Electrostatic Potential and Capacitance

CHAPTER 2

EXAM DRILL

ANSWERS

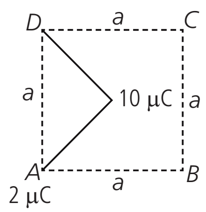
1. **(d) :** The electric potential is same throughout the equipotential surface. Thus, $W = 0$.

2. **(b) :** 1 Giga = 10^9 $\quad\therefore\quad$ 1 GeV = 10^9 eV

3. **(a) :** Electric field always direct from high potential to low potential. For the given situation the electric potential is decreasing from left to right therefore, potential energy of the dipole will also decrease. Thus dipole will move towards the right.

4. **(a) :** $V_A = \dfrac{K(10\times10^{-6}\,\text{C})}{\dfrac{a}{\sqrt{2}}} = \dfrac{\sqrt{2}\,K(10^{-5})}{a}$

$$V_B = \frac{K(10\times10^{-6}\,\text{C})}{\dfrac{a}{\sqrt{2}}} = \frac{\sqrt{2}K(10^{-5})}{a}$$



Work done in moving charge of 2 μC
$= q\,(V_B - V_A)$

$$= 2\times10^{-6}\left(\frac{\sqrt{2}\,K(10^{-5})}{a} - \frac{\sqrt{2}\,K(10^{-5})}{a}\right) = 0$$

5. **(c) :** Electric field intensity E is zero within a conductor due to charge given to it.

Also, $E = -\dfrac{dV}{dx}$ or $\dfrac{dV}{dx} = 0$ (inside the conductor)

$\therefore\quad V$ = constant.

So potential remains same throughout the conductor.

6. **(d) :** The charges reside on the surface of a conductor. Therefore, no charge no field is present inside the conductor.

7. **(b) :** When charge of capacitor becomes four times, then capacitance remains same but voltage V also becomes four times, therefore, $k = 1$.

8. **(a) :** Equivalent capacitance of 'n' number of capacitors in parallel is

$$C_{eq} = C_1 + C_2 + \ldots\ldots C_n$$

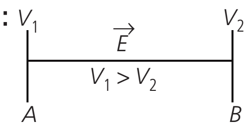
9. **(c) :** $C = \dfrac{k\varepsilon_0 A}{d}$. Therefore, $C \propto k$

10. **(d) :** Charge remains same in series and potential remains same in parallel configuration.

11. **(b) :** Both assertion and reason are true but reason is not the correct explanation of assertion.

If a material contain polar molecules, they will generally be in random orientations when no electric field is applied. An applied electric field will polarize the material by orienting the dipole moment of polar molecules.

12. **(d) :**

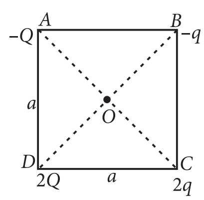


Suppose A and B are two regions having potentials V_1 and V_2 such that $V_1 > V_2$. So the electric field will be set up from A to B (*i.e.*, from higher potential to lower potential). Now since electron has got a negative charge, the force experienced by the electron will be in a direction opposite to the direction of the electric field. Hence the electrons move from a region of lower potential to a region of higher potential due to their negative charge.

13. **(i)** **(a) :** Let a be the side of the square $ABCD$.

$\therefore\quad AC = BD = \sqrt{a^2 + a^2} = a\sqrt{2}$

$OA = OB = OC = OD = \dfrac{a\sqrt{2}}{2} = \dfrac{a}{\sqrt{2}}$



Potential at the centre O due to given charge configuration is

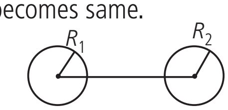
$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{(-Q)}{\left(\frac{a}{\sqrt{2}}\right)} + \frac{(-q)}{\left(\frac{a}{\sqrt{2}}\right)} + \frac{(2q)}{\left(\frac{a}{\sqrt{2}}\right)} + \frac{2Q}{\left(\frac{a}{\sqrt{2}}\right)}\right] = 0$$

$\Rightarrow\quad -Q - q + 2q + 2Q = 0$ or $Q + q = 0$ or $Q = -q$

(ii) **(a) :** Potential inside the sphere is the same as that on the surface *i.e.*, 80 V.

(iii) **(c) :** When the two spheres are connected by conducting wire, the potential of both the spheres becomes same.

$$V_1 = V_2 \Rightarrow \frac{kq_1}{R_1} = \frac{kq_2}{R_2}$$



$$\frac{q_1}{q_2} = \frac{R_1}{R_2} \qquad \ldots\text{(i)}$$

Ratio of surface charge densities

$$\frac{\sigma_1}{\sigma_2} = \frac{\dfrac{Q}{4\pi R_1^2}}{\dfrac{Q}{4\pi R_2^2}} = \frac{R_2^2}{R_1^2}\times\frac{q_1}{q_2} = \left(\frac{R_2}{R_1}\right)^2\times\frac{R_1}{R_2} \quad \text{(From (i))}$$

$$\Rightarrow\quad \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$

14. It increases the capacitance.

15. Infinity

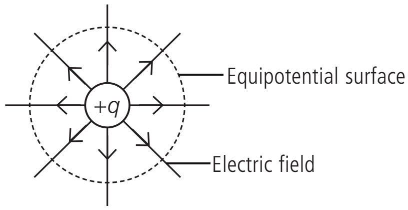
16. It is defined as the charge required to raise the potential by 1 V.

OR

It depends on the geometry of plates, distance between them and nature of dielectric medium.

17. Zero

18.

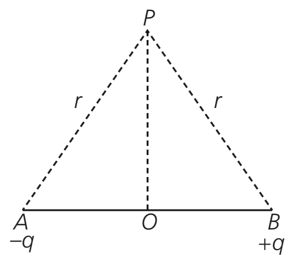


19. $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

$$E=-\frac{dV}{dr}=\frac{(-V_C-V_A)}{d}=\frac{V_A-V_C}{d}$$

$V_A - V_C = Ed.$

20. Electric potential at point P due to the dipole



$V = Vp_A + Vp_B$

$$V=\frac{K(-q)}{r}+\frac{K(+q)}{r}$$

$V = 0$

21. For air $C_0=\dfrac{A\in_0}{d}$

$C_0 = 8\text{ pF} = 8\times 10^{-12}\text{ F}$ $\quad\therefore\quad \dfrac{A\varepsilon_0}{d}=8\times10^{-12}\text{ F}$

Now $d' = d/2$ and $K = 6$

$$\Rightarrow C'=\frac{K\varepsilon_0 A}{d'}\times 12C_0$$

OR

For air $C=\dfrac{A\in_0}{d}$

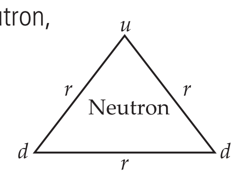
Thickness $t=\dfrac{d}{2}$ only when $K=\infty$

$$C_0=\frac{\in_0 A}{d};\ C'=\frac{\in_0 A}{(d-t)}=\frac{\in_0 A}{\left(d-\dfrac{d}{2}\right)}$$

$= 2\in_0 A$

$C' = 2\,C_0$, Hence capacitance will get doubled.

22. (a) Figure shows quark model of a neutron, where $r = 10^{-15}$ m.



Potential energy of neutron

$$U=\frac{1}{4\pi\in_0 r}[q_dq_d+q_dq_u+q_uq_d]$$

$$=\frac{9\times10^9}{10^{-15}}\left[\left(-\frac{e}{3}\right)\left(-\frac{e}{3}\right)+\left(\frac{2e}{3}\right)\left(-\frac{e}{3}\right)+\left(\frac{2e}{3}\right)\left(-\frac{e}{3}\right)\right]$$

$$U=\frac{9\times10^9}{10^{-15}}\left(\frac{1}{9}-\frac{4}{9}\right)(1.6\times10^{-19})^2$$

$= -7.68\times10^{-14}$ J

$$U=\frac{-7.68\times10^{-14}}{1.6\times10^{-19}}\text{ eV}=-4.8\times10^5\text{ eV}=-0.48\text{ MeV}$$

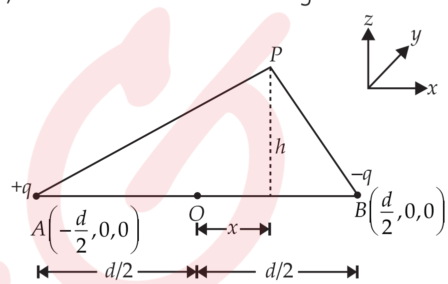
$m_nc^2 = 939$ MeV $\quad\therefore\ \dfrac{U}{m_nc^2}=\dfrac{0.48}{939}=5.11\times10^{-4}$

(b) Similarly for proton,

$$U=\frac{1}{4\pi\varepsilon_0 r}\left[\left(-\frac{e}{3}\right)\left(\frac{2e}{3}\right)+\left(-\frac{e}{3}\right)\left(\frac{2e}{3}\right)+\left(\frac{2e}{3}\right)\left(\frac{2e}{3}\right)\right]$$

$$=\frac{2e}{4\pi\varepsilon_0 r}\left[-\frac{2}{9}-\frac{2}{9}+\frac{4}{9}\right]=0$$

23. Figure shows two point charges, $+q$ at $A\left(-\dfrac{d}{2},0,0\right)$ and $-q$ at $B\left(\dfrac{d}{2},0,0\right)$ with centre at O. Let the equipotential surface (with $V = 0$) be at a distance x from the origin O.



The potential at any point P on the surface at a height h from AB is

$$V=\frac{q}{4\pi\in_0[(x+d/2)^2+h^2]^{1/2}}-\frac{q}{4\pi\in_0[(x-d/2)^2+h^2]^{1/2}}$$

If $V = 0$, then $\dfrac{q}{4\pi\in_0[(x+d/2)^2+h^2]^{1/2}}$

$$=\frac{q}{4\pi\in_0[(x-d/2)^2+h^2]^{1/2}}$$

or $\quad (x + d/2)^2 + h^2 = (x - d/2)^2 + h^2$

or $\quad x^2+\dfrac{d^2}{4}+xd=x^2+\dfrac{d^2}{4}-xd$

or $\quad 2xd = 0$ and $x = 0$.

24. Initially, potential difference across each capacitor is V.

$\therefore$ Total energy in capacitors $=\left(\dfrac{1}{2}CV^2\right)\times2=CV^2$

$\therefore$ Initial energy $(U_1) = CV^2$ ....(i)

Let the switch between A and B be opened.

Potential of A is still V. Capacity of A is $(3C)$.

$$\therefore\ U_A=\frac{1}{2}(3C)V^2=\frac{3CV^2}{2}$$

Potential of $B=\dfrac{\text{Initial charge stored}}{\text{New capacity}}=\dfrac{CV}{3C}=\dfrac{V}{3}$

$$\therefore\ U_B=\frac{1}{2}(3C)\left(\frac{V}{3}\right)^2=\frac{CV^2}{6}$$

$\therefore$ Final total energy $= U_A + U_B$

$$U_2=\frac{3CV^2}{2}+\frac{CV^2}{6},\quad U_2=\frac{5}{3}CV^2 \qquad ......\text{(ii)}$$

$$\therefore\ \frac{U_1}{U_2}=\frac{3CV^2}{5CV^2}=\frac{3}{5}\ \text{ or }\ \frac{U_1}{U_2}=\frac{3}{5}.$$

25. Here, $r = 1/2$ m = 0.5 m, $C = ?$
$C = 4\pi\varepsilon_0 r$, $C = 55.5\times10^{-12}$ farad = 55.5 pF

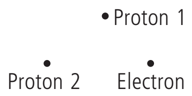
26. Here the two capacitors are in parallel
$\therefore$ Net capacitance $C = C_1 + C_2$

$$C_1=\frac{K_1\in_0 A/2}{d}=\frac{K_1\in_0 A}{2d}$$

$$C_2=\frac{K_2\in_0 A/2}{d}=\frac{K_2\in_0 A}{2d}$$

$$\Rightarrow C=\frac{K_1\in_0 A}{2d}+\frac{K_2\in_0 A}{2d};\ C=\frac{\in_0 A}{2d}(K_1+K_2)$$

27. The system of two protons and one electrons is represented in the given figure.



Charge on proton 1, $q_1 = 1.6\times10^{-19}$ C
Charge on proton 2, $q_2 = 1.6\times10^{-19}$ C
Charge on electron $q_3 = 1.6\times10^{-19}$ C
Distance between proton 1 and 2, $d_1 = 1.5\times10^{-10}$ m
Distance between proton 1 and electron, $d_2 = 1\times10^{-10}$ m
Distance between proton 2 and electron, $d_3 = 1\times10^{-10}$ m
The potential energy at infinity is zero.
Potential energy of the system,

$$U=\frac{q_1q_2}{4\pi\in_0 d_1}+\frac{q_2q_3}{4\pi\in_0 d_3}+\frac{q_3q_1}{4\pi\in_0 d_2}$$

$$V=\frac{9\times10^9\times10^{-19}\times10^{-19}}{10^{-10}}\left[-(16)^2+\frac{(1.6)^2}{1.5}+-(1.6)^2\right]$$

$= -30.7\times10^{-19}$ J $= -19.2$ eV
Therefore, the potential energy of the system is -19.2 eV.

OR

Let a be the radius of a sphere A, Q_A be the charge on the sphere, and C_A be the capacitance of the sphere. Let b be the radius of a sphere B, Q_B be the charge on the sphere, and C_B be the capacitance of the sphere. Since the two spheres are connected with a wire, their potential (V) will become equal.

Let E_A be the electric field of sphere A and E_B be the electric field of sphere B. Therefore their ratio,

$$\frac{E_A}{E_B}=\frac{Q_A}{4\pi\in_0\times a^2}\times\frac{b^2 4\pi\in_0}{Qs}$$

$$\frac{E_A}{E_B}=\frac{Q_A}{Q_S}\times\frac{b^2}{a^2} \quad ...(i)$$

However $\frac{Q_A}{Q_S}=\frac{C_AV}{C_SV}$ and $\frac{C_A}{C_B}=\frac{a}{b}$

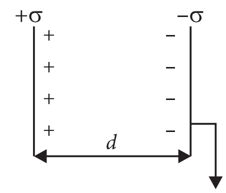
$$\therefore \frac{Q_A}{Q_B}=\frac{a}{b} \quad ...(ii)$$

Putting the value of (ii) and (i)

$$\frac{E_A}{E_B}\times\frac{a}{b}.\frac{b^2}{a^2}=b/a$$

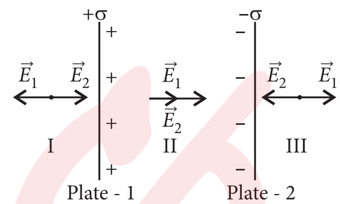
Thus, the ratio of electric fields at the surface is b/a.

28. Capacitor is based on the principle of electrostatic induction. The capacitance of an insulated conductor increases significantly by bringing an uncharged earthed conductor near to it. This combination forms parallel plate capacitor.



(a) Magnitude of electric field intensities

$$E_1 = E_2 = 2\ \frac{\sigma}{2\varepsilon_0}$$



(i) In region I (outside)

$$E_I = E_2 - E_1 = \frac{\sigma}{2\varepsilon_0}-\frac{\sigma}{2\varepsilon_0} = 0$$

(ii) In region II (inside)

$$E_{II} = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0}+\frac{\sigma}{2\varepsilon_0}=\frac{\sigma}{\varepsilon_0}$$

(iii) In region III (outside)

$$E_{III} = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0}-\frac{\sigma}{2\varepsilon_0} = 0$$

In the region II *i.e.*, in the space between the plates, resultant electric field $\vec{E}_{II}$ is directed normal to plates, from positive to negative charge plate.

(b) The potential difference between the plates is

$$V = E_{II}.d = \frac{\sigma}{\varepsilon_0}d \quad \text{or} \quad V=\frac{Q}{A\varepsilon_0}d$$

(c) Capacitance of the capacitor so formed is

$$C=\frac{Q}{V}=\frac{Q}{Qd/A\varepsilon_0} \quad \text{or} \quad C=\frac{\varepsilon_0 A}{d}$$

29. Potential rating of a parallel plate capacitor, $V = 1$ kV = 1000 V
Dielectric constant of a material, $\in = 3$
Dielectric strength = 10^{-7} Vm
For safety, the field intensity never exceeds 10 % of the dielectric strength. Hence, electric field intensity, $E = 10\%$ of $10^7 = 10^6$ V/m
Capacitance of the parallel plate capacitor,
$C = 50$ pF $= 50\times10^{-12}$ F
Distance between the plates is given by .

$$d=\frac{V}{E}=\frac{1000}{10^6}=10^{-3}\text{m}$$

Capacitance is given by the relation,

$$C=\frac{\in_0\in_r A}{d}$$

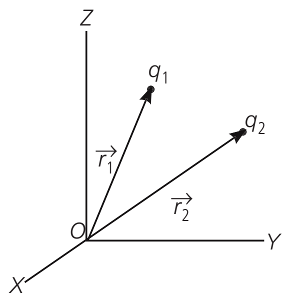
where, A = Area of each plate

So, $A=\frac{C\times d}{\in_0\in_r}$

$$A=\frac{50\times10^{-12}\times10^{-3}}{8.85\times10^{-12}\times3}=19\text{ cm}^2$$

Hence, area of each plate is about 19 cm^2.

30. The electrostatic potential energy of a system of point charges is defined as the work required to be done to bring the charges constituting the system to their respective locations from infinity.



Potential energy of system of two point charges. Consider two point charges q_1 and q_2 lying at points A and B, whose locations are $\vec{r}_1$ and $\vec{r}_2$ respectively, figure.

To calculate the electric potential energy of the two charges, remove the two charges to positions, such that they are at infinite distance from each other. First of all, bring the charge q_1 from infinity to its original position A. For this, no work is required. It is because, when charge q_1 is moved, no electrostatic force due to any other charge opposes it.

Now, move charge q_2 to its original position B, when charge q_2 is moved, the electric field due to the charge q_1 lying at point A, oppose it. Hence, work has to be done. The work done in moving charge q_2 from infinity to point B in the electric field of charge q_1 is given by

W = (electric potential due to charge q_1 at B) × q_2

$$=\left(\frac{2}{4\pi\varepsilon_0}\cdot\frac{q_1}{AB}\right)q_2$$

Since $AB = |\vec{r}_1-\vec{r}_2|$, we have

$$W=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{|\vec{r}_1-\vec{r}_2|}$$

The work done in bringing the two charges to their respective positions is stored as the potential energy of the configuration of two charges *i.e.*,

$$U=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{|\vec{r}_1-\vec{r}_2|}\qquad\text{...(i)}$$

If distance $|\vec{r}_1-\vec{r}_2|$ is denoted as r_{12}, then

$$U=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r_{12}}\qquad\text{...(ii)}$$

The equations (i) and (ii) give electric potential energy of a system of two point charges.

31. Electric field intensity of an infinite plane sheet of charge,

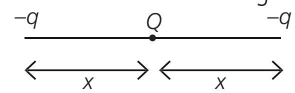
$$E=\frac{\sigma}{2\epsilon_0}$$

Let Δr be the separation between two equipotential surfaces having potential difference ΔV, then

$$E=\frac{\Delta V}{\Delta r}\ ;\ \frac{\sigma}{2\epsilon_0}=\frac{\Delta V}{\Delta r}\Rightarrow\Delta r=\frac{2\epsilon_0\,.\Delta V}{\sigma}$$

$$\Delta r=\frac{2\times8.854\times10^{-12}\times5}{10^{-8}}=8.85\times10^{-3}\text{ m}=8.85\text{ mm}$$

32. Let x is the distance between the charges as shown, then

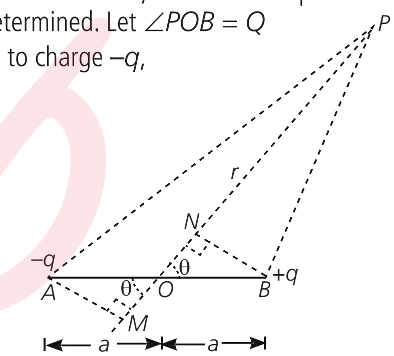


potential energy of the system of three charges is

$$\frac{K(-q)Q}{x}+\frac{KQ(-q)}{x}+\frac{K(-q)(-q)}{2x}=0$$

$$q=4Q\quad\Rightarrow\frac{Q}{q}=\frac{1}{4}$$

33. Consider an electric dipole AB having charge $-q$ at point A and charge $+q$ at point B. Let O be the centre of the dipole and P be any point at a distance r from its centre, where electric potential due to the dipole is to be determined. Let $\angle POB = Q$



The potential at point P due to charge $-q$,

$$V_1=-\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{PA}$$

and the potential at point P due to charge $+q$,

$$V_2=-\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{PB}$$

Therefore, net potential at point P due to the dipole,

$$V=V_1+V_2=-\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{PA}+\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{PB}$$

or $$V=\frac{1}{4\pi\varepsilon_0}\cdot q\left[\frac{1}{PB}-\frac{1}{PA}\right]\qquad\text{...(i)}$$

To find PB and PA, draw BN perpendicular to OP and AM perpendicular to MO after producing it. From right angled ΔAMO, we have

$$\cos\theta=\frac{OM}{OA}=\frac{OM}{a}\ \text{ or }\ OM=a\cos\theta$$

In case the length of th dipole is very small as compared to distance r, then

$$PA \approx PM = PO + OM = r + a\cos\theta$$

Similarly, it can be obtained that

$$PB = r - a\cos\theta$$

In the equation (i), substituting for PA and PB, we have

$$V=\frac{1}{4\pi\varepsilon_0}\cdot q\left[\frac{1}{r-a\cos\theta}-\frac{1}{r+a\cos\theta}\right]$$

$$=\frac{1}{4\pi\varepsilon_0}\cdot q\left[\frac{r+a\cos\theta-r+a\cos\theta}{r^2-a^2\cos^2\theta}\right]$$

$$=\frac{1}{4\pi\varepsilon_0}\cdot q\frac{2a\cos\theta}{(r^2-a^2\cos^2\theta)}$$

Since $q(2a) = p$, the electric dipole moment of the dipole, the above equation becomes

$$V=\frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{(r^2-a^2\cos^2\theta)}\qquad\text{...(ii)}$$

The equation (ii) gives electric potential due to the dipole at a distance r from its centre in a direction making an angle θ with the dipole.

OR

(a) No work done in moving a charge from one point on equipotential surface to the other. Therefore component of electric field intensity along the equipotential surface is zero it means, the electric field intensity is perpendicular to the equipotential surface. Thus, the surface is perpendicular to field lines.

(b) Here, $q_1 = 4\ \mu C$, $q_2 = -2\ \mu C$

Let the potential be zero at a point P, at a distance $r_1 = x$ from $4\ \mu C$ charge. So, $r_2 = 1 - x$.

Potential at $P = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = 0$

$$\frac{q_1}{r_1} + \frac{q_2}{r_2} \Rightarrow \frac{4}{x} = \frac{-(-2)}{1-x} \Rightarrow x = 2 - 2x$$

$x = 2/3$ from $4\ \mu C$ charge

34. Radius of the outer shell $= r_1$

Radius of the inner shell $= r_2$

The inner surface of the outer shell has charge $+Q$.

$$V = \frac{Q}{4\pi \in_0 b} - \frac{Q}{4\pi \in_0 a}$$

The outer surface of the inner shell has induced charge $-Q$. Potential difference between the two shells is given by,

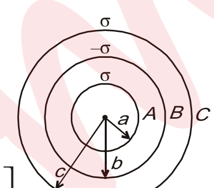
$$V = \frac{Q}{4\pi \in_0}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{Q(b-a)}{4\pi \in_0 ba}$$

Capacitance of the given system is given by,

$$C = \frac{\text{charge}(Q)}{\text{potential difference }(V)} = \frac{4\pi \in_0 ab}{b-a}$$

OR

(i) Potential of any shell will be due to charges residing on all the three shells.



$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{a} + \frac{q_B}{b} + \frac{q_C}{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{4\pi a^2(\sigma)}{a} + \frac{4\pi b^2(-\sigma)}{b} + \frac{4\pi c^2(\sigma)}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}[a - b + c]$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{b} + \frac{q_B}{b} + \frac{q_C}{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{4\pi a^2(\sigma)}{b} + \frac{4\pi b^2(-\sigma)}{b} + \frac{4\pi c^2(\sigma)}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$$

$$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{4\pi a^2(\sigma)}{c} + \frac{4\pi b^2(-\sigma)}{c} + \frac{4\pi c^2(\sigma)}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{c} - \frac{b^2}{c} + c\right] = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2 + c^2}{c}\right]$$

(ii) Given : $V_A = V_C$

$$\therefore\ \frac{\sigma}{\varepsilon_0}(a - b + c) = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$

or $a - b + c = \frac{a^2}{c} - \frac{b^2}{c} + c$

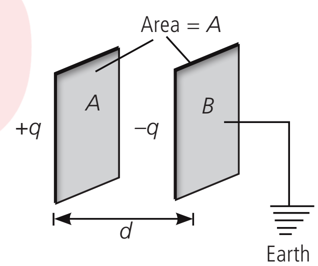
or $a - b = \frac{(a+b)(a-b)}{c}$ or $(a + b) = c$.

35. Parallel plate capacitor is most commonly used capacitor. It consists of two conducting plates placed parallel to each other figure. The separation d between the plates is very small compared to the area of the plates. Due to small separation between the plates, the fringing of electric field at the boundaries is negligible.

If charge $+q$ is given to plate A, then charge $-q$ is induced on the left face of plate B and charge $+q$ on its right face. When plate B is earthed, the charge $+q$ on the right face flows to earth. Due to charge $+q$ on plat A and $-q$ on plate B, electric field is set up between the two plates.

The electric field between the two plates is related to the potential gradient as

$E = \frac{dV}{dr}$ (in magnitude)



Between the two parallel plates, the electric field is uniform and perpendicular to the plates. Therefore, if V is potential difference between the two plates, then

$E = \frac{V}{d}$ $\left(\text{For uniform field, } \frac{dV}{dr} = \frac{V}{d}\right)$

or $V = Ed$ ...(i)

If σ is surface charge density of the plates, then the electric field between the two plates is given by

$$E = \frac{\sigma}{\varepsilon_0}$$

Here ε_0 is absolute permittivity of the free space. (It is assumed that medium between the plates is vacuum or air).

Substituting for E in equation (i), we have

$$V = \frac{\sigma}{\varepsilon_0}d$$

If A is the area of each plate, then

$$\sigma = \frac{q}{A} \quad \therefore \quad V = \frac{qd}{\varepsilon_0 A}$$

If C is the capacitance of the parallel plate capacitor, then

$$C = \frac{q}{V} = \frac{q}{qd/\varepsilon_0 A}$$

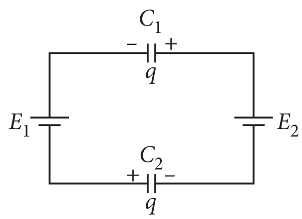
or $\quad C = \dfrac{\varepsilon_0 A}{d}$ ...(ii)

It gives the capacitance of a parallel plate capacitor, when its plates are held in air or vacuum.

OR

(a) $\dfrac{-q}{C_1} - E_1 - \dfrac{q}{C_2} + E_2 = 0$

or $\dfrac{q}{C_1} + \dfrac{q}{C_2} = E_2 - E_1$



Now, $V_1 = \dfrac{q}{C_1}, V_2 = \dfrac{q}{C_2}$

(b) On connecting the two given capacitors, let the final voltage be V.

If capacity of capacitor without the dielectric is C, then the charge on this capacitor is $q_1 = CV$

The other capacitor with dielectric has capacity εC. Therefore, charge on it is $q_2 = \varepsilon CV$

As $\varepsilon = \alpha V$, therefore, $q_2 = \alpha CV^2$

The initial charge on the capacitor (without dielectric) that was charged is

$$q_0 = CV_0$$

From the conservation of charge,

$$q_0 = q_1 + q_2$$

$$CV_0 = CV + \alpha CV^2 \text{ or } \alpha V^2 + V - V_0 = 0$$

$$V = \frac{-1 \pm \sqrt{1 + 4\alpha V_0}}{2\alpha}$$

using $\alpha = 7\text{ V}^{-1}$ and $V_0 = 26$ V, we get

$$V = \frac{-1 \pm \sqrt{1 + (4 \times 7 \times 26)}}{2 \times 7} = \frac{-1 \pm \sqrt{729}}{14}$$

As V is positive, therefore, $V = \dfrac{\sqrt{729} - 1}{14} = \dfrac{26}{14} = 1.86$ V.