Electric charges and Fields

EXAM DRILL

CHAPTER 1

ANSWERS

1. **(c)** : The SI unit of volume charge density is C/m^3

2. **(b)** : $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$, where the value of $k = 2$.

3. **(b)** : The dimensional formula of electric flux is $[ML^3T^{-3}A^{-1}]$.

4. **(b)** : The electric field, $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

5. **(a)** : In free space the value of electrostatic force constant is one.

6. **(d)** : Charge on silk is equal and opposite to charge on glass rod, *i.e.*, $q = -1.6 \times 10^{-12}$ C.

7. **(a)** : $\phi = q/\varepsilon_0$, $q = 1$ C

$\therefore \quad \phi = \dfrac{1}{8.85\times10^{-12}} = 1.13\times10^{11}\ \text{N m}^2\ \text{C}^{-1}$.

8. **(b) :** As $\sigma_1 = \sigma_2$ (Given)

$\therefore \quad \dfrac{q_1}{4\pi r_1^2} = \dfrac{q_2}{4\pi r_2^2}$, or, $\dfrac{q_1}{q_2} = \dfrac{r_1^2}{r_2^2}$

[Let r_1 and r_2 be two different radii]

Then the ratio of electric field intensities near the surface of spherical conductors,

$$\frac{E_1}{E_2} = \frac{q_1}{4\pi\varepsilon_0 r_1^2}\times\frac{4\pi\varepsilon_0 r_2^2}{q_2} = \frac{q_1}{q_2}\times\frac{r_2^2}{r_1^2} = \frac{q_1}{q_2}\times\frac{q_2}{q_1} = 1$$

i.e. $E_1 = E_2$

9. **(c) :** If the charged particle is initially at rest in an electric field, it will move along the electric line of force. But when the initial velocity of charged particle makes some angle with the line of force then the resultant path is not along the line of force. Because electric line of force may not coincide with the line of velocity of the charge.

10. **(c) :** When high energy X-ray beam falls on the ball, the metal will emit photoelectrons, thus leaving the positive charge on the ball. As a result of this, ball is deflected in the direction of electric field.

11. **(a) :** Work done will be zero because in rotating the charge in a circle, force is along the radius and direction of motion is perpendicular to it.

$\therefore$ Work done, $W = \vec{F}.\vec{S} = FS\cos\theta = FS\cos90° = 0$

12. **(i)** **(d) :** If there is only one type of charge in the universe then it will produce electric field somehow. Hence Gauss's law is valid.

(ii) **(c)**

(iii) **(c) :** According to Gauss's theorem,

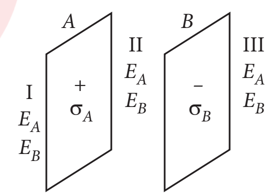
Electric flux through the sphere $= \dfrac{q}{\varepsilon_0}$

$\therefore$ Electric flux through the hemisphere $= \dfrac{1}{2}\dfrac{q}{\varepsilon_0}$

$$= \frac{10\times10^{-6}}{2\times8.854\times10^{-12}} = 0.56 \times 10^6\ \text{N m}^2\ \text{C}^{-1}$$

$\approx 0.6 \times 10^6\ \text{N m}^2\ \text{C}^{-1} = 6 \times 10^5\ \text{N m}^2\ \text{C}^{-1}$

13. **(i)** **(d) :** There are two plates A and B having surface charge densities, $\sigma_A = 17.0 \times 10^{-22}\ \text{C/m}^2$ on A and $\sigma_B = -17.0 \times 10^{-22}\ \text{C/m}^2$ on B, respectively.



According to Gauss' theorem, if the plates have same surface charge density but having opposite signs, then the electric field in region I is zero.

$$E_I = E_A + E_B = \frac{\sigma}{2\varepsilon_0} + \left(-\frac{\sigma}{2\varepsilon_0}\right) = 0$$

(ii) **(d) :** The electric field in region III is also zero.

$$E_{III} = E_A + E_B = \frac{\sigma}{2\varepsilon_0} + \left(-\frac{\sigma}{2\varepsilon_0}\right) = 0$$

(iii) **(c) :** In region II or between the plates , the electric field

$$E_{II} = E_A - E_B = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma(\sigma_A \text{ or } \sigma_B)}{\varepsilon_0} = \frac{17.0\times10^{-22}}{8.85\times10^{-12}}$$

$E = 1.9 \times 10^{-10}\ \text{N C}^{-1}$

14. Gauss's theorem of electrostatics is equivalent of Coloumb's law and superposition principle.

15. The SI unit of electric field intensity is N/C.

Torque, $\tau = PE\sin\theta$

16. Electrostatic forces are conservative.

17. Lightning is an example of electric discharge.

18. It experiences same net force and same net torque.

OR

Electric line of force is curved in the field of more than one point charges.

19. $q_1 = 2\ \mu C$, $q_2 = 6\ \mu C$, $F = 12$ N

$q'_1 = 2 - 4 = -2\ \mu C$, $q'_2 = 6 - 4 = -2\ \mu C$, $F' = ?$

$$\therefore \quad \frac{F'}{F} = \frac{q_1' q_2'}{q_1 q_2} = \frac{(-2)(2)}{(2)(6)} = \frac{-1}{3}$$

So, $F' = -12/3 = -4$ N (attractive).

20. Properties of electric charges are following:

(i) Like charges repel each other and unlike charges attract each other.

(ii) Charges are additive in nature *i.e.*, $Q = q_1 + q_2 + ... + q_n$

(iii) A charge is a conserved quantity *i.e.*, net charge $q = 0$

(iv) Electric charge is quantised *i.e.*, $Q = n \times e$.

21. No, the direction of electric field is from positive to negative charge so line of force can be regarded starting from positive charge and ending on a negative charge. Thats why lines of force cannot form closed loops.

OR

If charged particle was initially at rest, it will move along the direction of electric field. If initial velocity of the charged particle makes a certain angle with electric field, then the charged particles will not move along the line of force.

22. The intensity of electric field at a distance r meter in vacuum from a charge q coulomb is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

Here q (proton charge) = 1.6×10^{-19} C, $r = 0.53 \times 10^{-10}$ m and $1/4\pi\varepsilon_0 = 9.0 \times 10^9$ N m^2/C^2.

$$\therefore \ E = \left(9.0 \times 10^9 \frac{\text{N m}^2}{\text{C}^2}\right) \times \frac{1.6 \times 10^{-19}\text{C}}{(0.53 \times 10^{-10}\text{m})^2}$$

$= 5.13 \times 10^{11}$ N C^{-1}.

23. $$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q(Q-q)}{r^2}$$

As Q and r fixed, so F is a function of q.

force F will be maximum, only if $\frac{dF}{dq} = 0$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{d}{dq}[q(2Q-q)] = 0$$

$\frac{d}{dq}[q(2Q-q)] = 0$ or $q(-1) + 2Q - q = 0$

$2Q - 2q = 0$ or $2Q = 2q$; $\frac{Q}{q} = 1$

OR

Total flux through the sphere S_1, $\phi_1 = \frac{Q}{\varepsilon_0}$

Charge on the line enclosed by the sphere S_2

$$q = \int_0^l \lambda dx = \int_0^l kx dx = k\frac{l^2}{2}$$

Total charge enclosed by the sphere S_2, $Q' = Q + k\frac{l^2}{2}$

Total flux through the sphere S_2, $\phi_2 = \dfrac{Q + k\frac{l^2}{2}}{\varepsilon_0}$

24. The electric field strength at any point is the strength of electric field at that point.

If $\vec{F}$ is the force acting on a small test charge $+q_0$ at any $\vec{r}$, then electric field strength at that point is $\vec{E} = \frac{\vec{F}}{q_0}$

$$\vec{F} = \vec{E}q_0$$

25. Absolute permittivity. It is defined as the measure of permittivity in a vaccuum. It is denote by ε_0.

$$F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

S.I. unit of $\varepsilon_0 = \frac{1}{\text{N}} \cdot \frac{\text{C.C}}{m^2} = \text{C}^2\,\text{N}^{-1}\text{m}^{-2}$

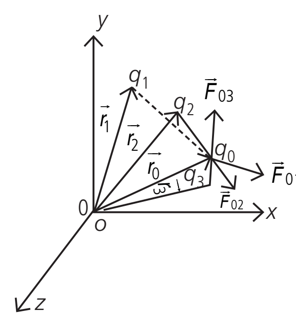
Its value in free space is 8.85×10^{-12} C^{2}N^{-1}m^{-2}.

Dielectric Constant : Dielectric constant for a medium is the ratio of absolute electrical permittivity of the medium to the absolute electric permittivity of free space. It is denoted by 'K'. Its value depends only on the nature of medium.

Relation between absolute permittivity and dielectric constant is given by,

$$\frac{\varepsilon}{\varepsilon_0} = K \Rightarrow \varepsilon = \varepsilon_0 K$$

26. According to the superposition principle, total force on any charge due to a number of other charges at rest is the vector sum of all the forces on that charge due to other charges, taken one at a time. The forces due to individual charges are unaffected by the presence or absence of other charges.



Let there are q_1, q_2 ...... q_n point charges situated at points with position vectors $\vec{r}_1, \vec{r}_2 .... \vec{r}_n$ with respect to origin O.

The total force $\vec{F}_0$ on a test charge q_0 at position vector $\vec{r}_0$ due to all n charges can be written as

$$\vec{F}_0 = \vec{F}_{O_1} + \vec{F}_{O_2} + ... + \vec{F}_{O_n}$$

Here, $\vec{F}_{O_1}$ is force on q_0 due to $q_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_0 q_1}{r_{10}^2}\hat{r}_{10}$

and $\vec{F}_{O_2}$ is force on q_0 due to $q_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_0 q_2}{r_{20}^2}\hat{r}_{20}$

Similarly $\vec{F}_{O_n}$ is force on q_0 due to $q_n = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_0 q_n}{r_{n0}^2}\hat{r}_{n0}$

$$\text{so, } \vec{F}_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_0 q_1}{r_{10}^2}\hat{r}_{10} + \frac{q_0 q_2}{r_{20}^2}\hat{r}_{20} + ... + \frac{q_0 q_n}{r_{no}^2} \cdot \hat{r}_{n0}$$

$$\therefore \quad \vec{F}_0 = \frac{1}{4\pi\varepsilon_0} \cdot \sum_{i=1}^{n} \frac{q_i}{|\vec{r}_0 - \vec{r}_i|^3}(\vec{r}_0 - \vec{r}_i)$$

27. Suppose the charge q is placed in equilibrium between the charges $+9e$ and $+e$ at a distance x from $+9e$. In addition to the charge q_1 the free charges $+9e$ and $+e$ are also be in equilibrium. Due to the charge $+e$, a repulsive force F will act on the charge $+9e$. Hence, for its equilibrium, the force F' on it due to the charge q must be and 'opposite' to the force F. This is possible only when the charge q is negative. The same is necessary for the equilibrium of the charge $+e$.

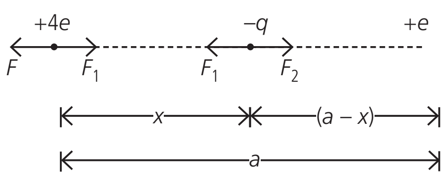
$$F = \frac{1}{4\pi\varepsilon_0}\frac{9e \times e}{a^2}.$$

Similarly, the force (of attraction) on the charge $+9e$ due to the charge $-q$ is

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{9e \times q}{x^2}.$$

For the equilibrium of charge +9e, $F = F'$.

$$\therefore \quad \frac{9e^2}{a^2} = \frac{9eq}{x^2}. \qquad \text{...(i)}$$



For the equilibrium of the charge $-q$, the force (of attraction) F_1 due to the charge $+4e$ on it must be equal to the force (of attraction) F_2 due to the charge $+e$, *i.e.*,

$$F_1 = F_2$$

or $$\frac{1}{4\pi\varepsilon_0}\frac{9e \times q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{e \times q}{(a-x)^2} \quad \text{or} \quad \frac{9eq}{x^2} = \frac{eq}{(a-x)^2}$$

$$\therefore \quad x = \frac{3}{4}a$$

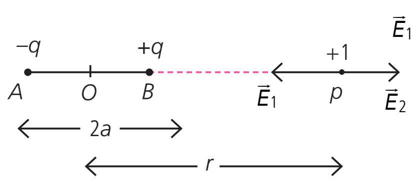
Substituting this value of x in eq. (i), and solving, we get

$$q = \frac{9}{16}e$$

For the equilibrium of the entire system, a negatively charge of magnitude $\frac{9}{16}e$ should be placed at a distance of $\frac{3}{4}a$ from the charge +9e. In this condition the net force on each charge will be zero. The equilibrium is unstable.

OR

Consider an electric dipole consisting of two point charges $-q$ and $+q$ separated by a small distance $2a$. We have to calculate electric field intensity $\vec{E}$ at a point P on the axis line of the dipole, and at a distance $OP = r$ from the centre O of the dipole, as shown,



If $\vec{E}_1$ is the electric intensity at P due to charge $-q$ at A. then

$$|\vec{E}_1| = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2} \text{ ; is along } PA$$

Suppose $\vec{E}_2$ is the electric intensity at P due to charge $+q$ at B, then

$$|\vec{E}_2| = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} \text{ ; along } BP \text{ produced.}$$

As $\vec{E}_1$ and $\vec{E}_2$ are collinear vectors acting in opposite directions and $|\vec{E}_2| > |\vec{E}_1|$ therefore, the resultant intensity $\vec{E}$ at P will be difference of two, acting along BP produced.

$$|\vec{E}| = |\vec{E}_2| - |\vec{E}_1| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2-(r-a)^2}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{4ar}{(r^2-a^2)^2}\right] = \frac{q \times 2a \times 2r}{4\pi\varepsilon_0 (r^2-a^2)^2}$$

But $q \times 2a = |\vec{p}|$, the dipole moment

$$\therefore \quad |\vec{E}| = \frac{|\vec{p}|}{4\pi\varepsilon_0}\cdot\frac{2r}{(r^2-a^2)^2}$$

If dipole is short, $2a << r$, then $|\vec{E}| = \frac{|\vec{p}|}{4\pi\varepsilon_0}\cdot\frac{2r}{r^4} = \frac{2|\vec{p}|}{4\pi\varepsilon_0 r^3}$

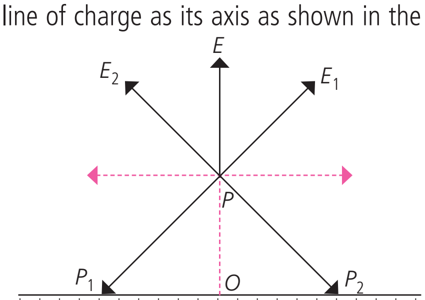
The direction of $\vec{E}$ is along BP produced *i.e.*, along the direction of $\vec{p}$.

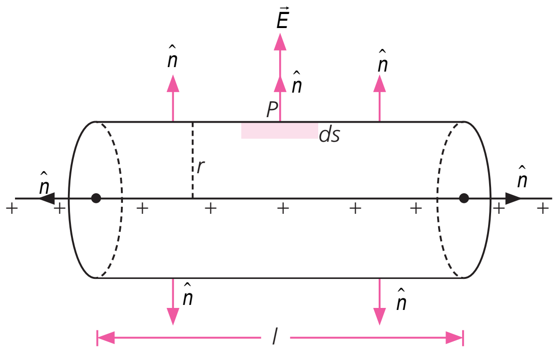
Clearly, $|\vec{E}| \propto \frac{1}{r^3}$

28. Consider an infinitely long thin wire with uniform linear charge density λ. To calculate field due to this wire at any point P, we consider a pair of line elements P_1 and P_2 of the wire at equal distances on either side of an arbitrary origin O.

Electric fields $\vec{E}_1$ and $\vec{E}_2$ at point P due to the two line elements are shown in figure. Their components normal to the radius vector $OP = r$ cancel out being equal and opposite. However, the components along OP add. Therefore, resultants electric field is radial. This is true for any such pair of line elements. Therefore, total field at any point P is radial. As the wire is infinite, electric field does depend upon distance of point from the wire, but not on the position of P. Thus, electric field at every point in the plane cutting the wire normally is radial and its magnitude depends only on the radial distance r.

Let us consider a right circular closed cylinder of radius r and length l with the infinitely long line of charge as its axis as shown in the figure. The magnitude of electric intensity $\vec{E}$ at every point on the curved surface of the cylinder is the same, because all such points are the same distance from the line charge.





Also, $\vec{E}$ and unit vector $\hat{n}$ along outward normal to curved surface are in the same directions, so that $\theta = 0°$.

∴ Electric flux over the curved surface of the cylinder,

$$\oint_S \vec{E}.\vec{ds} = \oint_S \vec{E}.\hat{n}\,ds = \oint_S E(1)\cos 0° ds = E\oint_S ds = E(2\pi rl)$$

where $(2\pi\, rl)$ is area of the curved surface of the cylinder.

On the ends of the cylinder, angle between electric field intensity $\vec{E}$ and outward normal $\hat{n}$ is 90°.

Therefore, these ends make no contribution to electric flux of the cylinder.

∴ Total electric flux over the whole cylinder, $\phi_E = E(2\pi\, rl)$

Charge enclosed in the cylinder

= linear charge density × length

$q = \lambda l$

According to Gauss's theorem, $\phi_E = \frac{q}{\varepsilon_0}$ ∴ $E(2\pi rl) = \frac{\lambda l}{\varepsilon_0}$

$E = \frac{\lambda}{2\pi\varepsilon_0 r}$ ∴ $E \propto \frac{1}{r}$

29. Electric field due to line charge, $E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2\lambda}{4\pi\varepsilon_0 r}$

force exerted by this field on electron

$$F = qE = \frac{e.2\lambda}{4\pi\epsilon_0 r}$$

force due to electric field = centripetal force

$$F = \frac{mv^2}{r}$$

or $\frac{e.2\lambda}{4\pi\varepsilon_0 r} = \frac{mv^2}{r}$ or $v^2 = \frac{e.2\lambda}{4\pi\varepsilon_0 m}$

$$\therefore \text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}\text{m}\left(\frac{2\lambda e}{4\pi\varepsilon_0 m}\right) = \frac{e\lambda}{4\pi\varepsilon_0}$$

$= 9 \times 10^9 \times 1.6 \times 10^{-19} \times 2 \times 10^{-8} = 2.88 \times 10^{-17}$ J

30. (a) It is the property by virtue of which the charge on a body is integral multiple of a basic unit of charge of an electron and it is represented by e. Thus, the charge of a body is always given by

$$q = ne$$

Where, n is any integer (positive or negative).

The value of basic unit of charge is the charge that an electron or proton carries. By convention, the charge on an electron is taken to be negative and is written as '$-e$' and charge on an proton is taken to be positive and is written as '$+e$'. Here $e = 1.6 \times 10^{-19}$ C. SI unit of charge is coulomb (C).

(b) It is the property by virtue of which total charge of an isolated system always remains constant or conserved. For example, if we rub two insulating bodies, one body gains the charge. So, it is not possible to create or destroy net charge carried by any isolated system. Charges can be created or destroyed in equal and unlike pairs only. For example, in the phenomenon of pair production, X-ray photon converts into an electron and a positron having total charge $- e + e = 0$.

$y = e^- + e^+$ (pair production)

In annihilation of matter, an electron and a positron combine with each other to produce two γ-ray photons with zero charge.

$e^- + e^+ = y + y$ (annihilation)

We have said that magnitude of charge on a body does not change, whatever be the speed of the charge. Also, magnitude of charge on a body does not depend on speed of observer.

i.e., charge at rest = charge in motion

31. **(a) :** $E = \frac{\sigma}{4\pi\varepsilon_0}(2\pi)\left[1 - \frac{x}{\sqrt{R^2 + x^2}}\right]$

$= 9 \times 10^9 \times 10 \times 10^{-9} \times 6.28\left[1 - \frac{2}{\sqrt{4+9}}\right]$

∴ $E = 90 \times 6.28\left[1 - \frac{2}{\sqrt{13}}\right] = 251$ N/C

32. The negative charge on the drop is

$q = 12\,e = 12 \times (16 \times 10^{-19}) = 1.92 \times 10^{-18}$ C

The density of oil is $\rho = 1.26$ g cm^{-3} = 1.26×10^4 kg m^{-1}.

The mass of the drop is m = volume × density = $\frac{4}{3}\pi r^3 \rho$,

where r is the radius of the drop in meter. The electric field is $E = 2.00 \times 10^4$ N C^{-1}.

The field E is directed vertically downwards so that the negatively charge drop experiences a upward electric force qE which balances the downward gravity force mg on the drop. Thus,

$$qE = mg = \frac{4}{3}\pi r^3 \rho g \quad \text{or} \quad r = \left(\frac{3qE}{4\pi\rho g}\right)^{1/3}$$

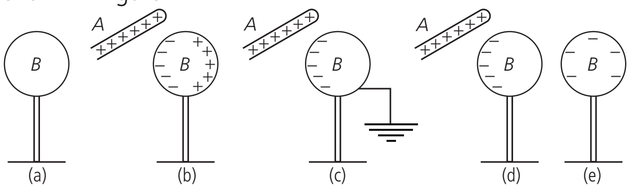
Substituting the given values, we get

$$r = \left(\frac{3\times(1.92\times10^{-18})\times2.00\times10^4}{4\times3.14\times(1.30\times10^3)\times9.8}\right)^{1/3} \text{m}$$

$= (0.719 \times 10^{-18})^{1/3}$ m $= 0.895 \times 10^{-6}$ m

33. When a charged body A is brought near an uncharged metallic body B, then equal charge of opposite sign appears on the near face of body B and an equal charge of same sign appears on the farther face of body B. The toal charge on body B is zero. In this process, body A does not lose any charge as if is not in direct contact with body B. This phenomenon of charging body B without actual contact is called charging by induction.

The steps involve in charging a metallic sphere by induction are shown in figure.



Let sphere B on an insulating stand is uncharged as shown in figure (a).

When a positively charged glass rod A is brought near the uncharged metallic sphere, free electrons of the sphere are attached and start piling up at the near end. This end therefore, becomes negatively charged and the farther end of the sphere becomes positively charged to the same extent. Due to deficit of electrons the sphere on the whole remains neutral as shown in figure (b). The redistribution of charge is almost instantaneous and stops as soon as net force on free electrons in the metallic sphere becomes zero.

When the sphere is grounded, *i.e.,* it is connected to earth by a conducting wire, electrons flow from the ground to the sphere and neutralise the positive charge on the father end of the sphere. The negative charge at the near end of the sphere remains bound there due to attractive force of glass rod as shown in figure (c).

When the sphere is disconnected from the ground, the negative charge continues to be held on the near end as shown in figure.

When the glass rod is removed, the negative charge spreads uniformly over the sphere B as shown in figure.

Thus, phenomenon of charging an uncharged conducting body, by bringing a charged body near it, without making a direct contact between the two bodies is called charging by induction.

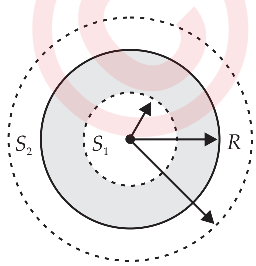
In this process of electrical induction, the positively charged glass rod does not lose any charge.

OR

(a) The given charge density distribution of the sphere of radius R is

$\rho(r) = kr$ for $r \le R$

$= 0$ for $r > R$



(i) For point $r < R$

let us consider a spherical Gaussian surface S_1 of radius r. Then on the surface,

$$\oint \vec{E}\cdot\overrightarrow{dS} = \frac{1}{\epsilon_0}\int \rho\, dV$$

As $\quad V = \frac{4}{3}\pi r^3$, $dV = 4\pi r^2 dr$ and $\rho(r) = kr$

$$\therefore \quad \oint \vec{E}\cdot\overrightarrow{dS} = \frac{1}{\epsilon_0}4\pi k\int_0^r r\, r^2\, dr\ ;\ (E)4\pi r^2 = \frac{4\pi k}{\epsilon_0}\frac{r^4}{4}$$

$$\vec{E} = \frac{1}{4\epsilon_0}kr^2\hat{r} \qquad \ldots \text{(i)}$$

The direction of $\vec{E}$ is radially outwards (for positive charge density)

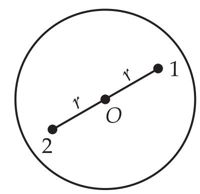
(ii) For points $r > R$, let us consider a spherical Gaussian surface S_2 of radius r. Then $\oint \vec{E}\cdot\overrightarrow{dS} = \frac{1}{\epsilon_0}\int \rho \cdot dV$

$$E(4\pi r^2) = \frac{4\pi k}{\epsilon_0}\int_0^R r^3 dr = \frac{4\pi k}{\epsilon_0}\frac{R^4}{4} \qquad \ldots \text{(ii)}$$

$$\vec{E} = \frac{k}{4\varepsilon_0}\frac{R^4}{r^2}\hat{r}$$

The direction of $\vec{E}$ is radially outwards (for positive charge density).

(b) From symmetry, we find that the two protons must be on the opposite sides of the centre, along a diameter of the sphere as shown in figure.



Proceeding as above, charge on the sphere,

$$q = \int_0^R \rho\, dV = \int_0^R (kr)\ 4\pi r^2\, dr\ ;\ q = 4\pi k\frac{R^4}{4} = 2e$$

$$\therefore \quad k = \frac{2e}{\pi R^4} \qquad \ldots \text{(iii)}$$

If protons 1 and 2 are embedded at distance r from the centre of the sphere as shown, then attractive force on proton 1 due to charge distribution is

$$F_1 = -eE = -e\frac{kr^2}{4\epsilon_0} \qquad \ldots \text{using (i)}$$

Repulsive force on proton 1 due to proton 2 is

$$F_2 = \frac{e^2}{4\pi\epsilon_0 (2r)^2} = \frac{e^2}{16\pi\varepsilon_0 r^2}$$

Net force on proton 1

$$F = F_1 + F_2$$

$$F = -e\frac{kr^2}{4\epsilon_0} + \frac{e^2}{16\pi\epsilon_0 r^2}$$

Using (iii), $F = \left[-\frac{er^2}{4\epsilon_0}\frac{2e}{\pi R^4} + \frac{e^2}{16\pi\epsilon_0 r^2}\right] = 0$

This force on proton 1 will be zero, when

$$\frac{er^2.2e}{4\epsilon_0\pi R^4} = \frac{e^2}{16\pi\epsilon_0 r^2} \quad \text{or} \quad r^4 = \frac{R^4}{8} \quad \text{or} \quad r = \frac{R}{(8)^{1/4}}$$

This is the distance of each of the two protons from the centre of the sphere.

34. (a) From Coulomb's law $\left(F = \frac{Qq}{r^2}\right)$, in cgs system,

$$1 \text{ dyne} = \frac{(1\,\text{esu of charge})\,(1\,\text{esu of charge})}{(1\,\text{cm})^2}$$

hence, 1 esu of charge = 1 $(\text{dyne})^{1/2}$ (cm)

Further, [esu of charge] = $[F]^{1/2}[L] = [MLT^{-2}]^{1/2}[L] = [M^{1/2}L^{3/2}T^{-1}]$
Thus, esu of charge is given in terms of fractional powers (1/2 and 3/2) of M and L, respectively.

(b) Since 1 dyne = 10^{-5} N and 1 esu of charge = x C (given),

$$10^{-5}\,\text{N} = \frac{1}{4\pi\epsilon_0}\frac{(x\,\text{C})^2}{(10^{-2}\,\text{m})^2} \qquad \text{(using SI units)}$$

or $$\frac{1}{4\pi\epsilon_0} = \frac{10^{-9}}{x^2}\ \text{N m}^2\,\text{C}^{-2}$$

When $x = \dfrac{1}{[3]\times 10^9}$

we get, $$\frac{1}{4\pi\varepsilon_0} = 10^{-9}\times[3]^2\times 10^{18}\ \text{N m}^2\,\text{C}^{-2}$$

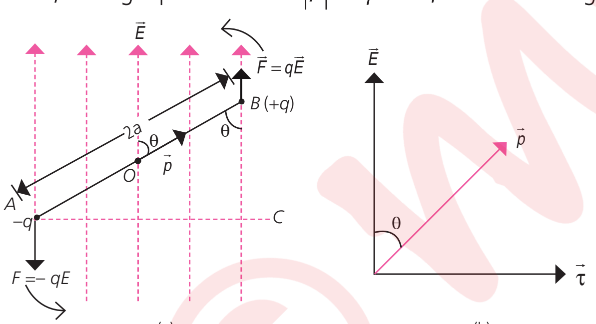
or $$\frac{1}{4\pi\varepsilon_0} = [3]^2\times 10^9\ \text{Nm}^2\,\text{C}^{-2}$$

With [3] ⟶ 2.99792458, we obtain

$$\frac{1}{4\pi\varepsilon_0} = (2.99792458)^2\times 10^9$$
$$= 8.98755\times 10^9\ \text{N m}^2\,\text{C}^{-2}\text{(exactly)}.$$

OR

Consider an electric dipole consisting of two equal and opposite point charges $-q$ at A and $+q$ at B separated by a small distance $AB = 2a$, having dipole moment $|\vec{p}| = q\times 2a$, as shown in figure.



(a) (b)

Let this dipole be held in a uniform external electric field $\vec{E}$ at an angle θ with the direction of $\vec{E}$.

Force on charge + q at $A = -q\,\vec{E}$, along the direction of $\vec{E}$.

Force on charge $-q$ at $B = q\,\vec{E}$, in a direction opposite to $\vec{E}$.

Since $\vec{E}$ is uniform, therefore, net force on the dipole is $(qE - qE) = 0$. However, as the forces are equal, unlike and parallel, acting at different points, therefore, they form a couple which rotates the dipole in the anticlockwise direction, as shown in figure. Thus, the couple tends to align the dipole axis along the direction of field $\vec{E}$.

Draw $AC \perp \vec{E}$

∴ perpendicular distance between the forces = arm of couple = AC

As torque = movement of the couple

$\vec{\tau}$ = force × arm of couple

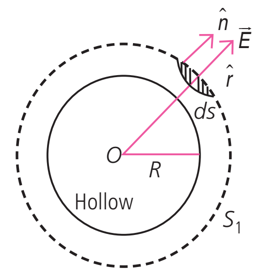
$\vec{\tau} = \vec{F}\times AC = \vec{F}\times AB\sin\theta = \vec{F}\times 2\vec{a}\sin\theta = (q\vec{E})\times 2\vec{a}\sin\theta =$ $(q\times 2\vec{a})\,\vec{E}\sin\theta$

As $q\times 2\vec{a} = \vec{p}$, therefore, $\tau = pE\sin\theta$

∴ In vector form, we can rewrite this eqn. as $\vec{\tau} = \vec{p}\times\vec{E}$

The direction of $\vec{\tau}$ is given by right hand screw rule and is perpendicular to $\vec{p}$ and $\vec{E}$, *i.e.*, perpendicular to the plane of the paper and outwards.

35. Consider a thin spherical shell of radius R with centre O. Let a charge $+q$ be distributed uniformly over the surface of the shell. To calculate electric field intensity at any point P, where $OP = r$, imagine a sphere S_1 with centre O and radius r as shown in the figure. The surface of this sphere is a Gaussian surface at every point of which electric intensity $\vec{E}$ is the same, directed radially outwards (as) is unit vector $\hat{n}$, So, that θ = 0°.



According to Gauss's theorem

$$\oint_S \vec{E}.\vec{ds} = \oint_S \vec{E}.\hat{n}\,ds = \frac{q}{\varepsilon_0} \quad\text{or}\quad E\oint_S ds = \frac{q}{\varepsilon_0}$$

∴ $$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$
$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

This is exactly the field produced by a charge q placed at the centre O. Hence we conclude that for points outside the spherical shell, the field due to uniformly charged shell is as if the entire charge of the shell is concentrated at the centre of the shell.

At a point on the surface of the shell

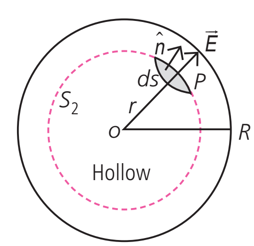
$r = R,\ E = q/4\pi\varepsilon_0 R^2$ = Maximum

If σ is surface density of charge on the shell, then

$q = 4\pi R^2\,.\,\sigma$

∴ $$E = \frac{4\pi R^2\sigma}{4\pi\varepsilon_0 R^2} = \frac{\sigma}{\varepsilon_0} = \text{constant},\ \textit{i.e.},\ E = \frac{\sigma}{\varepsilon_0};$$ this is the expression for field outside the shell.

In figure, the point P where we have to find electric intensity is inside the shell. The Gaussian surface is the surface of a sphere S_2 passing through P and with centre at O. The radius of sphere S_2 is $r < R$.



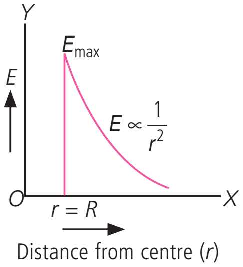
The electric flux through the Gaussian surface, as calculated above is $E\times 4\pi r^2$. As charge inside a spherical shell is zero, the Gaussian surface enclose no charge. The Gauss's theorem gives

$$E\times 4\pi r^2 = \frac{q}{\varepsilon_0} = 0$$

∴ $E = 0$ for $r < R$.

Hence, the field due to uniformly charged spherical shell is zero at all points inside the shell.

The variation of electric field intensity E with distance from the centre of a uniformly charged spherical shell is shown in figure.



OR

(a) Total charge inside the surface, $q = -6.7\ \mu C + 3.2\ \mu C$

$= -3.5\ \mu C$

$\therefore \quad q = -3.5\ \mu C$;

$$\phi_E = \frac{q}{\varepsilon_0} = \frac{-3.5\times10^{-6}}{8.854\times10^{-12}} = -3.95\times10^{5}\ \text{N m}^2/\text{C}$$

ϕ_E is negative, so the flux is inwards.

(b) The absence of electric field inside charged conductors means that the electric field lines cannot enter the inner empty space of any hollow conductor. Thus, conductors can acts as electrostatics sheilds. In order to save an electrical instruments from external fields, the instrument may be external fields will pass along outside of the surface, they will not enter inside and the process is known as electrostatic shielding.

(c) Area (100 m^2) in xy plane so area vector in $\hat{k}$

So flux $= \int \vec{E}_z \cdot d\vec{S} = \sqrt{3}\hat{k}.100\hat{k} = 173.2$

$= \int \vec{E}_z \cdot d\vec{S} = \sqrt{3}\hat{k}.100\hat{k} = 173.2$