

# Electric Charges and Fields



## ANSWERS

### Topic 1

1. (a) The net charge possessed by a body is an integral multiple of charge of an electron *i.e.*,  $q = \pm ne$ , where  $n = 0, 1, 2, 3, \dots$  is the number of electrons lost or gained by the body and  $e = 1.6 \times 10^{-19}$  C is charge of an electron. This is called law of quantization of charge.

(b) At macroscopic level, charges are enormously large as compared to the charge of an electron,  $e = 1.6 \times 10^{-19}$  C. Even a charge of 1 nC contains nearly  $10^{13}$  electronic charges. So, at this large scale, charge can have a continuous value rather than discrete integral multiple of  $e$ , and hence, the quantization of electric charge can be ignored.

2. When a glass rod is rubbed with a silk cloth, electrons from the glass rod are transferred to the piece of silk cloth. Due to this, the glass rod acquires positive (+) charge whereas the silk cloth acquires negative (–) charge. Before rubbing, both the glass rod and silk cloth are neutral and after rubbing the net charge on both of them is also equal to zero. Such similar phenomenon is observed with many other pairs of bodies. Thus, in an isolated system of bodies, charge is neither created nor destroyed, it is simply transferred from one body to the other. So, it is consistent with the law of conservation of charge.

$$3. (a) q = ne \text{ or } n = \frac{q}{e} = \frac{3 \times 10^{-7}}{1.6 \times 10^{-19}} = 1.875 \times 10^{12}$$

so,  $1.875 \times 10^{12}$  electrons have transferred from wool to polythene, as polythene acquires negative charge.

(b) Yes, mass of  $1.875 \times 10^{12}$  electrons *i.e.*,  $m = nm_e = 1.875 \times 10^{12} \times 9.1 \times 10^{-31}$  kg  $= 1.71 \times 10^{-18}$  kg has transferred from wool to polythene.

$$4. \text{ As } u = +\frac{2}{3}e \text{ and } d = -\frac{1}{3}e$$

$$\text{Charge of proton} = +e = \frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e$$

So, configuration of proton is 'uud'

$$\text{Charge of neutron} = +\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e$$

So, configuration of neutron is 'udd'.

### Topic 2

$$1. F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = 9 \times 10^9 \times \frac{2 \times 10^{-7} \times 3 \times 10^{-7}}{(30 \times 10^{-2})^2}$$

or  $F = 6 \times 10^{-3}$  N (repulsive in nature, as the two charges are like charges.)

$$2. (a) \text{ Given, } F_{12} = 0.2 \text{ N, } q_1 = 0.4 \times 10^{-6} \text{ C, } q_2 = -0.8 \times 10^{-6} \text{ C}$$

$$\text{As, } F_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\text{or } r^2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{F_{12}} = 9 \times 10^9 \times \frac{0.4 \times 10^{-6} \times 0.8 \times 10^{-6}}{0.2}$$

$$\text{or } r^2 = 14.4 \times 10^{-3} = 1.44 \times 10^{-2}$$

$$\text{or } r = 1.2 \times 10^{-1} \text{ m} = 12 \text{ cm}$$

(b) By Newton's third law,  $F_{21} = F_{12} = 0.2$  N attractive

3. The ratio of electrostatic force to the gravitational force between an electron and a proton separated by a distance  $r$  from each other is

$$\frac{F_{\text{electrostatic}}}{F_{\text{gravitational}}} = \frac{Ke \cdot e / r^2}{Gm_e m_p / r^2} = \frac{Ke^2}{Gm_e m_p}$$

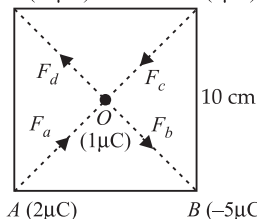
$$\text{So, } \left[ \frac{Ke^2}{Gm_e m_p} \right] = \left[ \frac{F_{\text{electrostatic}}}{F_{\text{gravitational}}} \right] = \frac{\text{MLT}^{-2}}{\text{MLT}^{-2}} = 1$$

Thus, the ratio  $\frac{Ke^2}{Gm_e m_p}$  is dimensionless.

$$\frac{Ke^2}{Gm_e m_p} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \text{ kg} \times 1.67 \times 10^{-27} \text{ kg}} = 0.23 \times 10^{40} = 2.3 \times 10^{39}$$

This ratio signifies that electrostatic forces are  $10^{39}$  times stronger than gravitational forces.

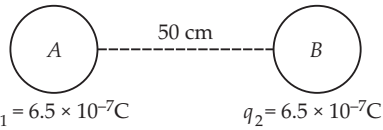
$$4. (a) \begin{matrix} D (-5\mu\text{C}) & C (2\mu\text{C}) \\ & \nearrow & \nwarrow \\ & O (1\mu\text{C}) & \\ & \nwarrow & \nearrow \\ A (2\mu\text{C}) & B (-5\mu\text{C}) \end{matrix}$$



Forces of repulsion on  $1 \mu\text{C}$  charge at  $O$  due to  $2 \mu\text{C}$  charge, at  $A$  and  $C$  are equal and opposite. Therefore, they cancel. Similarly, forces of attraction on  $1 \mu\text{C}$  charge at  $O$ , due to  $-5 \mu\text{C}$  charges at  $B$  and at  $D$  are also equal and opposite. Therefore, these also cancel.

Hence, the net force on the charge of  $1 \mu\text{C}$  at  $O$  is zero.

5. (a)  $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$



or  $F = 9 \times 10^9 \frac{(6.5 \times 10^{-7})^2}{(0.5)^2}$

or  $F = 1.520 \times 10^{-5} \text{ N}$

(b)  $q'_1 = 2q_1, q'_2 = 2q_2$  and  $r' = \frac{r}{2}$

So,  $F' = \frac{1}{4\pi\epsilon_0} \frac{q'_1 \times q'_2}{(r')^2} = \frac{1}{4\pi\epsilon_0} \frac{2q_1 \times 2q_2}{\left(\frac{r}{2}\right)^2}$

or  $F' = 16 \times 1.520 \times 10^{-5} = 24.32 \times 10^{-5} \text{ N}$

or  $F' = 2.432 \times 10^{-3} \text{ N}$ .

6.  $\textcircled{A} \textcircled{C} \quad \textcircled{B}$   
 $q_1 = 6.5 \times 10^{-7} \text{ C}, \quad q_2 = 6.5 \times 10^{-7} \text{ C}$

$\textcircled{A} \quad \textcircled{C} \textcircled{B}$   
 $q'_1 = 3.25 \times 10^{-7} \text{ C}, \quad q_{\text{net}} = (3.25 + 6.5) \times 10^{-7} \text{ C}$   
 $= 9.75 \times 10^{-7} \text{ C}$

$\textcircled{A} \quad \textcircled{B}$   
 $q''_1 = 3.25 \times 10^{-7} \text{ C}, \quad q''_2 = 4.875 \times 10^{-7} \text{ C}$

The new force of repulsion between A and B is

$F' = \frac{1}{4\pi\epsilon_0} \frac{q''_1 \times q''_2}{(r'')^2}$

or  $F' = 9 \times 10^9 \times \frac{3.25 \times 10^{-7} \times 4.875 \times 10^{-7}}{(0.5)^2}$

or  $F'' = 5.7 \times 10^{-3} \text{ N}$

### Topic 3

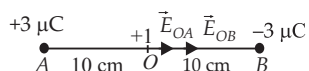
1. A field line cannot have sudden breaks because the moving test charge never jumps from one position to the other.

(b) Two field lines never cross each other at any point, because if they do so, we will obtain two tangents pointing in two different directions of electric field at a point, which is not possible.

2. (a) Electric fields at O due to the charges at A and B are

$E_{OA} = E_{OB} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = 9 \times 10^9 \times \frac{3 \times 10^{-6} \text{ C}}{(20/2 \times 10^{-2} \text{ m})^2}$

or  $E_{OA} = E_{OB} = 27 \times 10^5 \text{ N C}^{-1}$



As, they are directed in same direction, so net electric field at midpoint O is

$E = E_{OA} + E_{OB} = 2E_{OB} = 2 \times 27 \times 10^5$

or  $E = 5.4 \times 10^6 \text{ N C}^{-1}$  directed along OB.

(b) When test charge  $q_0 = -1.5 \times 10^{-9} \text{ C}$  is placed at point O, it experiences a force

$F = q_0 E = 1.5 \times 10^{-9} \times 5.4 \times 10^6$

or  $F = 8.1 \times 10^{-3} \text{ N}$

In direction opposite to that of  $E$  i.e., along OA.

3. (a) Area of square,  $A = a^2 = 10^2 = 100 \text{ cm}^2$   
 $= 100 \times 10^{-4} \text{ m}^2 = 10^{-2} \text{ m}^2$

As the plane is along yz plane, so area vector  $\vec{A}$  is directed along x-axis

i.e.,  $\vec{A} = (10^{-2} \hat{i}) \text{ m}^2$

∴ Electric Flux through the square is

$\phi = \vec{E} \cdot \vec{A} = (3 \times 10^3 \hat{i}) \cdot (10^{-2} \hat{i})$

or  $\phi = 30 \text{ V-m}$

(b) When normal to plane i.e.,  $\vec{A}$  makes an angle of  $60^\circ$  with  $\vec{E}$ , then

$\phi = EA \cos 60^\circ = 3 \times 10^3 \times 10^{-2} \times 1/2 = 1.5 \times 10$

or  $\phi' = 15 \text{ V-m}$ .

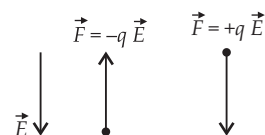
4.  $\phi_{\text{net}} = 0$ , As the net electric flux with closed surface like cube in uniform electric field is equal to zero, because the number of lines entering the cube is the same as the number of lines leaving the cube.

5. Torque on dipole is,  $\tau = pE \sin 30^\circ$

or  $\tau = 4 \times 10^{-9} \times 5 \times 10^4 \times \frac{1}{2}$  or  $\tau = 10 \times 10^{-5}$

or  $\tau = 1 \times 10^{-4} \text{ N m}$ .

6. Particles 1 and 2 are negatively charged as they experience forces in direction opposite to that of electric field  $\vec{E}$ , whereas particle 3 is positively charged as it experience force in the direction of electric field  $\vec{E}$ .



Particle-3 has the highest charge to mass ratio, as it shows maximum deflection in the electric field.

7. In equilibrium, force due to electric field on the drop balances the weight  $mg$  of drop

i.e.  $qE = mg$  or  $qE = V\rho g$

or  $qE = \frac{4}{3}\pi r^3 \rho g$  or  $r^3 = \frac{3qE}{4\pi \rho g} = \frac{3 \times 12e \times E}{4\pi \rho g}$

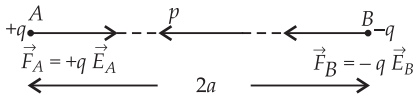
or  $r^3 = \frac{36 \times 1.6 \times 10^{-19} \times 2.55 \times 10^4}{4 \times 3.14 \times 1.26 \times 10^3 \times 9.8}$   
 $= 0.947 \times 10^{-18}$

or  $r = 0.981 \times 10^{-6} \text{ m} = 9.81 \times 10^{-7} \text{ m}$   
 $= 9.81 \times 10^{-4} \text{ mm}$ .

8. (a) It is wrong, because electric field lines must be normal to the surface of conductor outside it.

- (b) It is wrong because field lines cannot start or originate from negative charge, and also cannot end or submerge into positive charge.
- (c) It is correct.
- (d) It is wrong because electric field lines never intersect each other.
- (e) It is wrong because electric field lines cannot form closed loops.

9. As electric field increases in positive  $z$ -direction from  $A$  to  $B$ , So



$$E_B = E_A + \frac{dE}{dr} \cdot 2a$$

So, the net force on the electric dipole in electric field is

$$F_{\text{net}} = F_B - F_A = q(E_B - E_A)$$

$$\text{or } F_{\text{net}} = q \left[ E_A + \frac{dE}{dr} \cdot 2a - E_A \right]$$

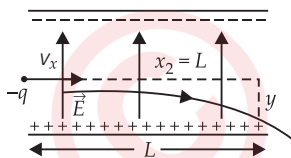
$$\text{or } F_{\text{net}} = q \cdot 2a \frac{dE}{dr} = p \cdot \frac{dE}{dr}$$

or  $F_{\text{net}} = 10^{-7} \times 10^5$  or  $F_{\text{net}} = 10^{-2} \text{ N}$  directed in direction of  $\vec{E}_B$  i.e., from  $B$  to  $A$  or along  $-z$  direction.

As the two forces on charges of electric dipole are collinear and opposite, so net torque on it is equal to zero.

10. Let the point at which the charged particle enters the electric field, be origin  $O(0, 0)$ , then after travelling a horizontal displacement  $L$ , it gets deflected by displacement  $y$  in vertical direction as it comes out of electric field.

So, co-ordinates of its initial position are  $x_1 = 0$  and  $y_1 = 0$  and final position on coming out of electric field are  $x_2 = L$  and  $y_2 = y$



Components of its acceleration are  $a_x = 0$

$$\text{and } a_y = \frac{F}{m} = \frac{qE}{m}$$

and of initial velocity are  $u_x = v_x$  and  $u_y = 0$

so, by 2<sup>nd</sup> equation of motion in horizontal direction,

$$y_2 - y_1 = u_x t + \frac{1}{2} a_x t^2$$

$$\text{or } L - 0 = u_x t + 0$$

$$t = \frac{L}{u_x} = \frac{L}{v_x} \quad (\because u_x = v_x)$$

and by 2<sup>nd</sup> equation of motion in vertical direction,

$$y_2 - y_1 = u_y t + \frac{1}{2} a_y t^2$$

$$\text{or } y - 0 = 0 + \frac{1}{2} \cdot \frac{qE}{m} \cdot \left( \frac{L}{v_x} \right)^2 \quad \text{or } y = \frac{qEL^2}{2mv_x^2}$$

This gives the vertical deflection of the particle at the far edge of the plate.

11. If the electron is released just near the negatively charged plate, then  $y = 0.5 \text{ cm}$  and hence

$$y = \frac{qEL^2}{2mv_x^2} \quad \text{or } L^2 = \frac{2mv_x^2 y}{qE}$$

$$\frac{2 \times 9.1 \times 10^{-31} \text{ kg} \times (2 \times 10^6 \text{ m s}^{-1})^2 \times 0.5 \times 10^{-2} \text{ m}}{1.6 \times 10^{-19} \text{ C} \times 9.1 \times 10^2 \text{ NC}^{-1}}$$

$$= 2.5 \times 10^{-4} \text{ m}$$

$$\Rightarrow L = 1.6 \times 10^{-2} \text{ m} = 1.6 \text{ cm}$$

## Topic 4

1. (a)  $\phi = 8 \times 10^3 \text{ N m}^2 \text{ C}^{-1}$

$$\text{or } \frac{q}{\epsilon_0} = 8 \times 10^3 \text{ N m}^2 \text{ C}^{-1}$$

$$\text{or } q = 8 \times 10^3 \epsilon_0 = 8 \times 10^3 \times 8.85 \times 10^{-12}$$

$$\text{or } q = 7.08 \times 10^{-8} \text{ C}$$

(b) As  $\phi = 0$

$$\text{or } \frac{q_{\text{net}}}{\epsilon_0} = 0 \quad \text{or } q_{\text{net}} = 0$$

So, the net charge enclosed by that closed surface is zero, although it may have some charges inside it.

2. Let us assume that the given square be one face of the cube of edge  $10 \text{ cm}$ .

As charge of  $+10 \mu\text{C}$  is at a distance of  $5 \text{ cm}$  above the centre of a square, so it is enclosed by the cube. Hence by Gauss's theorem, electric flux linked with the cube is

$$\phi = \frac{q}{\epsilon_0} = \frac{10 \times 10^{-6}}{8.85 \times 10^{-12}} = 1.13 \times 10^6 \text{ N m}^2 \text{ C}^{-1}$$

So, the magnitude of the electric flux through the square is

$$\phi_{\text{sq}} = \frac{\phi}{6} = \frac{1.13}{6} \times 10^6 \text{ or } \phi_{\text{sq}} = 1.9 \times 10^5 \text{ N m}^2 \text{ C}^{-1}$$

$$3. \quad \phi = \frac{q}{\epsilon_0} = \frac{2 \times 10^{-6}}{8.85 \times 10^{-12}} = 0.23 \times 10^6$$

$$\text{or } \phi = 2.3 \times 10^5 \text{ N m}^2 \text{ C}^{-1}.$$

4. (a) On increasing the radius of the Gaussian surface, charge enclosed by it remains the same and hence the electric flux linked with Gaussian surface also remains the same.

$$(b) \quad \phi = \frac{q}{\epsilon_0} \quad \text{or } q = \phi \epsilon_0$$

$$\text{or } q = -1 \times 10^3 \times 8.85 \times 10^{-12}$$

$$\text{or } q = -8.85 \times 10^{-9} \text{ C} = -8.85 \text{ nC}.$$

$$5. \quad R = \frac{2.4 \text{ m}}{2} = 1.2 \text{ m},$$

$$\sigma = 80.0 \mu \text{C m}^{-2} = 80 \times 10^{-6} \text{C m}^{-2}$$

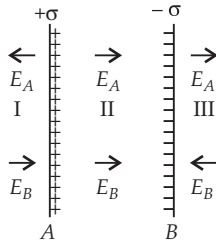
$$(a) \text{ As } \sigma = \frac{q}{4\pi R^2}$$

$$\text{So, } q = 4\pi R^2 \times \sigma = 4 \times 3.14 \times (1.2^2) \times 80 \times 10^{-6}$$

$$\text{or } q = 1.45 \times 10^{-3} \text{C}$$

$$(b) \phi = \frac{q}{\epsilon_0} = \frac{1.45 \times 10^{-3}}{8.85 \times 10^{-12}} = 1.64 \times 10^8 \text{N m}^2 \text{C}^{-1}$$

6. As both the plates have same surface charge density  $\sigma$ , so



$$E_A = E_B = \frac{\sigma}{2\epsilon_0}$$

(a) In region-I,

$$E_I = E_B - E_A = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} \text{ or } E_I = 0$$

$$(b) \text{ In region-III, } E_{III} = E_A - E_B = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} \text{ or } E_{III} = 0$$

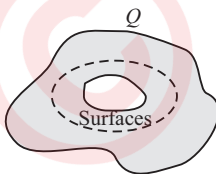
$$(c) \text{ In region-II } E_{II} = E_A + E_B = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

$$\text{or } E_{II} = \frac{17.0 \times 10^{-22}}{8.85 \times 10^{-12}} = 1.92 \times 10^{-10} \text{N C}^{-1}$$

7. (a) Let us consider a closed surface inside the conductor enclosing the cavity. As electric field inside the conductor is zero, so

$$\phi = \oint_s \vec{E} \cdot d\vec{s} = 0 \quad \dots(i)$$

By Gauss's theorem,



$$\phi = \frac{q_{\text{enclosed}}}{\epsilon_0} \quad \dots(ii)$$

By equations (i) and (ii), we found that

$$\frac{q_{\text{enclosed}}}{\epsilon_0} = 0 \text{ or } q_{\text{enclosed}} = 0$$

This shows that there cannot be any charge on the inner surface of

conductor at the cavity, so any charge  $Q$  given to the conductor will appear only on its outer surface.

(b) Let us consider that the charge  $-q_1$  is induced on the inner surface of conductor  $A$  at the cavity, when conductor  $B$  of charge  $q$  is kept in its cavity. Due to this charge of  $Q + q_1$  is induced on the outer surface of conductor.

Let us construct a closed Gaussian surface  $S$  inside the conductor  $A$  enclosing its cavity. As, electric field inside conductor is zero, so

$$\phi = \oint_s \vec{E} \cdot d\vec{s} = 0 \quad \dots(iii)$$

But, by Gauss's theorem

$$\phi = \frac{q - q_1}{\epsilon_0} \quad \dots(iv)$$

By equations (iii) and (iv), we get

$$\frac{q - q_1}{\epsilon_0} = 0 \text{ or } q - q_1 = 0 \text{ or } q_1 = q$$

i.e., equal and opposite charge,  $-q$  is induced on inner surface of conductor  $A$  at the cavity, when conductor  $B$  of charge,  $+q$  is kept in its cavity. Hence, the charge on outer surface of conductor is

$$Q + q_1 = Q + q$$

(c) As we found that electric field inside the cavity of conductor is zero, even on charging the conductor, so the sensitive instrument can be shielded from the strong electrostatic fields in its environment, by covering it with a metallic cover.

$$8. \quad r = 20 \text{ cm} = 0.20 \text{ m}, E = 1.5 \times 10^3 \text{N C}^{-1},$$

$$R = 10 \text{ cm} = 0.1 \text{ m}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \text{ or } 1.5 \times 10^3 = 9 \times 10^9 \times \frac{q}{(0.20)^2}$$

$$\text{or } q = \frac{1.5 \times 10^3 \times 0.04}{9 \times 10^9} = 6.67 \times 10^{-9}$$

Since electric field points radially inwards, so charge is negative i.e.

$$q = -6.67 \text{ nC}.$$

$$9. \quad E = 9 \times 10^4 \text{N C}^{-1}, \quad r = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

$$\text{As } E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\text{So, } \lambda = 2\pi\epsilon_0 r \cdot E = \frac{1}{2 \times 9 \times 10^9} \times 9 \times 10^4 \times 2 \times 10^{-2}$$

$$\text{or } \lambda = 1 \times 10^{-7} \text{C m}^{-1} = 0.1 \mu \text{C m}^{-1}.$$

