

Current Electricity

Topic 1

1. Due to negative charge on earth, an electric field is into the earth surface due to which the positive ions of atmosphere are constantly pumped in and an equivalent current of 1800 A is established across the globe.

Let us first calculate total negative charge on earth.

$$Q = \sigma A = 10^{-9}[4\pi R_e^2]$$

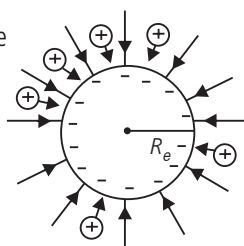
$$Q = 10^{-9}[4\pi(6.37 \times 10^6)^2]$$

$$Q = 509.65 \times 10^3 \text{ C}$$

Now time to neutralize

$$I = \frac{Q}{t}$$

$$t = \frac{Q}{I} = \frac{509.65 \times 10^3}{1800} \quad \text{or} \quad t = 283.14 \text{ s}$$



2. We can first calculate drift velocity of the electrons from the given data

$$I = Anev_d$$

$$3 = 2 \times 10^{-6} \times 8.5 \times 10^{28} \times 1.6 \times 10^{-19} v_d$$

$$v_d = 0.11 \times 10^{-3} \text{ m s}^{-1}$$

Now time taken by an electron to drift a length of 3 m with drift speed is

$$t = \frac{l}{v_d} = 27.27 \times 10^3 \text{ s.}$$

3. We can find resistance of the alloy manganin for all the readings as follows :

Current A	Voltage V	Resistance Ω
0.2	3.94	19.7 Ω
0.4	7.87	19.675 Ω
0.6	11.8	19.667 Ω
0.8	15.7	19.625 Ω
1.0	19.7	19.7 Ω
2.0	39.4	19.7 Ω
3.0	59.2	19.13 Ω
4.0	78.8	19.7 Ω
5.0	98.6	19.72 Ω
6.0	118.5	19.75 Ω
7.0	138.2	19.74 Ω
8.0	158.0	19.75 Ω

Here we can conclude that Ohm's law is valid to a high accuracy and resistance of the alloy is nearly constant at all currents.

Topic 2

1. As $R_2 = R_1 [1 + \alpha(t_2 - t_1)]$
 $\therefore 117 = 100 [1 + 1.7 \times 10^{-4}(t - 27)]$
 or $17 = 1.7 \times 10^{-2}(t - 27)$
 or $1000 = t - 27$ or $t = 1027^\circ\text{C}$.

2. We know the relation, $R = \rho \frac{l}{A}$
 $5 = \rho \frac{15}{6 \times 10^{-7}}$
 \therefore Resistivity, $\rho = \frac{30 \times 10^{-7}}{15} = 2 \times 10^{-7} \Omega \text{ m}$

3. $T_1 = 27.5^\circ\text{C}$, $R_1 = 2.1 \Omega$, $T_2 = 100^\circ\text{C}$ and $R_2 = 2.7 \Omega$

Using the relation,

$$R_2 = R_1 [1 + \alpha(T_2 - T_1)], \text{ we have}$$

Temperature coefficient of resistivity of silver,

$$\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)} = \frac{2.7 - 2.1}{2.1 \times (100 - 27.5)} = \frac{0.6}{152.25}$$

$$= 3.94 \times 10^{-3} \text{ or } 0.0039^\circ\text{C}^{-1}$$

4. At room temperature 27°C , the resistance of the heating element.

$$R_{27^\circ} = \frac{230}{3.2} = 71.875 \Omega$$

At the steady temperature $t^\circ\text{C}$, the resistance.

$$R_t = \frac{230}{2.8} = 82.143 \Omega$$

Now, $R_2 = R_1 [1 + \alpha(t_2 - t_1)]$

$$\text{or } 82.143 = 71.875 [1 + 1.7 \times 10^{-4}(t - 27)]$$

$$\text{or } 0.0840 \times 10^4 = t - 27$$

$$\text{or } 840 = t - 27 \quad \text{or} \quad t = 867^\circ\text{C}.$$

5. Two wires have same length, and resistance. As the specific resistances are unequal, the areas are different.

$$\text{For copper wire, } R_{\text{Cu}} = \rho_{\text{Cu}} \frac{l}{A_{\text{Cu}}}$$

$$\text{For aluminium wire, } R_{\text{Al}} = \rho_{\text{Al}} \frac{l}{A_{\text{Al}}}$$

$$\text{So, } \rho_{\text{Cu}} \frac{l}{A_{\text{Cu}}} = \rho_{\text{Al}} \frac{l}{A_{\text{Al}}}$$

$$\frac{A_{\text{Al}}}{A_{\text{Cu}}} = \frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}} = \frac{2.63 \times 10^{-8}}{1.72 \times 10^{-8}} = \frac{263}{172}$$

$$\text{Ratio of masses, } \frac{M_{\text{Al}}}{M_{\text{Cu}}} = \frac{d_{\text{Al}}/A_{\text{Al}}}{d_{\text{Cu}}/A_{\text{Cu}}}$$

$$= \frac{2.7 \times 263}{8.9 \times 172} = 0.46$$

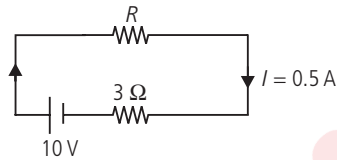
$$M_{Al} < M_{Cu}$$

Thus, the aluminium wire for the same resistance is very light than copper and that is why aluminium wires are preferred for overhead power cables.

6. (a) Alloys of metals usually have greater resistivity than that of their constituent metals.
 (b) Alloys usually have much lower temperature coefficients of resistance than pure metals.
 (c) The resistivity of the alloy manganin is nearly independent of increasing temperature.
 (d) The resistivity of a typical insulator (e.g. amber) is greater than that of a metal by factor of the order of 10^{22} .

Topic 3

1. $I = \frac{E}{R+r}$
 $0.5 = \frac{10}{R+3}$
 $R+3 = 20$
 $R = 17 \Omega$

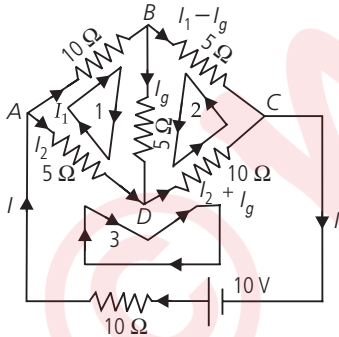


Terminal voltage of the battery

$$V = E - Ir$$

$$V = 10 - 0.5 \times 3 = 8.5 \text{ V}$$

2. Let us first distribute the current in different branches. Now, equations for different loops using Kirchoff's IInd law,



Loop 1

$$\Sigma E = \Sigma IR$$

$$10I_1 + 5I_g - 5I_2 = 0 \quad \text{or} \quad 2I_1 + I_g - I_2 = 0 \quad \dots(i)$$

Loop 2

$$\Sigma E = \Sigma IR$$

$$5I_g + 10[I_2 + I_g] - 5[I_1 - I_g] = 0$$

$$10I_2 + 20I_g - 5I_1 = 0 \quad \text{or} \quad 2I_2 + 4I_g - I_1 = 0 \quad \dots(ii)$$

Loop 3

$$5I_2 + 10(I_2 + I_g) + 10I = 10$$

$$15I_2 + 10I_g + 10I = 10$$

$$\text{or} \quad 3I_2 + 2I_g + 2I = 2 \quad \dots(iii)$$

Solving equations (i) and (ii)

$$2I_1 + I_g - I_2 = 0$$

$$[-I_1 + 4I_g + 2I_2 = 0] \times 2$$

$$\text{or} \quad 9I_g + 3I_2 = 0 \quad \text{or} \quad I_2 = -3I_g \quad \dots(iv)$$

In the loop ABCDA

$$10I_1 + 5[I_1 - I_g] - 10[I_2 + I_g] - 5I_2 = 0$$

$$15I_1 - 15I_2 - 15I_g = 0 \quad \text{or} \quad I_1 - I_2 - I_g = 0 \quad \dots(v)$$

Solving equations (ii) and (v)

$$2I_2 + 4I_g - I_1 = 0 \quad \text{or} \quad 2(I_1 - I_2 - I_g) = 0$$

$$\text{or} \quad 2I_g + I_1 = 0 \quad \text{or} \quad I_1 = -2I_g \quad \dots(vi)$$

Now using the result of (iv) and (vi) in equation (iii)

$$3I_2 + 2I_g + 2I = 2$$

$$-3[3I_g] + 2I_g + 2I = 2 \quad \text{or} \quad 2I - 7I_g = 2 \quad \dots(vii)$$

Using Kirchoff's law, $I = I_1 + I_2$

$$I = -5I_g$$

So, equation (vii)

$$2[-5I_g] - 7I_g = 2 \quad \text{or} \quad -17I_g = 2$$

So, finally $I_g = -2/17 \text{ A}$ and $I = \frac{+10}{17} \text{ A}$

$$\text{Also} \quad I_1 = \frac{4}{17} \text{ A}, \quad I_2 = \frac{6}{17} \text{ A}$$

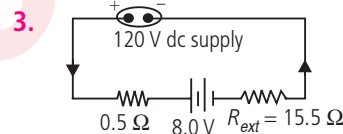
$$\text{Current in branch AB} = \frac{4}{17} \text{ A}$$

$$\text{Current in branch AD} = \frac{6}{17} \text{ A}$$

$$\text{Current in branch BD} = \frac{-2}{17} \text{ A}$$

$$\text{Current in branch BC} = \frac{6}{17} \text{ A}$$

$$\text{Current in branch DC} = \frac{4}{17} \text{ A}$$



During charging, the electric current is sent into the 8.0 V battery. The current in the circuit

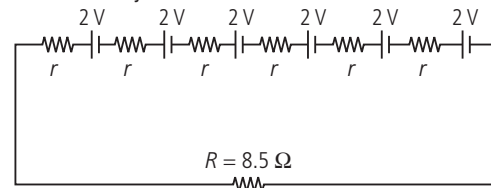
$$I = \frac{E_1 - E_2}{R_{\text{ext}} + r} = \frac{120 - 8}{15.5 + 0.5} \quad \text{or} \quad I = \frac{112}{16} = 7 \text{ A}$$

Now, terminal voltage of the battery during charging

$$V = E + Ir = 8 + 7(0.5) = 11.5 \text{ V}$$

A series resistance is joined in the charging circuit to limit the excessive current so that charging is slow and permanent.

4. (a) Six cells are joined in series.

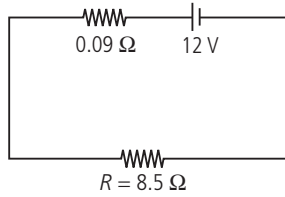


Equivalent emf is $2 \times 6 = 12 \text{ V}$

Equivalent internal resistance is $0.015 \times 6 = 0.09 \Omega$

Current drawn from supply

$$I = \frac{12}{8.5 + 0.09} = \frac{12}{8.59} = 1.39 \text{ A}$$



Terminal voltage of battery,

$$V = E - Ir = 12 - 1.39 \times 0.09$$

$$\text{or } V = 12 - 0.1251 = 11.8 \text{ V}$$

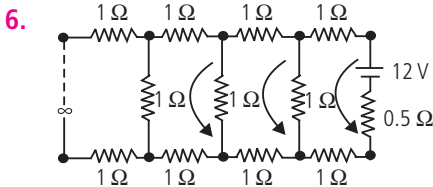
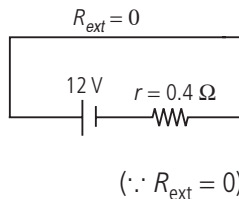
(b) Maximum current is drawn from a battery when external resistance is treated to be zero.

$$I_{\text{max}} = \frac{1.9}{380} = 0.005 \text{ A}$$

To start a car, a current of the order of 100 A is needed, so the battery mentioned above can not drive the starting motor.

5. The maximum current will be obtained when no external resistance is offered by wire joining the two terminals.

$$I = \frac{E}{R_{\text{ext}} + r} \Rightarrow I = \frac{12}{0.4} = 30 \text{ A}$$



As shown in diagram the loop of resistance shown by arrow is repeated at times, let us assume the equivalent resistance is x , so by adding one more loop the resistance will remain as x .

$$x = 2 + \frac{x}{1+x}$$

$$(1+x)x = 2(1+x) + x$$

$$x + x^2 = 2 + 3x$$

$$x^2 - 2x - 2 = 0$$

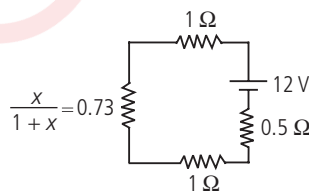
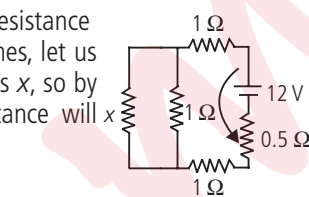
$$x = \frac{2 \pm \sqrt{4 - 4 \times 1 \times (-2)}}{2}$$

$$x = (1 + \sqrt{3}) \Omega = 2.732 \Omega$$

So, the equivalent circuit can be

Now, current drawn

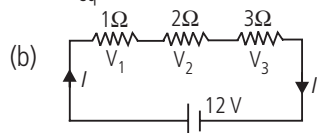
$$I = \frac{12}{3.23} = 3.71 \text{ A}$$



7. (a) Total resistance of the combination in series

$$R_{\text{eq}} = R_1 + R_2 + R_3$$

$$R_{\text{eq}} = 1 \Omega + 2 \Omega + 3 \Omega = 6 \Omega$$



Current in the circuit, $I = \frac{12}{6} = 2 \text{ A}$

In series, same current flows through each resistor.

Now potential drop across 1 Ω,

$$V_1 = IR_1 = 2 \times 1 = 2 \text{ V}$$

Potential drop across 2 Ω,

$$V_2 = IR_2 = 2 \times 2 = 4 \text{ V}$$

$$IR_2 = 2 \times 2 = 4 \text{ V}$$

Potential drop across 3 Ω, $V_3 = IR_3 = 2 \times 3 = 6 \text{ V}$

8. (a) Total resistance of the combination in parallel

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

or $\frac{1}{R_{\text{eq}}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$

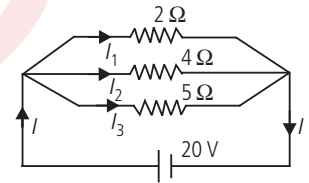
$$R_{\text{eq}} = \frac{20}{19} = 1.05 \Omega$$

(b) Potential of 20 V will be same across each resistor, so current

$$I_1 = \frac{V}{R_1} = \frac{20}{2} = 10 \text{ A}$$

$$I_2 = \frac{V}{R_2} = \frac{20}{4} = 5 \text{ A}$$

$$I_3 = \frac{V}{R_3} = \frac{20}{5} = 4 \text{ A}$$



Total current drawn from the cell

$$I = I_1 + I_2 + I_3 \text{ or } I = 19 \text{ A.}$$

9. (a) The current will be constant because it is given to be steady.

Current density, $J = \frac{I}{A}$ will not be constant in non-uniform cross-section.

The electric field E is also variable in non uniform cross-section.

$$J = \sigma E$$

$$\frac{I}{A} = \sigma E \Rightarrow E = \frac{I}{\sigma A}. \text{ So } E \propto \frac{1}{A}$$

Drift speed also changes in non uniform cross-section.

$$J = nev_d$$

$$v_d = \frac{I}{Ane}, \text{ so } v_d \propto \frac{1}{A}$$

(b) Ohm's law is not a fundamental law in nature. It is not universally followed. Semiconductor diodes, transistors, thermistors, vacuum tubes etc. do not follow Ohm's law.

(c) If emf of supply battery is E and internal resistance r , then current through an external resistance R is given by

$$I = \frac{E}{R+r}$$

so, internal resistance r should be least to supply high currents.

(d) In high tension supply, the internal resistance is made large. Because, if accidentally the short circuiting take place, the excessive current produced should not cross the safety limits.

