

Nuclei



ANSWERS

Topic 1

1. (a) Abundance of ${}^6_3\text{Li}$ is 7.5% and abundance of ${}^7_3\text{Li}$ is 92.5%.

Hence atomic mass of lithium,

$$A = \frac{7.5(6.01512 \text{ u}) + 92.5(7.01600 \text{ u})}{100}$$

$$A = \frac{451134 + 648.98}{100} \text{ u} = 6.941 \text{ u}$$

- (b) Let abundance of ${}^{10}_5\text{B}$ is $x\%$ than abundance of ${}^{11}_5\text{B}$ will be $(100 - x)\%$.

Atomic mass of boron

$$= \frac{x[10.01294 \text{ u}] + (100 - x)[11.00931 \text{ u}]}{100}$$

$$\Rightarrow 100 \times 10.811 \text{ u} = 1100.931 \text{ u} - 0.99637x \text{ u}$$

$$\text{Solving we get, } x = \frac{19.831}{0.99637} = 19.9\%$$

So, relative abundance of ${}^{10}_5\text{B}$ isotope = 19.9%

Relative abundance of ${}^{11}_5\text{B}$ isotope = 80.1%

2. Average atomic mass of neon with the given abundances,

$$A = \frac{90.51(19.99 \text{ u}) + 0.27(20.99 \text{ u}) + 9.22(21.99 \text{ u})}{100}$$

$$A = \frac{2017.7}{100} \text{ u} = 20.18 \text{ u}$$

3. The ${}^{14}_7\text{N}$ nucleus contains 7 protons and 7 neutrons.

Mass of 7 protons = $7 \times 1.00783 = 7.05481 \text{ amu}$

Mass of 7 neutrons = $7 \times 1.00867 = 7.06069 \text{ amu}$

Total mass = 14.11550 amu

Mass of ${}^{14}_7\text{N}$ nucleus = 14.00307 amu

Mass defect, $= \Delta m = 0.11243 \text{ amu}$

B.E. of nitrogen nucleus = $0.11243 \times 931 = 104.67 \text{ MeV}$.

4. Let us first find the binding energy of ${}^{56}_{26}\text{Fe}$.

No. of protons in Fe = $Z = 26$

Mass of protons = $26 \times 1.007825 \text{ u} = 26.203450 \text{ u}$

No. of neutrons in Fe, $n = A - Z = 56 - 26 = 30$

Mass of neutrons = $30 \times 1.008665 \text{ u} = 30.259950 \text{ u}$

Total theoretical mass of nucleus

$$= 26.203450 \text{ u} + 30.259950 \text{ u} = 56.463400 \text{ u}$$

Actual mass of Fe nucleus 55.934939 u

Mass defect $\Delta m = \text{Total mass} - \text{Actual mass} = 0.528461 \text{ u}$

B.E. of ${}^{56}_{26}\text{Fe}$ nucleus $E = \Delta mc^2 = \Delta m \times 931.5 \text{ MeV}$

$$= 0.528461 (931.5) \text{ MeV} = 492.26 \text{ MeV}$$

$$\frac{\text{B.E.}}{\text{nucleon}} \text{ of } {}^{56}_{26}\text{Fe} = \frac{492.26}{56} \text{ MeV} = 8.79 \text{ MeV}$$

Now binding energy of ${}^{209}_{83}\text{Bi}$

No. of protons in Bi = $Z = 83$

No. of neutrons in Bi $n = A - Z = 209 - 83 = 126$

Mass of protons = $83 \times 1.007825 \text{ u} = 83.649475 \text{ u}$

Mass of neutrons = $126 \times 1.008665 \text{ u} = 127.091790 \text{ u}$

Total theoretical mass of nucleus = 210.741265 u

Actual mass of Bi nucleus = 208.980388 u

Mass defect, $\Delta m = 210.741260 - 208.980388 = 1.760877 \text{ u}$

$$\Rightarrow \text{B.E. of } {}^{209}_{83}\text{Bi} \text{ nucleus} \Rightarrow \Delta mc^2$$

$$\Rightarrow \Delta m (931.5 \text{ MeV})$$

$$\Rightarrow 1.760877 \times 931.5 \text{ MeV} \Rightarrow 1640.3 \text{ MeV}$$

$$\frac{\text{B.E.}}{\text{nucleon}} \text{ of } {}^{209}_{83}\text{Bi} = \frac{1640.3}{209} \text{ MeV} = 7.85 \text{ MeV}$$

So, ${}^{56}_{26}\text{Fe}$ is much more stable than ${}^{209}_{83}\text{Bi}$, due to more binding energy per nucleon.

5. Let us first find the B.E. of each copper nucleus and then we can find binding energy of 300 g of ${}^{63}_{29}\text{Cu}$.

Mass of 29 protons = $29 \times 1.00783 = 29.22707 \text{ u}$

Mass of 34 neutrons = $34 \times 1.00867 = 34.29478 \text{ u}$

Total theoretical mass = 63.52185 u

Actual mass of Cu nucleus = 62.92960 u

Mass of defect = Theoretical mass - Actual mass = 0.59225 u

B.E. of each Cu nucleus = $\Delta m [931.5 \text{ MeV}]$

$$= 0.59225 [931.5 \text{ MeV}] = 551.385 \text{ MeV}$$

Number of atoms in 3 g of copper

$$n = \frac{\text{Avogadro number}}{\text{Mass number}} \times 3$$

$$\text{or } n = \frac{6.023 \times 10^{23} \times 3}{63} = 2.86 \times 10^{22}$$

Total binding energy in 3 g of copper

$$= 2.86 \times 10^{22} \times 551.385 \text{ MeV} = 1.6 \times 10^{25} \text{ MeV}$$

So, the energy required to separate all the neutrons and protons from each other in 3 g copper coin will be $1.6 \times 10^{25} \text{ MeV}$.

Topic 2

1. Number of atoms present in 2 g of deuterium
 $= 6.023 \times 10^{23}$

Total number of atoms present in 2000 g of deuterium

$$= \frac{6.023 \times 10^{23} \times 2000}{2} = 6.023 \times 10^{26}$$

Energy released in the fusion of 2 deuterium atoms = 3.27 MeV

Total energy released in the fusion of 2.0 kg of deuterium atoms

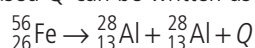
$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} = 9.81 \times 10^{26} \text{ MeV} = 15.696 \times 10^{13} \text{ J}$$

Energy consumed by the bulb per second = 100 J

Time for which the bulb will glow

$$t = \frac{15.69 \times 10^{13}}{100} \text{ s or } t = \frac{15.69 \times 10^{11}}{3.15 \times 10^7} \text{ years} = 4.9 \times 10^4 \text{ years.}$$

2. The fission of Fe-56 into two fragments of ${}^{28}_{13}\text{Al}$ with energy released Q can be written as



$$Q = [m({}^{56}_{26}\text{Fe}) - 2m({}^{28}_{13}\text{Al})]c^2$$

$$= [55.93494 - 2 \times 27.98191] \times 931.5 \text{ MeV}$$

$$= -0.02888 \times 931.5 = -26.90 \text{ MeV}$$

As the Q -value is negative, the fission is not possible energetically.

3. Number of atoms present in 1 mole i.e.,

$$239 \text{ g of } {}^{239}_{94}\text{Pu} = 6.023 \times 10^{23}$$

Number of atoms present in 1000 g of ${}^{239}_{94}\text{Pu}$

$$= \frac{6.023 \times 10^{23} \times 1000}{239} = 2.52 \times 10^{24}$$

Energy released per fission = 180 MeV

Total energy released = $2.52 \times 10^{24} \times 180 \text{ MeV} = 4.54 \times 10^{26} \text{ MeV}$.

4. In the fission of one nucleus of ${}^{235}_{92}\text{U}$, energy generated is 200 MeV.

\therefore Energy generated in fission of 1 kg of ${}^{235}_{92}\text{U}$

$$= 200 \times \frac{6.023 \times 10^{23}}{235} \times 1000 \text{ MeV} = 5.106 \times 10^{26} \text{ MeV}$$

$$= 5.106 \times 10^{26} \times 1.6 \times 10^{-13} \text{ J} = 8.17 \times 10^{13} \text{ J}$$

Time for which reactor operates

$$= \frac{80}{100} \times 5 \text{ yr} = 4 \text{ yr.}$$

Total energy generated in 5 years

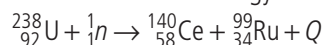
$$= 1000 \times 10^6 \times 60 \times 60 \times 24 \times 365 \times 4 \text{ J}$$

Amount of ${}^{235}_{92}\text{U}$ consumed in 5 years

$$= \frac{1000 \times 10^6 \times 60 \times 60 \times 24 \times 365 \times 4}{8.17 \times 10^{13}} \text{ kg} = 1544 \text{ kg}$$

Initial amount of ${}^{235}_{92}\text{U} = 2 \times 1544 \text{ kg} = 3088 \text{ kg}$

5. The fission of U-238 by fast neutrons into fragments Ce-140 and Ru-99 with energy released Q can be written as



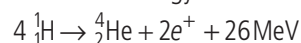
The Q value,

$$Q = [m(\text{U-238}) + m_n - m(\text{Ce-140}) - m(\text{Ru-99})]c^2$$

$$Q = [238.05079 + 1.00867 - 139.90543 - 98.90594] \text{ amu} \times c^2$$

$$Q = 0.24809 \times 931.5 \text{ MeV} = 231.09 \text{ MeV} = 231.1 \text{ MeV.}$$

6. (a) In the fusion reactions taking place within core of Sun, 4 hydrogen nuclei combines to form a helium nucleus with the release of 26 MeV of energy.



Number of atoms in 1 kg of ${}^1_1\text{H}$,

$$n = \frac{1000 \text{ g} \times 6 \times 10^{23}}{\text{Atomic mass}} = \frac{1000 \text{ g}}{1 \text{ g}} \times 6 \times 10^{23}$$

$$= 6 \times 10^{26} \text{ atoms}$$

Energy released in the fusion of 1 kg of ${}^1_1\text{H}$,

$$E_1 = \frac{6 \times 10^{26} \times 26}{4} \text{ MeV} = 39 \times 10^{26} \text{ MeV}$$

(b) Energy released per fission of U-235 is 200 MeV.

Number of atoms in 1 kg of U-235,

$$n = \frac{1000 \text{ g} \times 6 \times 10^{23}}{235 \text{ g}} = 25.53 \times 10^{23} \text{ atoms}$$

Total energy released for fission of 1 kg of uranium,

$$E_2 = 25.53 \times 10^{23} \times 200 \text{ MeV} = 5.1 \times 10^{26} \text{ MeV}$$

$$\frac{E_1}{E_2} = \frac{39 \times 10^{26}}{5.1 \times 10^{26}} = 7.65 \approx 8$$

So, the energy released in fusion of 1 kg of Hydrogen is nearly 8 times the energy released in fission of 1 kg of Uranium-235.

7. 10% of total power 200,000 MW to be obtained from nuclear power plant by 2020 AD.

So, power from nuclear plants

$$= 2 \times 10^5 \times 0.1 \text{ MW} = 2 \times 10^4 \text{ MW} = 2 \times 10^{10} \text{ W}$$

With efficiency of power plants 25% only, the energy converted

$$\text{into electrical energy per fission} = \frac{25}{100} \times 200 = 50 \text{ MeV}$$

$$= 50 \times 1.6 \times 10^{-13} \text{ Joule} = 8 \times 10^{-12}$$

Total energy to be produced

$$= 2 \times 10^4 \text{ MW} = 2 \times 10^{10} \text{ joule/sec}$$

$$= 2 \times 10^{10} \times 60 \times 60 \times 24 \times 365 \text{ joule / year}$$

$$= \frac{2 \times 10^{24} \times 36 \times 24 \times 365}{8}$$

Mass of 6.023×10^{23} atoms of 235 U = 235 g

$$= 235 \times 10^{-3} \text{ kg}$$

Mass of $\frac{2 \times 36 \times 24 \times 365}{8} \times 10^{24}$ atoms

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times \frac{2 \times 36 \times 24 \times 365 \times 10^{24}}{8}$$

$$= 3.08 \times 10^4 \text{ kg}$$

Hence, mass of uranium needed per year = $3.08 \times 10^4 \text{ kg}$

