Electrostatic Potential and Capacitance

CHAPTER 2

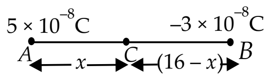
NCERT FOCUS

ANSWERS

Topic 1

1. (i) Let C be the point on the line joining the two charges, where electric potential is zero, then

$V_C = 0$



or $V_{CA} + V_{CB} = 0$

or $V_{CA} = -V_{CB}$

or $$\frac{1}{4\pi\varepsilon_0}\frac{q_A}{r_{CA}} = -\frac{1}{4\pi\varepsilon_0}\frac{q_B}{r_{CB}}$$

or $$\frac{5\times10^{-8}\,\text{C}}{x\times10^{-2}\,\text{m}} = -\frac{(-3\times10^{-8}\,\text{C})}{[(16-x)\times10^{-2}\,\text{m}]} \quad \text{or} \quad \frac{5}{x} = \frac{3}{16-x}$$

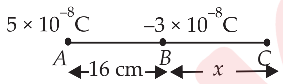
or $80 - 5x = 3x$ or $80 = 8x$

or $x = \dfrac{80}{8}$ or $x = 10$ cm

So, electric potential is zero at distance of 10 cm from charge of 5×10^{-8} C on line joining the two charges between them.

If point C is not between the two charges, then

$V_{CA} + V_{CB} = 0$ or $V_{CA} = -V_{CB}$



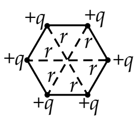
or $$\frac{1}{4\pi\varepsilon_0}\frac{q_A}{r_{CA}} = \frac{-1}{4\pi\varepsilon_0}\frac{q_B}{r_{CB}}$$

or $$\frac{5\times10^{-8}\ \text{C}}{\left[(16+x)\times10^{-2}\,\text{m}\right]} = \frac{-(-3\times10^{-8}\,\text{C})}{[x\times10^{-2}\ \text{m}]} = \frac{5}{16+x} = \frac{3}{x}$$

or $5x = 48 + 3x$ or $2x = 48$ or $x = 24$ cm

So, electric potential is also equal to zero at a distance of 24 cm from charge of -3×10^{-8} C and at a distance of $(24 + 16) = 40$ cm from charge of 5×10^{-8} C, on the side of charge of -3×10^{-8} C.

2. $V = 6 \times \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$



or $V = 6 \times 9 \times 10^9 \times \dfrac{5\times10^{-6}}{10\times10^{-2}}$

or $V = 2.7 \times 10^6$ volts.

3. The length of diagonal of the cube of each side b is

$$\sqrt{3b^2} = b\sqrt{3}$$

Distance of any of the vertices from the centre of cube,

$$r = \frac{\sqrt{3}}{2}b$$

$$V = 8 \times \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = 8\times\frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{3}\,\frac{b}{2}} \quad \text{or} \quad V = \frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$$

$E = 0$, as electric field at centre due to a charge at any corner of cube is just equal and opposite to that of another charge at diagonally opposite corner of cube.

Topic 2

1. (a) Since it is an electric dipole, so a plane normal to AB and passing through its mid-point has zero potential everywhere.

(b) Normal to the plane in the direction AB.

Topic 3

1. $C_0 = \dfrac{\varepsilon_0 A}{d} = 8$ pF

$$C = \frac{K\,\varepsilon_0 A}{d/2} = \frac{2K\varepsilon_0 A}{d} = 2\,KC_0 = 2\times6\times8 \text{ pF}$$

or $C = 96$ pF

2. $A = 6 \times 10^{-3}\ \text{m}^2$, $d = 3$ mm $= 3 \times 10^{-3}$ m, $V = 100$ V

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}}$$

$= 17.7 \times 10^{-12}$ F or $C_0 = 17.7$ pF

On connecting the capacitor across 100 V supply, charge on each plate of the capacitor is

$Q_0 = C_0V = 17.7 \times 10^{-12} \times 100$ or $Q_0 = 1.77 \times 10^{-9}$ C.

3. $C = KC_0 = 6 \times 17.7 = 106.2$ pF

(a) If the voltage supply remained connected, then the potential difference across the capacitor will remain the same *i.e.* $V = 100$ V and hence charge on the capacitor becomes

$Q = CV = 106.2 \times 10^{-12} \times 100$

or $Q = 1.062 \times 10^{-8}$ C

(b) If the voltage supply was disconnected, then charge on the capacitor remains the same

i.e. $Q = 1.77 \times 10^{-9}$ C

and hence potential difference across the capacitor becomes

$$V = \frac{Q}{C} = \frac{1.77\times10^{-9}\,\text{C}}{106.2\times10^{-12}\,\text{F}} \quad \text{or} \quad V = 16.7\ \text{V}$$

4. $r_1 = 13$ cm, $r_2 = 12$ cm, $K = 32$, $Q = 2.5$ μC

(a) Capacitance of capacitor is

$$C = \frac{4\pi\varepsilon_0 k r_1 r_2}{r_1 - r_2} = \frac{1\times 32}{9\times 10^9}\times\frac{13\times 10^{-2}\times 12\times 10^{-2}}{(13-12)\times 10^{-2}}$$

or $C = 5.5 \times 10^{-9}$ F

(b) Electric potential of inner sphere is

$$V_B = V_{BB} + V_{BA}$$

$$= \frac{1}{4\pi\varepsilon_0 k}\left[+\frac{Q}{r_2}-\frac{Q}{r_1}\right] = \frac{Q}{4\pi\varepsilon_0 k}\left[\frac{r_1 - r_2}{r_1 r_2}\right]$$

$$= \frac{9\times 10^9}{32} 2.5\times 10^{-6}\left[\frac{13-12}{13\times 12}\right]\times\frac{10^{-2}}{10^{-4}} = 4.5\times 102 \text{ V.}$$

(c) Capacitance of isolated sphere of radius 12 cm is

$$C_0 = 4\pi\varepsilon_0 r_2 = \frac{1}{9\times 10^9}\times 12\times 10^{-2}$$

or $C_0 = 1.3 \times 10^{-11}$ F

Here $C > C_0$, because a single conductor A can be charged to a electric potential till it reaches the breakdown value of surroundings. But when another earthed metallic conductor B is brought near it, negative charge induced on it decreases the electric potential on A, hence more charge can not be stored on A.

Topic 4

1. (a) $C = \frac{C_1}{3} = \frac{9}{3}$ pF or $C = 3$ pF

(b) Net charge stored in combination of capacitors is

$Q = CV = 3 \times 10^{-12} \times 120 = 360$ pC

So, potential difference across each capacitor is

$$V_1 = \frac{Q}{C_1} = \frac{360 \text{ pC}}{9 \text{ pF}} \quad \text{or} \quad V_1 = 40 \text{ volts.}$$

2. (a) $C = C_1 + C_2 + C_3 = 2 + 3 + 4$ or $C = 9$ pF

(b) Since the capacitors are in parallel, so potential difference across each of them is same *i.e.*

$V_1 = V_2 = V_3 = 100$ V

So, charges stored on capacitors are

$Q_1 = C_1V_1 = 2 \times 100 = 200$ pC

$Q_2 = C_2V_2 = 3 \times 100 = 300$ pC

$Q_3 = C_3V_3 = 4 \times 100 = 400$ pC

3. C_2 and C_3 are in series, so, $C = 100$ pF

$\because$ C_1 and C' are parallel,

so $C'' = C_1 + C' = 100 + 100$ or $C'' = 200$ pF

C_4 and C'' are in series, so net capacitance of the network is

$$\frac{1}{C} = \frac{1}{C''} + \frac{1}{C_4} = \frac{1}{200} + \frac{1}{100} = \frac{1+2}{200}$$

$$\text{or} \quad C_2 = \frac{200}{3} \text{ pF} = 66.7 \text{ pF}$$

Net charge stored on the combination is

$$\because \quad = CV = \frac{200}{3}\times 10^{-12}\times 300 = 2\times 10^{-8} \text{ C}$$

As C'' and C_4 are in series, so

$Q'' = Q_4 = Q$ or $Q'' = Q_4 = 2 \times 10^{-8}$ C

$$\text{and hence } V'' = \frac{Q''}{C''} = \frac{2\times 10^{-8}\text{C}}{200\times 10^{-12}\text{F}} = 100 \text{ V}$$

$$\text{and } V_4 = \frac{Q_4}{C_4} = \frac{2\times 10^{-8}\text{C}}{100\times 10^{-12}\text{F}} = 200 \text{ V}$$

$\because$ C_1 and C' are in parallel, so

$V_1 = V' = V''$ or $V_1 = V' = 100$ V

and hence $Q_1 = C_1V_1 = 100 \times 10^{-12} \times 100 = 1 \times 10^{-8}$ C

and $Q' = C'V' = 100 \times 10^{-12} \times 100 = 1 \times 10^{-8}$ C

C_2 and C_3 are in parallel, so $Q_2 = Q_3 = Q'$

or $Q_2 = Q_3 = 1 \times 10^{-8}$ C

$$\text{and hence } V_2 = \frac{Q_2}{C_2} = \frac{1\times 10^{-8}\text{C}}{200\times 10^{-12}\text{F}} = 50 \text{ V}$$

$$\text{and } V_3 = \frac{Q_3}{C_3} = \frac{1\times 10^{-8}\text{C}}{200\times 10^{-12}\text{F}} = 50 \text{ V.}$$

Topic 5

1. $U = \frac{1}{2}CV^2 = \frac{1}{2}\times 12\times 10^{-12}\times 50^2$

$= 15000 \times 10^{-12}$ or $U = 1.5 \times 10^{-8}$ J.

2. $C_1 = 600$ pF, $V_1 = 200$ V, $C_2 = 600$ pF, $V_2 = 0$

On connecting charged capacitor to uncharged capacitor, the common potential V across the capacitors is

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{600\times 10^{-12}\times 200 + 0}{(600+600)\times 10^{-12}}$$

or $V = 100$ V

Energy stored in capacitors before connection is

$$U_i = \frac{1}{2}C_1V_1^2 + 0 = \frac{1}{2}\times 600\times 10^{-12}\times 200^2 \quad \text{or} \quad U_i = 12 \ \mu\text{J}$$

and energy stored in capacitors after connection is

$$U_f = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(600+600)\times 10^{-12}\times 100^2$$

or $U_f = 6$ μJ

Hence the energy lost in the process is

$\Delta = U_f - U_i = (6 - 12)$ μJ or $\Delta U = -6$ μJ.

3. $A = 90$ cm$^2 = 90 \times 10^{-4}$ m^2, $d = 2.5$ mm

$= 2.5 \times 10^{-3}$ m

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times 10^{-12}\times 9\times 10^{-3}}{2.5\times 10^{-3}} \quad \text{or} \quad C = 32 \text{ pF}$$

(a) $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 32 \times 10^{-12} \times 400^2$

or $U = 2.56$ μJ

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2}\times\frac{\varepsilon_0 A}{d} \times (E.d)^2$

$$= \frac{1}{2}\varepsilon_0 AE^2 d \quad \text{or} \quad \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

or Energy per unit volume, $u = \frac{1}{2}\varepsilon_0 E^2$

4. $C_1 = 4\ \mu F$, $V_1 = 200\ V$, $C_2 = 2\ \mu F$, $V_2 = 0$

So, common potential difference across the two capacitors after connection is

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{4\times10^{-6}\times200+0}{(4+2)\times10^{-6}} = 133.33\ V$$

Initially, total energy stored in capacitors before connection is

$$U_i = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times 4\times10^{-6}\times 200^2 = 0.08\ J$$

and total energy stored in capacitors after connection is

$$U_f = \frac{1}{2}(C_1+C_2)V^2 = \frac{1}{2}(4+2)\times10^{-6}\times133.33^2$$

or $U_f = 0.053\ J$

So, energy lost due to connection is

$$\Delta U = U_f - U_i = 0.053 - 0.08$$

or $\Delta U = -0.027\ J$.