

Electrostatic Potential and Capacitance

CHAPTER 2



NCERT FOCUS

ANSWERS

Topic 1

1. (i) Let C be the point on the line joining the two charges, where electric potential is zero, then

$$\begin{aligned}
 V_C &= 0 \\
 \text{or } V_{CA} + V_{CB} &= 0 \\
 \text{or } V_{CA} &= -V_{CB} \\
 \text{or } \frac{1}{4\pi\epsilon_0} \frac{q_A}{r_{CA}} &= -\frac{1}{4\pi\epsilon_0} \frac{q_B}{r_{CB}} \\
 \text{or } \frac{5 \times 10^{-8} \text{ C}}{x \times 10^{-2} \text{ m}} &= -\frac{(-3 \times 10^{-8} \text{ C})}{[(16-x) \times 10^{-2} \text{ m}]} \quad \text{or} \quad \frac{5}{x} = \frac{3}{16-x} \\
 \text{or } 80 - 5x &= 3x \quad \text{or} \quad 80 = 8x \\
 \text{or } x &= \frac{80}{8} \quad \text{or} \quad x = 10 \text{ cm}
 \end{aligned}$$

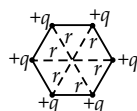
So, electric potential is zero at distance of 10 cm from charge of $5 \times 10^{-8} \text{ C}$ on line joining the two charges between them.

If point C is not between the two charges, then

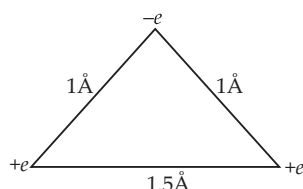
$$\begin{aligned}
 V_{CA} + V_{CB} &= 0 \quad \text{or} \quad V_{CA} = -V_{CB} \\
 \text{or } \frac{1}{4\pi\epsilon_0} \frac{q_A}{r_{CA}} &= -\frac{1}{4\pi\epsilon_0} \frac{q_B}{r_{CB}} \\
 \text{or } \frac{5 \times 10^{-8} \text{ C}}{[(16+x) \times 10^{-2} \text{ m}]} &= \frac{-(-3 \times 10^{-8} \text{ C})}{[x \times 10^{-2} \text{ m}]} = \frac{5}{16+x} = \frac{3}{x} \\
 \text{or } 5x &= 48 + 3x \quad \text{or} \quad 2x = 48 \quad \text{or} \quad x = 24 \text{ cm}
 \end{aligned}$$

So, electric potential is also equal to zero at a distance of 24 cm from charge of $-3 \times 10^{-8} \text{ C}$ and at a distance of $(24 + 16) = 40 \text{ cm}$ from charge of $5 \times 10^{-8} \text{ C}$, on the side of charge of $-3 \times 10^{-8} \text{ C}$.

$$\begin{aligned}
 2. \quad V &= 6 \times \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\
 \text{or } V &= 6 \times 9 \times 10^9 \times \frac{5 \times 10^{-6}}{10 \times 10^{-2}} \\
 \text{or } V &= 2.7 \times 10^6 \text{ volts.}
 \end{aligned}$$



3. It is a system of three point charges and the potential energy stored in this system of charges is



$$U = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right]$$

$$U = 9 \times 10^9 \left[\frac{e \times -e}{1} + \frac{e \times e}{1.5} + \frac{e \times -e}{1} \right] \times \frac{1}{10^{-10} \text{ m}}$$

$$\text{or } U = 9 \times 10^9 \times (1.6 \times 10^{-19})^2 \times 10^{10} \left[-1 + \frac{1}{1.5} - 1 \right] \text{ J}$$

$$\text{or } U = \frac{9 \times 10^{19} \times (1.6 \times 10^{-19})^2}{1.6 \times 10^{-19}} \times \left(\frac{-2}{1.5} \right) \text{ eV}$$

$$\text{or } U = -19.2 \text{ eV}$$

with zero potential energy at infinity.

$$4. \quad (a) \quad C = \frac{C_1}{3} = \frac{9}{3} \text{ pF} \quad \text{or} \quad C = 3 \text{ pF}$$

- (b) Net charge stored in combination of capacitors is

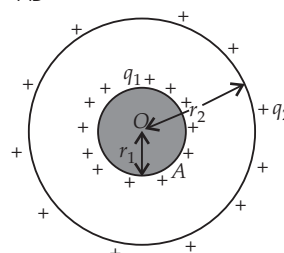
$$Q = CV = 3 \times 10^{-12} \times 120 = 360 \text{ pC}$$

So, potential difference across each capacitor is

$$V_1 = \frac{Q}{C_1} = \frac{360 \text{ pC}}{9 \text{ pF}} \quad \text{or} \quad V_1 = 40 \text{ volts.}$$

5. The potential on inner small sphere is

$$V_A = V_{AA} + V_{AB}$$



$$\text{or } V_A = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} \right]$$

whereas the potential on the outer shell B is

$$V_B = V_{BA} + V_{BB} = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} \right]$$

$$\text{So, } V_A - V_B = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_1} - \frac{q_1}{r_2} \right]$$

$$= \frac{q_1}{4\pi\epsilon_0} \left[\frac{r_2 - r_1}{r_1 r_2} \right]$$

As $r_2 > r_1$, so $V_A > V_B$ i.e. inner sphere A is at higher potential than outer conducting shell B, for any value of charge q_1 . So, when inner sphere A is connected to outer shell B, then charge will flow

from inner sphere A to outer shell B , until electric potentials on them is same i.e.

$$V_A - V_B = 0 \text{ or } q_1 = 0 \quad [\text{As } r_1 \neq r_2]$$

So, charge q_1 given to sphere A will flow on the shell B , no matter what the charge on the shell B is.

6. The length of diagonal of the cube of each side b is

$$\sqrt{3b^2} = b\sqrt{3}$$

Distance of any of the vertices from the centre of cube,

$$r = \frac{\sqrt{3}}{2} b$$

$$V = 8 \times \frac{1}{4\pi\epsilon_0} \frac{q}{r} = 8 \times \frac{1}{4\pi\epsilon_0} \frac{q}{\frac{\sqrt{3}}{2} b} \text{ or } V = \frac{4q}{\sqrt{3}\pi\epsilon_0 b}$$

$E = 0$, as electric field at centre due to a charge at any corner of cube is just equal and opposite to that of another charge at diagonally opposite corner of cube.

7. $R = 12 \text{ cm}$, $q = 1.6 \times 10^{-7} \text{ C}$

(a) $E_{\text{in}} = 0$

$$(b) E_{\text{out}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = 9 \times 10^9 \times \frac{1.6 \times 10^{-7}}{(12 \times 10^{-2})^2}$$

$$\text{or } E_{\text{out}} = 1.0 \times 10^5 \text{ N C}^{-1}$$

$$(c) E_{\text{on}} = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} = 9 \times 10^9 \times \frac{1.6 \times 10^{-7}}{(18 \times 10^{-2})^2}$$

$$\text{or } E_{\text{on}} = 4.44 \times 10^4 \text{ N C}^{-1}$$

Topic 2

1. (a) Since it is an electric dipole, so a plane normal to AB and passing through its mid-point has zero potential everywhere.

(b) Normal to the plane in the direction AB .

$$2. (a) U = \frac{1}{4\pi\epsilon_0} \frac{+e \times -e}{r} = -9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{0.53 \times 10^{-10}} \text{ J}$$

$$\text{or } U = \frac{-9 \times 10^9 \times (1.6 \times 10^{-19})^2}{0.53 \times 10^{-10} \times 1.6 \times 10^{-19}} \text{ eV or } U = -27.2 \text{ eV}$$

$$(b) \text{K.E.} = +\frac{|U|}{2} = 13.6 \text{ eV}$$

$$\text{Total energy } E = \text{P.E.} + \text{K.E.} = -27.2 + 13.6 = -13.6 \text{ eV}$$

$$\text{Now } W = \Delta U = 0 - (-13.6)$$

$$\text{or } W = +13.6 \text{ eV}$$

(c) P.E. at $1.06 \times 10^{-10} \text{ m}$ separation,

$$U = \frac{9 \times 10^9 \times (-1.6 \times 10^{-19}) \times (1.6 \times 10^{-19})}{1.06 \times 10^{-10}} = -21.74 \times 10^{-19} \text{ J}$$

$$\text{or } U = \frac{-21.74 \times 10^{-19}}{1.06 \times 10^{-19}} = -13.585 \text{ eV}$$

For (a): Taking -13.585 eV as zero of P.E., then

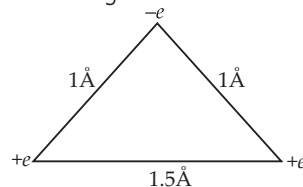
$$\text{P.E. of the system} = -27.17 - (-13.585)$$

$$= -13.585 \text{ eV} = -13.6 \text{ eV.}$$

$$\text{For (b): } \Delta U = -13.6 \text{ eV} - (-13.6 \text{ eV}) = 0$$

$$\therefore W = 0$$

3. It is a system of three point charges and the potential energy stored is this system of charges is



$$U = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right]$$

$$U = 9 \times 10^9 \left[\frac{e \times -e}{1} + \frac{e \times e}{1.5} + \frac{e \times -e}{1} \right] \times \frac{1}{10^{-10} \text{ m}}$$

$$\text{or } U = 9 \times 10^9 \times (1.6 \times 10^{-19})^2 \times 10^{10} \left[-1 + \frac{1}{1.5} - 1 \right] \text{ J}$$

$$\text{or } U = \frac{9 \times 10^{19} \times (1.6 \times 10^{-19})^2}{1.6 \times 10^{-19}} \times \left(\frac{-2}{1.5} \right) \text{ eV}$$

$$\text{or } U = -19.2 \text{ eV}$$

with zero potential energy at infinity.

$$4. V_P = V_{PA} + V_{PB} + V_{PC} + V_{PD}$$

$$\text{or } V_P = \frac{1}{4\pi\epsilon_0} \left[\frac{+q}{r+a} - \frac{q}{r} - \frac{q}{r} + \frac{q}{r-a} \right]$$

$$\text{or } V_P = \frac{q}{4\pi\epsilon_0} \left[\frac{r(r-a) - 2(r^2 - a^2) + r(r+a)}{r(r^2 - a^2)} \right]$$

$$\text{or } V_P = \frac{q}{4\pi\epsilon_0} \left[\frac{r^2 - ra - 2r^2 + 2a^2 + r^2 + ra}{r(r^2 - a^2)} \right]$$

$$\text{or } V_P = \frac{1}{4\pi\epsilon_0} \frac{q \cdot 2a^2}{r(r^2 - a^2)} = \frac{1}{4\pi\epsilon_0} \frac{p \cdot a}{r(r^2 - a^2)}$$

For $\frac{r}{a} \gg 1$ or $r \gg a$

$$V_P \approx \frac{1}{4\pi\epsilon_0} \frac{pa}{r^3} \text{ or } V_P \propto \frac{1}{r^3}.$$

However, electric potential at any point on axis of electric dipole is

$$V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2} \text{ or } V \propto \frac{1}{r^2} \text{ and due to point charge is}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \text{ or } V \propto \frac{1}{r}$$

5. (a) Planes parallel to x-y plane or normal to the electric field in z-direction.

(b) Planes parallel to x-y plane or normal to the electric field in z-direction, but the planes having different fixed potentials will become closer with increase in electric field intensity.

(c) Concentric spherical surfaces with their centres at origin.

(d) A time dependent changing shape nearer to grid, and at far off distances from the grid, it slowly becomes planar and parallel to the grid.

6. (a) The two point charges form an electric dipole of moment $p = q \cdot 2a$ directed along + z-axis. Point $A(0, 0, z)$ lies on the axis of electric dipole, so electric potential at point A is

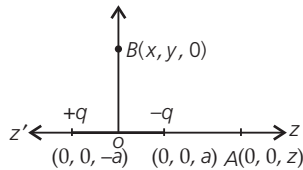
$$V_A = \frac{1}{4\pi\epsilon_0} \left[\frac{-q}{z+a} + \frac{q}{z-a} \right] = \frac{q}{4\pi\epsilon_0} \left[\frac{-z+a+z+a}{z^2-a^2} \right]$$

$$\text{or } V_A = \frac{1}{4\pi\epsilon_0} \frac{p}{(z^2-a^2)}$$

Point $B(x, y, 0)$ lies on the equatorial plane of electric dipole, so electric potential at point B is zero i.e. $V_B = 0$

(b) Electric potential at any point on the axis of electric dipole at distance r from its centre is

$$V = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2-a^2)}$$



when $r \gg a$, then electric potential becomes

$$V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2}$$

so in that case $V \propto \frac{1}{r^2}$

(c) As both the points $C(5, 0, 0)$ and $D(-7, 0, 0)$ lie on the perpendicular bisector of electric dipole, so electric potential at both the points is zero. Hence work done in moving the charge from C to D is

$$W_{CD} = q_0 [V_D - V_C] = q_0 \times 0$$

$$\text{or } W_{CD} = 0$$

This work done will remain equal to zero even if the path of the test charge between the same points is changed, as electric field is conservative field and work done in moving a charge between the two points in electric field is independent of the path chosen to move the charge.

Topic 3

$$1. \quad C_0 = \frac{\epsilon_0 A}{d} = 8 \text{ pF}$$

$$C = \frac{K\epsilon_0 A}{d/2} = \frac{2K\epsilon_0 A}{d} = 2KC_0 = 2 \times 6 \times 8 \text{ pF}$$

$$\text{or } C = 96 \text{ pF}$$

$$2. \quad A = 6 \times 10^{-3} \text{ m}^2, d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}, V = 100 \text{ V}$$

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$

$$= 17.7 \times 10^{-12} \text{ F} \quad \text{or } C_0 = 17.7 \text{ pF}$$

On connecting the capacitor across 100 V supply, charge on each plate of the capacitor is

$$Q_0 = C_0 V = 17.7 \times 10^{-12} \times 100 \text{ or } Q_0 = 1.77 \times 10^{-9} \text{ C.}$$

$$3. \quad C = KC_0 = 6 \times 17.7 = 106.2 \text{ pF}$$

(a) If the voltage supply remained connected, then the potential difference across the capacitor will remain the same i.e. $V = 100 \text{ V}$ and hence charge on the capacitor becomes

$$Q = CV = 106.2 \times 10^{-12} \times 100$$

$$\text{or } Q = 1.062 \times 10^{-8} \text{ C}$$

(b) If the voltage supply was disconnected, then charge on the capacitor remains the same

$$\text{i.e. } Q = 1.77 \times 10^{-9} \text{ C}$$

and hence potential difference across the capacitor becomes

$$V = \frac{Q}{C} = \frac{1.77 \times 10^{-9} \text{ C}}{106.2 \times 10^{-12} \text{ F}} \quad \text{or } V = 16.7 \text{ V}$$

4. Magnitude of electric field between the plates of charged capacitor is $E = \frac{\sigma}{\epsilon_0}$

However, magnitude of electric field of one plate on the other plate of charged capacitor is $E_1 = \frac{\sigma}{2\epsilon_0}$. So, force on the one plate of charged capacitor due to the other is

$$F = QE_1 = Q \cdot \frac{\sigma}{2\epsilon_0} = \frac{1}{2} \times Q \times \frac{\sigma}{\epsilon_0} \quad \text{or } F = \frac{1}{2} QE$$

The factor $\frac{1}{2}$ is because the electric field of one plate on the other plate of charged capacitor is $\frac{1}{2}$ of the resultant electric field E between the plates of charged capacitor.

$$5. \quad r_1 = 13 \text{ cm}, r_2 = 12 \text{ cm}, K = 32, Q = 2.5 \mu\text{C}$$

(a) Capacitance of capacitor is

$$C = \frac{4\pi\epsilon_0 K r_1 r_2}{r_1 - r_2} = \frac{1 \times 32}{9 \times 10^9} \times \frac{13 \times 10^{-2} \times 12 \times 10^{-2}}{(13-12) \times 10^{-2}}$$

$$\text{or } C = 5.5 \times 10^{-9} \text{ F}$$

(b) Electric potential of inner sphere is

$$V_B = V_{BB} + V_{BA}$$

$$= \frac{1}{4\pi\epsilon_0 k} \left[\frac{Q}{r_2} - \frac{Q}{r_1} \right] = \frac{Q}{4\pi\epsilon_0 k} \left[\frac{r_1 - r_2}{r_1 r_2} \right]$$

$$= \frac{9 \times 10^9}{32} \times 2.5 \times 10^{-6} \left[\frac{13-12}{13 \times 12} \right] \times \frac{10^{-2}}{10^{-4}} = 4.5 \times 10^2 \text{ V.}$$

(c) Capacitance of isolated sphere of radius 12 cm is

$$C_0 = 4\pi\epsilon_0 r_2 = \frac{1}{9 \times 10^9} \times 12 \times 10^{-2}$$

$$\text{or } C_0 = 1.3 \times 10^{-11} \text{ F}$$

Here $C > C_0$, because a single conductor A can be charged to a electric potential till it reaches the breakdown value of surroundings. But when another earthed metallic conductor B is brought near it, negative charge induced on it decreases the electric potential on A , hence more charge can not be stored on A .

$$6. \quad V = 1 \times 10^3 \text{ V}, K = 3, C = 50 \times 10^{-12} \text{ F}$$

Given $E = 10\%$ of dielectric strength

$$\text{or } E = \frac{10}{100} \times 10^7 \text{ V m}^{-1} = 10^6 \text{ V m}^{-1}$$

$$\text{As } V = E.d,$$

$$\text{so } d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3} \text{ m. and } C = \frac{k\epsilon_0 A}{d}$$

$$\text{or } A = \frac{C \cdot d}{k\epsilon_0} = \frac{50 \times 10^{-12} \times 10^{-3}}{3 \times 8.85 \times 10^{-12}}$$

$$\text{or } A = 1.9 \times 10^{-3} \text{ m}^2 = 19 \text{ cm}^2.$$

7. Let $+Q$ be the charge on outer spherical shell A of radius r_1 and $+Q$ be the charge on inner spherical shell B of radius r_2 . Then electric potential on shell A is

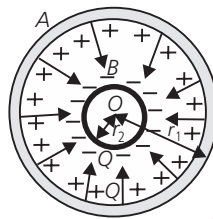
$$V_A = V_{AA} + V_{AB} = \frac{1}{4\pi\epsilon_0} \left[\frac{+Q}{r_1} - \frac{Q}{r_1} \right]$$

$$\text{or } V_A = 0$$

and electric potential on shell B is

$$V_B = V_{BA} + V_{BB} = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{r_1} - \frac{Q}{r_2} \right]$$

$$\text{or } V_B = \frac{Q}{4\pi\epsilon_0} \left[\frac{r_2 - r_1}{r_1 r_2} \right]$$



So, the potential difference between the two spherical shells A and B is

$$V = V_A - V_B = 0 - \frac{Q}{4\pi\epsilon_0} \left[\frac{r_2 - r_1}{r_1 r_2} \right] \text{ or } V = \frac{Q}{4\pi\epsilon_0} \left[\frac{r_1 - r_2}{r_1 r_2} \right]$$

$$\text{or } \frac{Q}{V} = \frac{4\pi\epsilon_0 r_1 r_2}{r_1 - r_2} \text{ or } C = \frac{4\pi\epsilon_0 r_1 r_2}{r_1 - r_2}$$

This gives the capacitance of the spherical capacitor.

8. Capacitance of cylindrical capacitor is

$$C = \frac{2\pi\epsilon_0 L}{2.303 \log_{10} \frac{b}{a}} = \frac{2 \times 15 \times 10^{-2}}{9 \times 10^9 \times 2.303 \times \log_{10} \frac{1.5}{1.4}}$$

$$\text{or } C = 1.21 \times 10^{-10} \text{ F}$$

Electric potential of the inner cylinder is

$$V = \frac{Q}{C} = \frac{3.5 \times 10^{-6} \text{ C}}{1.21 \times 10^{-10} \text{ F}} \text{ or } V = 2.89 \times 10^4 \text{ V.}$$

Topic 4

1. (a) $C = \frac{C_1}{3} = \frac{9}{3} \text{ pF}$ or $C = 3 \text{ pF}$

(b) Net charge stored in combination of capacitors is

$$Q = CV = 3 \times 10^{-12} \times 120 = 360 \text{ pC}$$

So, potential difference across each capacitor is

$$V_1 = \frac{Q}{C_1} = \frac{360 \text{ pC}}{9 \text{ pF}} \text{ or } V_1 = 40 \text{ volts.}$$

2. (a) $C = C_1 + C_2 + C_3 = 2 + 3 + 4$ or $C = 9 \text{ pF}$

(b) Since the capacitors are in parallel, so potential difference across each of them is same i.e.

$$V_1 = V_2 = V_3 = 100 \text{ V}$$

So, charges stored on capacitors are

$$Q_1 = C_1 V_1 = 2 \times 100 = 200 \text{ pC}$$

$$Q_2 = C_2 V_2 = 3 \times 100 = 300 \text{ pC}$$

$$Q_3 = C_3 V_3 = 4 \times 100 = 400 \text{ pC}$$

3. C_2 and C_3 are in series, so, $C = 100 \text{ pF}$

$\therefore C_1$ and C' are parallel,

$$\text{so } C'' = C_1 + C' = 100 + 100 \text{ or } C'' = 200 \text{ pF}$$

C_4 and C'' are in series, so net capacitance of the network is

$$\frac{1}{C} = \frac{1}{C''} + \frac{1}{C_4} = \frac{1}{200} + \frac{1}{100} = \frac{1+2}{200}$$

$$\text{or } C_2 = \frac{200}{3} \text{ pF} = 66.7 \text{ pF}$$

Net charge stored on the combination is

$$\therefore CV = \frac{200}{3} \times 10^{-12} \times 300 = 2 \times 10^{-8} \text{ C}$$

As C'' and C_4 are in series, so

$$Q'' = Q_4 = Q \text{ or } Q'' = Q_4 = 2 \times 10^{-8} \text{ C}$$

$$\text{and hence } V'' = \frac{Q''}{C''} = \frac{2 \times 10^{-8} \text{ C}}{200 \times 10^{-12} \text{ F}} = 100 \text{ V}$$

$$\text{and } V_4 = \frac{Q_4}{C_4} = \frac{2 \times 10^{-8} \text{ C}}{100 \times 10^{-12} \text{ F}} = 200 \text{ V}$$

$\therefore C_1$ and C' are in parallel, so

$$V_1 = V' = V'' \text{ or } V_1 = V' = 100 \text{ V}$$

$$\text{and hence } Q_1 = C_1 V_1 = 100 \times 10^{-12} \times 100 = 1 \times 10^{-8} \text{ C}$$

$$\text{and } Q' = C' V' = 100 \times 10^{-12} \times 100 = 1 \times 10^{-8} \text{ C}$$

C_2 and C_3 are in parallel, so $Q_2 = Q_3 = Q'$

$$\text{or } Q_2 = Q_3 = 1 \times 10^{-8} \text{ C}$$

$$\text{and hence } V_2 = \frac{Q_2}{C_2} = \frac{1 \times 10^{-8} \text{ C}}{200 \times 10^{-12} \text{ F}} = 50 \text{ V}$$

$$\text{and } V_3 = \frac{Q_3}{C_3} = \frac{1 \times 10^{-8} \text{ C}}{200 \times 10^{-12} \text{ F}} = 50 \text{ V.}$$

4. Minimum number of capacitors that must be connected in series in a row are

$$n = \frac{1000 \text{ V}}{400 \text{ V}} = 2.5 \approx 3$$

capacitance of 3 capacitors in series in a row is

$$C' = \frac{1}{3} \mu\text{F}$$

Minimum number of rows of 3 capacitors each to be connected in parallel to obtain net capacitance of $2 \mu\text{F}$ are

$$m = \frac{2 \mu\text{F}}{\frac{1}{3} \mu\text{F}} = 6$$

So, minimum number of capacitors required are

$$m \times n = 6 \times 3 = 18$$

Topic 5

1. $U = \frac{1}{2} CV^2 = \frac{1}{2} \times 12 \times 10^{-12} \times 50^2$
 $= 15000 \times 10^{-12} \text{ or } U = 1.5 \times 10^{-8} \text{ J.}$

2. $C_1 = 600 \text{ pF}$, $V_1 = 200 \text{ V}$, $C_2 = 600 \text{ pF}$, $V_2 = 0$

On connecting charged capacitor to uncharged capacitor, the common potential V across the capacitors is

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{600 \times 10^{-12} \times 200 + 0}{(600 + 600) \times 10^{-12}}$$

or $V = 100 \text{ V}$

Energy stored in capacitors before connection is

$$U_i = \frac{1}{2} C_1 V_1^2 + 0 = \frac{1}{2} \times 600 \times 10^{-12} \times 200^2 \text{ or } U_i = 12 \text{ } \mu\text{J}$$

and energy stored in capacitors after connection is

$$U_f = \frac{1}{2} (C_1 + C_2) V^2 = \frac{1}{2} (600 + 600) \times 10^{-12} \times 100^2$$

or $U_f = 6 \text{ } \mu\text{J}$

Hence the energy lost in the process is

$$\Delta = U_f - U_i = (6 - 12) \text{ } \mu\text{J} \text{ or } \Delta U = -6 \text{ } \mu\text{J}.$$

3. $A = 90 \text{ cm}^2 = 90 \times 10^{-4} \text{ m}^2$, $d = 2.5 \text{ mm}$
 $= 2.5 \times 10^{-3} \text{ m}$

$$C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 90 \times 10^{-3}}{2.5 \times 10^{-3}} \text{ or } C = 32 \text{ pF}$$

(a) $U = \frac{1}{2} C V^2 = \frac{1}{2} \times 32 \times 10^{-12} \times 400^2$

or $U = 2.56 \text{ } \mu\text{J}$

(b) $U = \frac{1}{2} C V^2 = \frac{1}{2} \times \frac{\epsilon_0 A}{d} \times (E \cdot d)^2$
 $= \frac{1}{2} \epsilon_0 A E^2 d \text{ or } \frac{U}{A d} = \frac{1}{2} \epsilon_0 E^2$

or Energy per unit volume, $u = \frac{1}{2} \epsilon_0 E^2$

4. $C_1 = 4 \text{ } \mu\text{F}$, $V_1 = 200 \text{ V}$, $C_2 = 2 \text{ } \mu\text{F}$, $V_2 = 0$

So, common potential difference across the two capacitors after connection is

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{4 \times 10^{-6} \times 200 + 0}{(4 + 2) \times 10^{-6}} = 133.33 \text{ V}$$

Initially, total energy stored in capacitors before connection is

$$U_i = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 4 \times 10^{-6} \times 200^2 = 0.08 \text{ J}$$

and total energy stored in capacitors after connection is

$$U_f = \frac{1}{2} (C_1 + C_2) V^2 = \frac{1}{2} (4 + 2) \times 10^{-6} \times 133.33^2$$

or $U_f = 0.053 \text{ J}$

So, energy lost due to connection is

$$\Delta U = U_f - U_i = 0.053 - 0.08$$

or $\Delta U = -0.027 \text{ J}.$

