) :** Isotopes have same number of electrons and protons but different neutron number. That's why the mass number of isotopes are different and can be separated by using a mass spectrometer.

6. **(b) :** Nitrogen is a stable element. Those elements which have an odd number of protons and an odd number of neutrons are generally unstable. $^{14}_{7}N$ has 7 protons and 7 neutrons *i.e.*, odd number of both protons and neutrons but it is an exception. It is a stable element.

7. **(b) :** A neutron is slightly heavier than a proton. As neutrons are charge less particles they penetrate matter more than protons. Therefore, they are more effective than protons in fission reactions.

8. **(c) :** In both fission and fusion large amounts of energy is released. The reason is correct. Charge, mass, momentum and energy, all are conserved.

9. Fusion, $R = R_0 A^{\frac{1}{3}} \Rightarrow \frac{R_{Al}}{R_{Cu}} = \left(\frac{A_{Al}}{A_{Cu}}\right)^{\frac{1}{3}}$

$$\Rightarrow \quad R_{Cu} = R_{Al}\left(\frac{A_{Cu}}{A_{Al}}\right)^{\frac{1}{3}} = 3.6\left(\frac{64}{27}\right)^{\frac{1}{3}} = 4.8 \text{ fermi}$$

10. Heavy water used as a moderator to cool the reactions and absorb excess neutrons.

OR

$${}_{92}U^{235} + {}_0n^1 = {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + 200 \text{ MeV}$$

11. **(i) (d)**

(ii) **(d) :** In nuclear fission or fusion both energy and mass are conserved.

(iii) (b) : As only 0.1% of the original mass is converted into energy, hence out of 1 kg mass 1 g is converted into energy.

$\therefore$ Energy released during fission, $E = \Delta mc^2$

$$= 1 \text{ g} \times (3 \times 10^8 \text{ m s}^{-1})^2 = 10^{-3} \times 9 \times 10^{16} \text{ J} = 9 \times 10^{13} \text{ J}$$

12. Nucleus was first discovered in 1911 by Rutherford and his associates by experiments on scattering of α-particle by atoms. He found that the scattering result could be explained, if atoms consists of a small, central, massive and positive core surrounded by orbiting electron. The experiment results indicated that the size of the nucleus is of the order of 10^{-14} metres and it thus 10,000 times smaller than the size of atom.

Relation between the radius and mass number of the nucleus

$R = R_0 A^{1/3}$

If m is the average mass of a nucleon and R is the nuclear radius, then mass of nucleus $= mA$, where A is the mass number of the element.

Volume of the nucleus, $V = \frac{4}{3}\pi R^3$

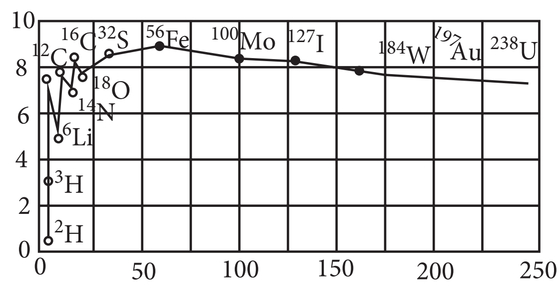
$$\therefore \quad V = \frac{4}{3}\pi (R_0 A^{1/3})^3 \Rightarrow \quad V = \frac{4}{3}\pi R_0^3 A$$

Density of nuclear matter, $\rho = \frac{mA}{V}$

$$\Rightarrow \quad \rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A} \Rightarrow \rho = \frac{3m}{4\pi R_0^3}$$

This shows that the nuclear density is independent of A.

13. (i)



The constancy of binding energy in the range $30 < A < 170$ is a consequence of the fact that the nuclear force is short ranged. Consider a particular nucleon inside a sufficiently large nucleus. It will be under the influence of only some of its neighbours, which come within the range of the nuclear force. If any other nucleon is at a distance more than the range of the nuclear force from the particular nucleon it will have no influence on the binding energy of the nucleon under consideration. If a nucleon can have a maximum of p neighbours within the range of nuclear force, its binding energy would be proportional to p. Let the binding energy of the nucleus be pk, where k is a constant having the dimensions of energy. If we increase A by adding nucleons they will not change the binding energy of a nucleon inside. Since most of the nucleons in a large nucleus reside inside it and not on the surface, the change in binding energy per nucleon would be small. The binding energy per nucleon is a constant and is approximately equal to pk.

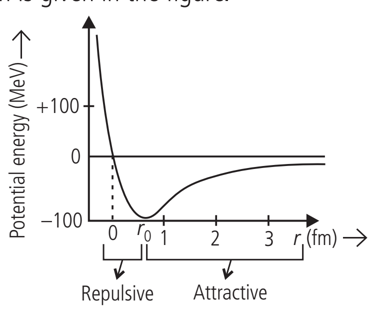
(ii) A very heavy nucleus with A = 240, has lower binding energy per nucleon compared to that of a nucleus with A = 120. When a heavy nucleus with mass number A = 240 breaks into two nuclei, A = 120, energy is released in this process.

OR

(i) Yes, because they have same atomic number 3.

(ii) $_3X^7$, because neutron to proton ratio in $_3X^7$ is 1.33 which is ≈ 1, whereas in $_3Y^4$ is 0.33 which is much less than 1. For greater stability neutron to proton ratio must be nearly equal to 1.

14. Plot of potential energy of a pair of nucleons as a function of their separation is given in the figure.



Conclusions: (i) The nuclear force is much stronger than the coulomb force acting between charges or the gravitational forces between masses.

(ii) The nuclear force between two nucleons falls rapidly to zero as their distance is more than a few fermies.

(iii) For a separation greater than r_0, the force is attractive and for separation less than r_0, the force is strongly repulsive.

15. A certain number of neutrons and protons are brought together to form a nucleus of a certain charge and mass, an energy ΔE_b will be released in this process.

The energy ΔE_b is called the binding energy of the nucleus. If we separate a nucleus into its nucleons we would have to transfer a total energy equal to ΔE_b, to the nucleons.

Example : $_{92}U^{235} + _0n^1 \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^1_0n + Q$

The energy (Q) released was estimated to be 200 MeV per fission (or about 0.9 MeV per nucleon) and is equivalent to the difference in masses of the nuclei before and after the fission.

16. Fusion reaction,

$${}^2_1H + {}^2_1H \longrightarrow {}^3_2He + n$$

Energy released = final $B.E.$ – initial $B.E.$
= 7.73 – (2.23 + 2.23) = 3.27 MeV.

17.

	Nuclear Fission		Nuclear Fusion
1.	The process of splitting of a heavy nucleus into two nuclei of nearly comparable masses with liberation of energy is called nuclear fission. Example: ${}^{235}_{92}U + {}^1_0n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^1_0n + Q$	1.	When two or more than two light nuclei fuse together to form heavy nucleus with the liberation of energy, the process is called nuclear fusion. Example: ${}^2_1H + {}^2_1H \rightarrow {}^4_2He + 24$ MeV
2.	A suitable bullet or projectile like neutron is needed.	2.	The lighter nuclei have to be brought very close to each other against electrostatic repulsion.
3.	The products of nuclear fission reaction are radioactive.	3.	The products of nuclear fusion are not radioactive.

OR

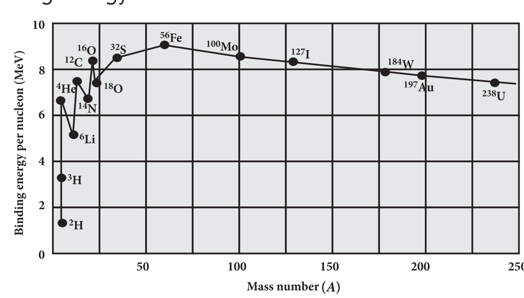
$_{92}U^{235} + _0n^1 \rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3_0n^1$

$\Delta m = m(_{92}U^{235}) + m(_0n^1) - m(_{56}Ba^{141}) - m(_{36}Kr^{92}) - 3\times m(_0n^1)$
$= 0.213503$ u

$E = \Delta m \times 931.5$ MeV = 198.88 MeV

18. In a nuclear reaction, the sum of the masses of the target nucleus (${}^2_1H$) and the bombarding particle (${}^2_1H$) may be greater than the product nucleus $\left({}^3_2He\right)$ and the outgoing neutron $_0{}^1n$. So from the law of conservation of mass-energy some energy (3.27 MeV) is evolved due to mass defect in the nuclear reaction. This energy is called Q-value of the nuclear reaction.

19. Binding energy curve:



Two salient features of the curve

(i) The binding energy per nucleon, E_{bn}, is practically constant, *i.e.* practically independent of the atomic number for nuclei of middle mass number (30 < A < 170). The curve has a maximum value of about 8.75 MeV for A = 56 and has a value of 7.6 MeV for A = 238.

(ii) E_{bn} is lower for both light nuclei (A < 30) and heavy nuclei (A > 170).

The binding energy curve can be used to explain the phenomena of nuclear fission and nuclear fusion.

Nuclear fission : Binding energy per nucleon is smaller for heavier nuclei than the middle ones, *i.e.*, heavier nuclei are less stable. When a heavier nucleus splits into the lighter nuclei, the B.E./nucleon changes from about 7.6 MeV to 8.4 MeV. Greater binding energy of the product nuclei results in the liberation of energy. This is what happens in nuclear fission which is the basis of the atom bomb.

Nuclear fusion : The binding energy per nucleon is small for light nuclei, *i.e.*, they are less stable. So when two light nuclei combine to form a heavier nucleus, the higher binding energy per nucleon of the latter results in the release of energy. This is what happens in a nuclear fusion which is the basis of the hydrogen bomb.

OR

(a) Given $m = 2$ kg, $P = 600$ W.

Here two deuterium nuclei produce 3.27 MeV energy

$= 5.232 \times 10^{-13}$ J

$\therefore$ Energy per nuclei $= \frac{5.232\times10^{-13}}{2} = 2.616 \times 10^{-13}$ J

No. of deuterium atom in 2 kg

$= \frac{6.023\times10^{23}\times2000}{2} = 6.023 \times 10^{26}$ atom

$\therefore$ Total energy $= 6.023 \times 10^{26} \times 2.616 \times 10^{-13}$

$= 15.75 \times 10^{13}$ J

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} \Rightarrow t = \frac{15.75\times10^{13}}{600} = 2.6\times10^{11}\text{s}$$

$$= \frac{2.6\times10^{11}}{365\times24\times60\times60} = 8.2\times10^{3} \text{ years}$$

(a) In a nuclear reaction, the sum of the masses of the target nucleus ($^2_1$H) and the bombarding particle ($^2_1$H) may be greater than the product nucleus $\left(^3_2\text{He}\right)$ and the outgoing neutron ^1_0n. So from the law of conservation of mass-energy some energy (3.27 MeV) is evolved due to mass defect in the nuclear reaction. This energy is called Q-value of the nuclear reaction.

20. Applying principle of conservation of energy,

$$E - B = K_n + K_p = \frac{p_n^2}{2m} + \frac{p_p^2}{2m} \quad \text{...(i)}$$

From law of conservation of momentum,

$p_n + p_p = E/c$

when $E = B$,

from equation (i), $p_n = p_p = 0$

$\therefore$ Process cannot take place.

For process to take place, let E be very slightly bigger than B so that $E = B + \lambda$, ($\lambda << B$.)

$$\lambda = \frac{p_n^2}{2m} + \frac{p_p^2}{2m}$$

$$\lambda = \frac{1}{2m}[p_p^2 + (p_p - E/c)^2]$$

$$p_p = \frac{E}{2c} \pm \sqrt{\frac{E}{4c^2} - \left(\frac{E^2}{2c^2} - m\lambda\right)}$$

For p_p (momentum of proton) to be real, the determinant must be positive.

$$\frac{E^2}{4c^2} - \left(\frac{E^2}{2c^2} - m\lambda\right) \geq 0 \text{ or } \lambda = \frac{E^2}{4mc^2} \simeq \frac{B^2}{4mc^2}$$