CHAPTER 13

Nuclei

NCERT FOCUS

ANSWERS

Topic 1

1. (a) Abundance of ${}^{6}_{3}\text{Li}$ is 7.5% and abundance of ${}^{7}_{3}\text{Li}$ is 92.5%.

Hence atomic mass of lithium,

$$A = \frac{7.5\,(6.01512\text{ u}) + 92.5\,(7.01600\text{ u})}{100}$$

$$A = \frac{451134 + 648.98}{100}\text{ u} = 6.941\text{ u}$$

(b) Let abundance of ${}^{10}_{5}\text{B}$ is x % than abundance of ${}^{11}_{5}\text{B}$ will be (100 – x)%.

Atomic mass of boron

$$= \frac{x[10.01294\text{ u}] + (100 - x)[11.00931\text{ u}]}{100}$$

$\Rightarrow$ 100 × 10.811 u = 1100.931 u – 0.99637x u

Solving we get, $x = \frac{19.831}{0.99637} = 19.9\%$

So, relative abundance of ${}^{10}_{5}\text{B}$ isotope = 19.9%

Relative abundance of ${}^{11}_{5}\text{B}$ isotope = 80.1%

2. Average atomic mass of neon with the given abundances,

$$A = \frac{90.51(19.99\text{ u}) + 0.27\,(20.99\text{ u}) + 9.22\,(21.99\text{ u})}{100}$$

$$A = \frac{2017.7}{100}\text{ u} = 20.18\text{ u}$$

3. The ${}^{14}_{7}\text{N}$ nucleus contains 7 protons and 7 neutrons.

Mass of 7 protons = 7 × 1.00783 = 7.05481 amu

Mass of 7 neutrons = 7 × 1.00867 = 7.06069 amu

Total mass = 14.11550 amu

Mass of ${}^{14}_{7}\text{N}$ nucleus = 14.00307 amu

Mass defect, = Δm = 0.11243 amu

B.E. of nitrogen nucleus = 0.11243 × 931 = 104.67 MeV.

4. Let us first find the binding energy of ${}^{56}_{26}\text{Fe}$.

No. of protons in Fe = Z = 26

Mass of protons = 26 × 1.007825 u = 26.203450 u

No. of neutrons in Fe, $n = A - Z$ = 56 – 26 = 30

Mass of neutrons = 30 × 1.008665 u = 30.259950 u

Total theoretical mass of nucleus

= 26.203450 u + 30.259950 u = 56.463400 u

Actual mass of Fe nucleus 55.934939 u

Mass defect Δm = Total mass – Actual mass = 0.528461 u

B.E. of ${}^{56}_{26}\text{Fe}$ nucleus $E = \Delta mc^2 = \Delta m$ 931.5 MeV

= 0.528461 (931.5) MeV = 492.26 MeV

$$\frac{\text{B.E}}{\text{nucleon}} \text{ of } {}^{56}_{26}\text{Fe} = \frac{492.26}{56}\text{ MeV} = 8.79\text{ MeV}$$

Now binding energy of ${}^{209}_{83}\text{Bi}$

No. of protons in Bi = Z = 83

No. of neutrons in Bi $n = A - Z$ = 209 – 83 = 126

Mass of protons = 83 × 1.007825 u = 83.649475 u

Mass of neutrons = 126 × 1.008665 u = 127.091790 u

Total theoretical mass of nucleus = 210.741265 u

Actual mass of Bi nucleus = 208.980388 u

Mass defect, Δm = 210.741260 – 208.980388 = 1.760877 u

$\Rightarrow$ B.E. of ${}^{209}_{83}\text{Bi}$ nucleus $\Rightarrow \Delta mc^2$

$\Rightarrow$ Δm (931.5 MeV)

$\Rightarrow$ 1.760877 × 931.5 MeV $\Rightarrow$ 1640.3 MeV

$$\frac{\text{B.E}}{\text{nucleon}} \text{ of } {}^{209}_{83}\text{Bi} = \frac{1640.3}{209}\text{ MeV} = 7.85\text{ MeV}$$

So, ${}^{56}_{26}\text{Fe}$ is much more stable than ${}^{209}_{83}\text{Bi}$, due to more binding energy per nucleon.

5. Let us first find the B.E. of each copper nucleus and then we can find binding energy of 300 g of ${}^{63}_{29}\text{Cu}$.

Mass of 29 protons = 29 × 1.00783 = 29.22707 u

Mass of 34 neutrons = 34 × 1.00867 = 34.29478 u

Total theoretical mass = 63.52185 u

Actual mass of Cu nucleus = 62.92960 u

Mass of defect = Theoretical mass – Actual mass = 0.59225 u

B.E. of each Cu nucleus = Δm [931.5 MeV]

= 0.59225 [931.5 MeV] = 551.385 MeV

Number of atoms in 3 g of copper

$$n = \frac{\text{Avogadro number}}{\text{Mass number}} \times 3$$

or $$n = \frac{6.023 \times 10^{23} \times 3}{63} = 2.86 \times 10^{22}$$

Total binding energy in 3 g of copper

= 2.86 × 10^{22} × 551.385 MeV = 1.6 × 10^{25} MeV

So, the energy required to separate all the neutrons and protons from each other in 3 g copper coin will be 1.6 × 10^{25} MeV.

Topic 2

1. Number of atoms present in 2 g of deuterium

= 6.023 × 10^{23}

Total number of atoms present in 2000 g of deuterium

$$= \frac{6.023 \times 10^{23} \times 2000}{2} = 6.023 \times 10^{26}$$

Energy released in the fusion of 2 deuterium atoms = 3.27 MeV

Total energy released in the fusion of 2.0 kg of deuterium atoms

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} = 9.81 \times 10^{26} \text{ MeV} = 15.696 \times 10^{13} \text{ J}$$

Energy consumed by the bulb per second = 100 J

Time for which the bulb will glow

$$t = \frac{15.69 \times 10^{13}}{100} \text{ s or } t = \frac{15.69 \times 10^{11}}{3.15 \times 10^{7}} \text{ years} = 4.9 \times 104 \text{ years.}$$

2. The fission of Fe-56 into two fragments of $^{28}_{13}\text{Al}$ with energy released Q can be written as

$$^{56}_{26}\text{Fe} \rightarrow {}^{28}_{13}\text{Al} + {}^{28}_{13}\text{Al} + Q$$

$$Q = \left[m(^{56}_{26}\text{Fe}) - 2m(^{28}_{13}\text{Al}) \right] c^2$$

$$= [55.93494 - 2 \times 27.98191] \times 931.5 \text{ MeV}$$

$$= -0.02888 \times 931.5 = -26.90 \text{ MeV}$$

As the Q-value is negative, the fission is not possible energetically.

3. Number of atoms present in 1 mole *i.e.*,

239 g of $^{239}_{94}\text{Pu} = 6.023 \times 10^{23}$

Number of atoms present in 1000 g of $^{239}_{94}\text{Pu}$

$$= \frac{6.023 \times 10^{23} \times 1000}{239} = 2.52 \times 10^{24}$$

Energy released per fission = 180 MeV

Total energy released = $2.52 \times 10^{24} \times 180 \text{ MeV} = 4.54 \times 10^{26}$ MeV.

4. In the fission of one nucleus of $^{235}_{92}\text{U}$, energy generated is 200 MeV.

∴ Energy generated in fission of 1 kg of $^{235}_{92}\text{U}$

$$= 200 \times \frac{6.023 \times 10^{23}}{235} \times 1000 \text{ MeV} = 5.106 \times 10^{26} \text{ MeV}$$

$$= 5.106 \times 10^{26} \times 1.6 \times 10^{-13} \text{ J} = 8.17 \times 10^{13} \text{ J}$$

Time for which reactor operates

$$= \frac{80}{100} \times 5 \text{ yr} = 4 \text{ yr.}$$

Total energy generated in 5 years

$$= 1000 \times 10^6 \times 60 \times 60 \times 24 \times 365 \times 4 \text{ J}$$

Amount of $^{235}_{92}\text{U}$ consumed in 5 years

$$= \frac{1000 \times 10^6 \times 60 \times 60 \times 24 \times 365 \times 4}{8.17 \times 10^{13}} \text{ kg} = 1544 \text{ kg}$$

Initial amount of $^{235}_{92}\text{U}$ = 2 × 1544 kg = 3088 kg

5. The fission of U-238 by fast neutrons into fragments Ce-140 and Ru-99 with energy released Q can be written as

$$^{238}_{92}\text{U} + {}^{1}_{1}n \rightarrow {}^{140}_{58}\text{Ce} + {}^{99}_{34}\text{Ru} + Q$$

The Q value,

$$Q = [m(\text{U-238}) + m_n - m(\text{Ce-140}) - m(\text{Ru-99})]c^2$$

$$Q = [238.05079 + 1.00867 - 139.90543 - 98.90594] \text{ amu} \times c^2$$

$$Q = 0.24809 \times 931.5 \text{ MeV} = 231.09 \text{ MeV} = 231.1 \text{ MeV.}$$

6. (a) In the fusion reactions taking place within core of Sun, 4 hydrogen nuclei combines to form a helium nucleus with the release of 26 MeV of energy.

$$4\,^{1}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + 2e^{+} + 26 \text{ MeV}$$

Number of atoms in 1 kg of $^{1}_{1}\text{H}$,

$$n = \frac{1000 \text{ g} \times 6 \times 10^{23}}{\text{Atomic mass}} = \frac{1000 \text{ g}}{1 \text{ g}} \times 6 \times 10^{23}$$

$$= 6 \times 10^{26} \text{ atoms}$$

Energy released in the fusion of 1 kg of $^{1}_{1}\text{H}$,

$$E_1 = \frac{6 \times 10^{26} \times 26}{4} \text{ MeV} = 39 \times 10^{26} \text{ MeV}$$

(b) Energy released per fission of U-235 is 200 MeV.

Number of atoms in 1 kg of U-235,

$$n = \frac{1000 \text{ g} \times 6 \times 10^{23}}{235 \text{ g}} = 25.53 \times 10^{23} \text{ atoms}$$

Total energy released for fission of 1 kg of uranium,

$$E_2 = 25.53 \times 10^{23} \times 200 \text{ MeV} = 5.1 \times 10^{26} \text{ MeV}$$

$$\frac{E_1}{E_2} = \frac{39 \times 10^{26}}{5.1 \times 10^{26}} = 7.65 \approx 8$$

So, the energy released in fusion of 1 kg of Hydrogen is nearly 8 times the energy released in fission of 1 kg of Uranium-235.

7. 10% of total power 200,000 MW to be obtained from nuclear power plant by 2020 AD.

So, power from nuclear plants

$$= 2 \times 10^5 \times 0.1 \text{ MW} = 2 \times 10^4 \text{ MW} = 2 \times 10^{10} \text{ W}$$

With efficiency of power plants 25% only, the energy converted into electrical energy per fission $= \frac{25}{100} \times 200 = 50$ MeV

$$= 50 \times 1.6 \times 10^{-13} \text{ Joule} = 8 \times 10^{-3}$$

Total energy to be produced

$$= 2 \times 10^4 \text{ MW} = 2 \times 10^{10} \text{ joule/sec}$$

$$= 2 \times 10^{10} \times 60 \times 60 \times 24 \times 365 \text{ joule / year}$$

$$= \frac{2 \times 10^{24} \times 36 \times 24 \times 365}{8}$$

Mass of 6.023×10^{23} atoms of 235 U = 235 g

$$= 235 \times 10^{-3} \text{ kg}$$

Mass of $\frac{2 \times 36 \times 24 \times 365}{8} \times 10^{24}$ atoms

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times \frac{2 \times 36 \times 24 \times 365 \times 10^{24}}{8}$$

$$= 3.08 \times 10^4 \text{ kg}$$

Hence, mass of uranium needed per year = 3.08×10^4 kg