CHAPTER 4

Moving Charges and Magnetism

EXAM DRILL

ANSWERS

1. (a) : Force on a charged particle due to circular motion

$F = \frac{mv^2}{r}$ ...(i)

Force on a charged particles due to magnetic field

$F_B = qvB$ ...(ii)

Using equation (i) and (ii)

$\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB}$

The time period $T = \frac{2\pi r}{v}$

$\Rightarrow T = \frac{2\pi}{v} \times \left(\frac{mv}{qB}\right) \Rightarrow T = \frac{2\pi m}{qB}$

Therefore, time period is independent of speed of particle.

2. (d) : A stationary charge does not experience any force in a magnetic field. Because, force increases if the speed v is increased, in this case speed $v = 0$, then $F = Bqv \Rightarrow F = 0$

3. (b) : Consider any small segment of the wire and find the direction in which the magnetic force acts on it.

By using $i d\vec{l} \times \vec{B}$.

The magnetic force acts radially outward. Now if we sum up the effects on all of the segments, it drill to stretch the wire.

4. (b) : Magnetic field due to linear portion. Any element $d\vec{l}$ of linear portions like PQ or ST will make angles 0° or π with the position vector $\vec{r}$. Therefore, field at O due to linear portion is

$B = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin\theta}{r^2} = 0$

Magnetic field due to semi-circular portion. Any element $d\vec{l}$ on this portion will be perpendicular to the position vector $\vec{r}$, therefore, field due to one such element at point will be

$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin\pi/2}{r^2} = \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$

Magnetic field due to the entire circular portion is given by

$B = \int dB = \frac{\mu_0 I}{4\pi r^2} \int dl = \frac{\mu_0 I}{4\pi r^2} \cdot \pi r = \frac{\mu_0 I}{4r}$

$\therefore$ Total magnetic field at point $O = \frac{\mu_0 I}{4r}$.

5. (c) : $\vec{F} = q(\vec{v} \times \vec{B})$

$\therefore F = qvB\sin\theta$ which shows magnetic force is velocity dependent due to which it differs from one inertial frame to another.

6. (c) : Magnetic field will be in same direction with equal magnitude because of the incoming and outgoing current. So, net effect will not be zero.

7. (d) : By Ampere's circuital law,

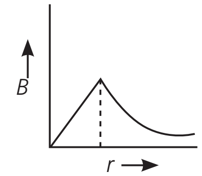
$\oint \vec{B} \cdot d\vec{l} = \mu_0 \times$ current enclosed

$\Rightarrow B \cdot 2\pi r = \mu_0 \times \frac{Ir^2}{a^2} \Rightarrow B = \frac{\mu_0 Ir}{2\pi a^2}$ (for $r < a$)

At surface of $r = a$, so $B = \frac{\mu_0 I}{2\pi a}$

$\therefore B = \frac{\mu_0 I}{2\pi r}$ (for $r > a$)

The variation of magnetic field with distance 'r' from the axis is given by



8. (a) : Here, $l = 50$ cm, $N = 100$, $i = 2.5$ A

Magnetic field inside the solenoid,

$B = \mu_0 ni = \frac{\mu_0 Ni}{l}$

$\Rightarrow B = \frac{4\pi \times 10^{-7} \times 100 \times 2.5}{0.5} = 6.28 \times 10^{-4}$ T

9. (a) : Magnetic moment is given by

$M = IA = \frac{q}{t}(\pi r^2)$

$= \frac{q}{(2\pi/\omega)}(\pi r^2) \quad \left[\because t = T = \frac{2\pi}{\omega}\right] = \frac{1}{2} q\omega r^2$

10. (a) : Here, $\vec{B} = B_0(\hat{i} + 2\hat{j} - 4\hat{k})$; $\vec{v} = v_0(3\hat{i} - \hat{j} + 2\hat{k})$

$\vec{F} = \vec{F}_e + \vec{F}_m \therefore \vec{F} = 0$ or $\vec{F}_e = -\vec{F}_m$

$\vec{F}_e = -q(\vec{v} \times \vec{B}) = -qv_0B_0[(3\hat{i} - \hat{j} + 2\hat{k}) \times (\hat{i} + 2\hat{j} - 4\hat{k})]$

$= -qv_0B_0(14\hat{j} + 7\hat{k})$

The electric field produced by the charge q, will be,

$\vec{E} = \frac{\vec{F}_e}{q} = -\frac{qv_0B_0(14\hat{j} + 7\hat{k})}{q}$ or $\vec{E} = -v_0B_0(14\hat{j} + 7\hat{k})$

11. (a) : Magnetic dipole moment of the current loop = Ampere turns × Area of the coil or $M = NI\pi r^2$

When radius of the coil becomes double, new magnetic moment will be $M_1 = NI\pi(2r)^2 = 4NI\pi r^2 = 4M$

Hence, magnetic moment becomes four times when radius is doubled.

12. (c) : Magnetic force on moving charge is always perpendicular to its velocity. $\vec{F} = q(\vec{v} \times \vec{B})$

Also, work done by magnetic force on a moving charge is zero.

13. (i) (d)

(ii) (a) : Using, $qvB\sin\theta = \dfrac{mv^2}{r}$

$r \propto \dfrac{1}{\sin\theta}$ for the same values of m, v, q and B

$\therefore \quad \dfrac{r_A}{r_B} = \dfrac{\sin 90°}{\sin 30°} = 2$ or $r_A = 2r_B$ or $r_B < r_A$

(iii) (d) : The radius of the helical path of the electron in the uniform magnetic field is

$$r = \frac{mv_\perp}{eB} = \frac{mv\sin\theta}{eB} = \frac{(2.4\times10^{-23}\ \text{kg m/s})\times\sin 30°}{(1.6\times10^{-19}\ \text{C})\times 0.15\ \text{T}}$$

$$= 5\times10^{-4}\ \text{m} = 0.5\times10^{-3}\ \text{m} = 0.5\ \text{mm}$$

14. The magnetic force experienced by the charge q moving with velocity $\vec{v}$ in magnetic field $\vec{B}$ is given by Lorentz force, $\vec{F} = q(\vec{v} \times \vec{B})$

The direction of the Lorentz force is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. Its direction is given by right-handed screw rule.

15. For undeflected charged particle

$F_m = F_e$ or $qvB \sin 90° = qE$

$\therefore \quad v = \dfrac{E}{B}$

As the electric field is switched off, the charged particle is moving x-axis. The perpendicular magnetic field will deflect it along a circular trajectory in the XY plane.

16. (i) It should have low torsional constant, *i.e.,* restoring torque per unit twist should be small.

(ii) It should be non-magnetic substances and good conductor of electricity.

OR

One tesla is defined as the magnitude of magnetic field which produces a force of 1 newton when a charge of 1 coulomb moves perpendicularly in the region of the magnetic field at a velocity of 1 m/s.

$F = qvB \Rightarrow B = \dfrac{F}{qv}$ or $1\ \text{T} = \dfrac{1\ \text{N}}{(1\ \text{C})(1\ \text{m/s})}$

17. Here, $N = 10$, $r = 8$ cm $= 0.08$ m, $I = 2$ A

$$\therefore \quad B = \frac{\mu_0 NI}{2r} = \frac{4\pi\times10^{-7}\times10\times2}{2\times0.08} = 1.57\times10^{-4}\ \text{T}$$

As the current flows clockwise when seen from above the coil, the magnetic field at the centre of the coil points vertically downwards.

18. Here $N = 1$, $A = 5\times10^{-2}$ m^2,
$B = 2\times10^{-2}$ Wb m^{-2}, $k = 4\times10^{-9}$ Nm deg^{-1}

Current sensitivity is

$$\frac{NBA}{k} = \frac{1\times2\times10^{-2}\times5\times10^{-2}}{4\times10^{-9}} = 0.25\times10^6\ \text{deg}\ A^{-1}$$

$= 0.25$ deg μ A^{-1}

19. When a charge particle enters in a region of uniform magnetic field, perpendicular to their path, they move in circular path.

The time period will be $T = \dfrac{2\pi m}{qB}$

But $f = \dfrac{1}{T}$, $f = \dfrac{qB}{2\pi m}$

Since B and q is same for electron and proton.

So, $f \propto \dfrac{1}{m}$

Since mass of electron is smaller than that of proton, hence it will have higher frequency.

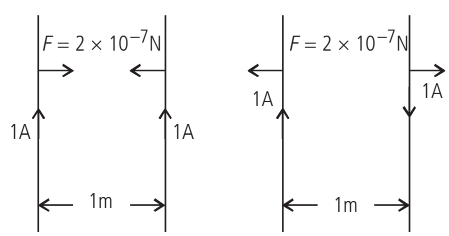
20. One ampere is the value of steady current which maintained in each of the two very long, straight, parallel conductors having negligible cross section and placed one metre apart in vacuum, would produce on each of these conductors a force of attractive or repulsive nature of magnitude 2×10^{-7} N m^{-1} on their unit length.

Force between two straight parallel current carrying conductors,

$F = \dfrac{\mu_0}{2\pi}\dfrac{I_1 I_2}{r}$ when,

$I_1 = I_2 = 1$ A, $r = 1$ m, then

$F = 2\times10^{-7}$ N m^{-1}



21. Ampere found that the distribution of magnetic lines of force around a finite, current carrying solenoid is similar to that produced by a bar magnet.

The magnetic induction at a point along the axis of a circular coil carry current is

$$B = \frac{\mu_0 n I a^2}{2\left(a^2 + x^2\right)^{3/2}}$$

Take $x >> a$, a^2 is small

$n = 1 \quad A = \pi a^2$

$$B = \frac{\mu_0 IA}{2\pi x^3} \qquad \text{...(i)}$$

The magnetic induction at a point along the axis line of bar magnet is

$$B = \frac{\mu_0}{4\pi}\left(\frac{2M}{x^3}\right) \qquad \text{...(ii)}$$

Compare equation (i) and (ii)

$M = IA$

Hence, a current loop is equivalent to a magnetic dipole.

22. The magnetic field due to a long straight line is

$$B = \frac{\mu_0 i}{2\pi d} \qquad \text{...(i)}$$

Where d is the distances from the wire.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \Rightarrow \mu_0 = \frac{1}{c^2 \varepsilon_0} \qquad \text{...(ii)}$$

Using equation (i) and (ii)

$$B = \frac{i}{2\pi d c^2 \varepsilon_0}$$

23. When two wires are connected in parallel, the current in both of them is in the same direction then the wire attract each other. Hence, the two wires will come near to each other.

Now, when two wire are connected in parallel, the current in both of them is in the opposite direction then the wires repel each other. Hence, the two wires will go away each other.

24. Voltmeter resistance being very high, when connected in series, it makes the effective resistance of the circuit very high. Due to it, the current in the circuit becomes extremely small. Since ammeter measures the current, hence the deflection of ammeter is almost zero. As voltmeter measure potential differences between the two points, it will show the reading due to voltage of the battery.

OR

Biot-Savart's law consider the contribution of each element of current in a conductor to determine the magnetic field, while in the Ampere's law one need to know the current passing through a given surfaces.

Ampere's Law is more convenient to use even though both laws are equally valid in all situations.

25. The main advantage of using radial magnetic field is that maximum torque is obtained and torque is uniform in all position of the moving coil galvanometer.

The angle between the plane of the coil and magnetic field is zero in every position of coil. As the radial magnetic field is parallel to the coil and provides constant torque. This make the deflection directly proportional to current and helps to measure the current easily.

OR

When a proton, a deuteron and an alpha particle are accelerated through potential difference V, then their energies are

$E_p = \text{eV}, E_d = \text{eV}, E_\alpha = 2\text{ eV}$

(i) $KE_p : KE_d : KE_\alpha = 1 : 1 : 2$

(ii) $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2mKE}}{qB}$

$$r_p : r_d : r_\alpha = \frac{\sqrt{m_p}}{e} : \frac{\sqrt{2m_p}}{e} : \frac{\sqrt{4m_p}}{2e} = 1 : \sqrt{2} : 1$$

As $r_p = 5$ cm $\therefore$ $rd = 5\sqrt{2}$ cm, $r_\alpha = 5$ cm

26. Magnetic field produced by current I_2 at any point on the wire AB is

$B_2 = \dfrac{\mu_0 I_2}{2\pi h}$, normally into the plane of paper

Force exerted by field B_2 on length l of wire AB,

$F_1 = I_1 l B_2 \sin 90°$

$$= I_2 l \frac{\mu_0 I_2}{2\pi h} = \frac{\pi_0 I_1 I_2 . l}{2\pi h}$$

= Force acting per unit length of wire AB,

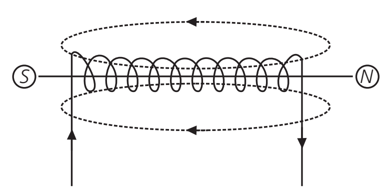
$\dfrac{F_1}{l} = \dfrac{\mu_0 I_1 I_2}{2\pi h}$, away from CD (repulsive)

For equilibrium of wire AB,

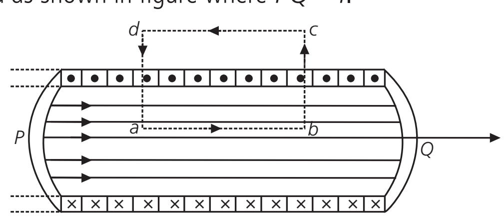
Weight per unit length of AB = Magnetic force per unit length

or $\quad \dfrac{mg}{l} = \dfrac{\mu_0 I_1 I_2}{2\pi h}$

27.



Consider a rectangular amperian loop $abcd$ near the middle of solenoid as shown in figure where $PQ = l$.



Let the magnetic field along the path ab be B and is zero along cd. As the paths bc and da are perpendicular to the axis of solenoid, the magnetic field component along these paths is zero. Therefore, the path bc and da will not contribute to the line integral of magnetic field B.

Total number of turns in length $l = nl$

The line integral of magnetic field induction B over the closed path $abcd$ is

$$\int_{abcd} \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} + \int_d^a \vec{B}\cdot d\vec{l}$$

$$\because \quad \int_a^b \vec{B}\cdot d\vec{l} = \int_a^b B\, dl \cos 0^\circ = Bl$$

and $\int_b^c \vec{B}\cdot d\vec{l} = \int_b^c B\, dl\, \cos 90^\circ = 0 = \int_d^a \vec{B}\cdot d\vec{l}$

Also $\int_c^d \vec{B}\cdot d\vec{l} = 0$ ($\because$ Outside the solenoid, $B = 0$)

$$\therefore \quad \int_{abcd} \vec{B}\cdot d\vec{l} = Bl + 0 + 0 + 0 = Bl \qquad \text{...(i)}$$

Using Ampere's circuital law

$\int_{abcd} \vec{B}\cdot d\vec{l} = \mu_0 \times$ total current in rectangle $abcd$

$= \mu_0 \times$ number of turns in rectangle $\times$ current

$= \mu_0 \times nl \times i = \mu_0\, nl\, i$ ...(ii)

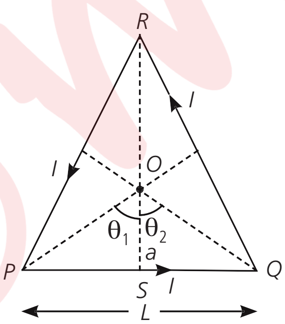
From (i) and (ii), we have

$Bl = \mu_0 nl\, i \;\; \therefore\; B = \mu_0 ni$

It gives magnetic field strength inside straight current carrying solenoid, directed along the axis of solenoid.

28. The situation is shown in figure. The magnetic field at the centre O due to the current through side PQ is given by

$$B_1 = \frac{\mu_0 I}{4\pi a}[\sin\theta_1 + \sin\theta_2]$$



where a is the distance of PQ from O and θ_1, θ_2 are the angles as shown. The magnetic field due to each of the three sides is the same in magnitude and direction, therefore, total magnetic field at O is

$$B = 3B_1 = \frac{3\mu_0 I}{4\pi a}[\sin\theta_1 + \sin\theta_2]$$

Here, $I = 1.0$ A, $\theta_1 = \theta_2 = 60^\circ$, $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}

$$\frac{PS}{OS} = \tan\theta_1 \;\; or \;\; \frac{L/2}{a} = \tan 60^\circ$$

$$\therefore \quad a = \frac{L}{2\tan 60^\circ} = \frac{4.5\times 10^{-2}}{2\sqrt{3}}\,\text{m}$$

$$\therefore \quad B = \frac{3\times 4\pi \times 10^{-7}\times 1.0\times 2\sqrt{3}}{4\pi \times 4.5\times 10^{-2}}[\sin 60^\circ + \sin 60^\circ]$$

$$= \frac{6\sqrt{3}\times 10^{-5}}{4.5}\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right] = 4\times 10^{-5}\,\text{T},$$

directed normally outwards.

OR

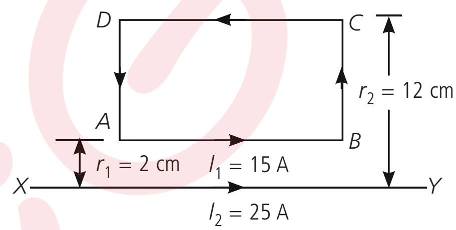
For the net magnetic field at the point O to be zero, the direction of current in loop l_2 should be opposite to that in loop l_1.

Magnitude of magnetic field due to current i_1 in l_2 = Magnitude of magnetic field due to current i_2 in l_2

$$\text{or} \quad \frac{\mu_0 i_1 (0.03)^2}{2[(0.03)^2 + (0.04)^2]^{3/2}} = \frac{\mu_0 i_2 (0.04)^2}{2[(0.04)^2 + (0.03)^2]^{3/2}}$$

$$\text{or} \quad i_2 = \frac{(0.03)^2}{(0.04)^2} i_1 = \frac{9}{16}\times 1\,\text{A} = 0.56\ \text{A}.$$

29. Consider the rectangular loop $ABCD$ placed near a long straight conductor XY, as shown in figure. The arm AB will get attracted, while CD will get repelled. Forces on arms BC and AD, being equal, opposite and collinear, will cancel each other.



Current through the rectangular loop, $I_1 = 15$ A

Current through the long wire XY, $I_2 = 25$ A

Force on AB,

$$F_1 = \frac{\mu_0}{4\pi}\cdot\frac{2I_1 I_2}{r_1}\times \text{ length of conductor } AB$$

$$= \frac{10^{-7}\times 2\times 15\times 25}{2.0\times 10^{-2}}\times 25\times 10^{-2}$$

$= 9.375 \times 10^{-4}$ N (Attractive)

Force on CD,

$$F_2 = \frac{\mu_0}{4\pi}\cdot\frac{2I_1 I_2}{r_1}\times \text{ length of conductor } CD$$

$$= \frac{10^{-7}\times 2\times 15\times 25}{12.0\times 10^{-2}}\times 25\times 10^{-2}$$

$= 1.5625 \times 10^{-4}$ (Repulsive)

$\therefore$ Net force on the loop,

$F = F_1 - F_2 = 9.375 \times 10^{-4} - 1.5625 \times 10^{-4}$

$= 7.8125 \times 10^{-4}$ N $\simeq 7.8 \times 10^{-4}$ N (Attractive)

Thus the force on the loop will act towards the long conductor (attractive), if the current in its closer side is in the same direction as the current in the long conductor, otherwise it will be repulsive.

30. (a) Current sensitivity : It is defined as the deflection of coil per unit current flowing in it, *i.e.*,

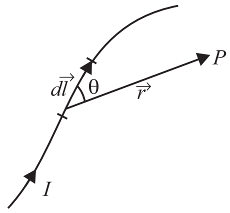
$$S = \frac{\theta}{I} = \frac{NAB}{k}$$

(b) $A = 16 \times 10^{-4}$ m^2, $N = 200$, $B = 0.2$ T,

$k = 10^{-6}$ Nm/degree, $\theta = 30°$,

$$I = \frac{k}{NBA}\theta = \frac{10^{-6} \times 30}{200 \times 0.2 \times 16 \times 10^{-4}} = 4.69 \times 10^{-4} \text{ A.}$$

31. A current carrying wire produces a magnetic field around it. Biot-Savart law states that magnitude of intensity of small magnetic field $d\vec{B}$ due to current I carrying element $d\vec{l}$ at any point P at distance r from it is given by



$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

where θ is the angle between $\vec{r}$ and $d\vec{l}$ and

$\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1} is called permittivity of free space.

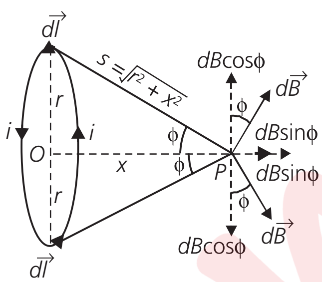
In vectorial form,

$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \vec{r}}{r^3}$$

So, the direction of $d\vec{B}$ is perpendicular to the plane containing $\vec{r}$ and $d\vec{l}$.

S.I. unit of magnetic field strength is tesla denoted by 'T' and CGS unit is gauss denoted by 'G', where 1 T = 10^4 G

Magnetic field on the axis of circular coil



Small magnetic field due to current element of circular coil of radius r at point P at distance x from its centre is

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin 90°}{s^2} = \frac{\mu_0}{4\pi}\frac{Idl}{(r^2 + x^2)}$$

Component $dB\cos\phi$ due to current element at point P is cancelled by equal and opposite component $dB\cos\phi$ of another diametrically opposite current element, whereas the sine components $dB\sin\phi$ add up to give net magnetic field along the axis. So, net magnetic field at point P due to entire loop is

$$B = \oint dB\sin\phi = \int_0^{2\pi r}\frac{\mu_0}{4\pi}\frac{Idl}{(r^2 + x^2)} \cdot \frac{r}{(r^2 + x^2)^{1/2}}$$

$$B = \frac{\mu_0 Ir}{4\pi(r^2 + x^2)^{3/2}}\int_0^{2\pi r} dl \quad \text{or} \quad B = \frac{\mu_0 Ir}{4\pi(r^2 + x^2)^{3/2}} 2\pi r$$

or $\quad B = \dfrac{\mu_0 Ir^2}{2(r^2 + x^2)^{3/2}}$ directed along the axis,

(a) towards the coil if current in it is in clockwise direction

(b) away from the coil if current in it is in anticlockwise direction.

The direction of the magnetic field produced is given by right hand thumb rule, where thumb gives direction of the current and curled fingers give the direction of magnetic field produced. Hence the current carrying circular loop behaves like a magnet.

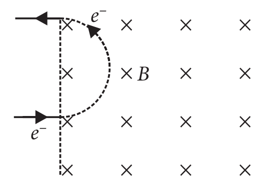
OR

Let the time taken by the electron to come out of the region of magnetic field be t.

Velocity of the electron, $v = 4 \times 10^4$ m/s

Magnetic field, $B = 10^{-5}$ T

Mass of the electron, $m = 9 \times 10^{-31}$ kg



Trajectory of electron

We know

$$t = \frac{\pi r}{v} \text{ where } r = \frac{mv}{qB}$$

$$\text{Now, } t = \frac{\pi m}{Bq} = \frac{3.14 \times 9 \times 10^{-31}}{10^{-5} \times 1.6 \times 10^{-19}}$$

$\Rightarrow \quad t = 17.66 \times 10^{-7}$ s = 1.77 μs

Thus, the time taken by the electron to come out of the region of magnetic field is 1.77 μs.

32. (a) No, that would require $\vec{\tau}$ to act in the vertical direction. But $\vec{\tau} = I(\vec{A} \times \vec{B})$ and for the horizontal loop $\vec{A}$ acts in the vertical direction, so $\vec{\tau}$ acts in the plane of the horizontal loop.

(b) The torque on a loop of area $\vec{A}$ and carrying current I, when placed in a magnetic field $\vec{B}$, is given by

$$\vec{\tau} = I(\vec{A} \times \vec{B})$$

Obviously, torque $\vec{\tau}$ will become zero if the area vector $\vec{A}$ is in the direction of external field $\vec{B}$. Hence in position of stable equilibrium, the current loop will orient itself with its plane perpendicular to the direction of $\vec{B}$ (because the direction of $\vec{A}$ is normal to the plane of current loop).

In this orientation, the magnetic field produced by the loop is in the same direction as external field, both normal to the plane of the loop, thus giving rise to maximum flux of the total field.

(c) The loop will assume circular shape with its plane perpendicular to the magnetic field so as to maximise magnetic flux. This is because for a given perimeter, a circle encloses greater area than any other shape.

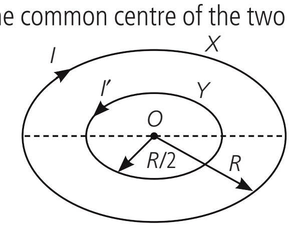
OR

(a) Magnetic field at the centre O due to the current I flowing clockwise in coil X (radius = R),

$$\vec{B}_X = \frac{\mu_0 I}{2R}, \text{ acting vertically downwards}$$

For total magnetic field to be zero at the common centre of the two coils,

$$\vec{B}_X + \vec{B}_Y = 0 \text{ or } \vec{B}_Y = -\vec{B}_X$$



So the current in coil Y must flow anticlockwise.

Also $B_Y = B_X$

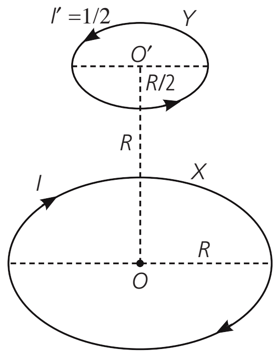
or $\dfrac{\mu_0 r}{2(R/2)} = \dfrac{\mu_0 I}{2R}$

or $I' = I/2$

(b) When the coil Y is lifted vertically upwards through a distance R, its centre O' lies on the axial line of the coil X.

$\therefore$ Magnetic field at point O' due to current I in coil X.

$$B'_X = \frac{\mu_0 I R^2}{2(R^2+R^2)^{3/2}} = \frac{\mu_0 I}{4\sqrt{2}R}$$



acting vertically downwards.

Magnetic field at point O' due to current $I/2$ in coil Y,

$$B'_Y = \frac{\mu_0 (I/2)}{2(R/2)} = \frac{\mu_0 I}{2R}$$

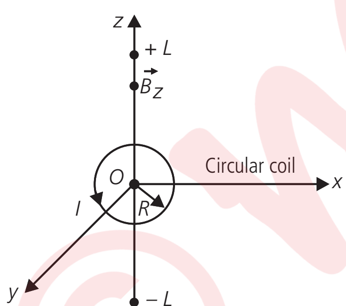
acting vertically upwards.

The net magnetic field at the centre O' of the coil Y,

$$\vec{B}' = \vec{B}'_Y + \vec{B}'_X = \frac{\mu_0 I}{2R} - \frac{\mu_0 I}{4\sqrt{2}R}$$

$$= 0.323\frac{\mu_0 I}{R} \text{ acting vertically upwards.}$$

33. (a) Magnetic field due to a circular current-carrying loop lying in the xy-plane acts along z-axis as shown in figure.



$$I(L) = J(L) = \left|\int_{-L}^{L} \vec{B}\cdot\vec{dl}\right| = \int_{-L}^{+L} Bdl\cos 0° = \int_{-L}^{+L} Bdl = 2BL$$

$\therefore$ $I(L)$ is monotonically increasing function of L.

(b) Consider an Amperian loop around the circular coil of such a large radius that $L \to \infty$. Since this loop encloses a current I, from Ampere's law

$$I(\infty) = \oint_{-\infty}^{+\infty} \vec{B}\cdot\vec{dl} = \mu_0 I$$

(c) As $B_z = \dfrac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}$

$$\int_{-\infty}^{+\infty} B_z dz = \int_{-\infty}^{+\infty} \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}} dz$$

Let $z = R\tan\theta$ so that $dz = R\sec^2\theta\, d\theta$

and $(z^2 + R^2)^{3/2} = (R^2\tan^2\theta + R^2)^{3/2}$

$= R^3\sec^3\theta$ (as $1 + \tan^2\theta = \sec^2\theta$)

Thus, $$\int_{-\infty}^{+\infty} B_z dz = \frac{\mu_0 I}{2}\int_{-\pi/2}^{+\pi/2} \frac{R^2(R\sec^2\theta d\theta)}{R^3\sec^3\theta}$$

$$\frac{\mu_0 I}{2}\int_{-\pi/2}^{+\pi/2} \cos\theta d\theta = \mu_0 I$$

(d) Since $(B_z)_{square} < (B_z)_{circular\ coil}$

For the same current, and side of the square equal to radius of the coil

$I(L)_{square} < I(L)_{circular\ coil}$

By using the same argument as in (b), it can be shown that

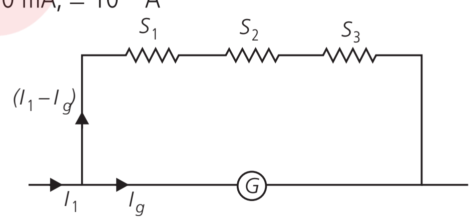
$I(\infty)_{square} = I(\infty)_{circular\ coil}$

(as $I(\infty) = \oint_{-\infty}^{+\infty} \vec{B}\cdot\vec{dl} = \mu_0 I$)

OR

Here, $G = 10\ \Omega$, $I_g = 1$ mA $= 10^{-3}$ A

For $I_1 = 10$ mA, $= 10^{-2}$ A



$I_g G = (S_1 + S_2 + S_3)(I_1 - I_g)$

$10^{-3} \times 10 = (S_1 + S_2 + S_3)(10^{-2} - 10^{-3})$

or $S_1 + S_2 + S_3 = \dfrac{10}{9}$ ...(i)

Similarly, for $I_2 = 100$ mA $= 0.1$ A

$I_g(G + S_1) = (I_2 - I_g)(S_2 + S_3)$

$10^{-3}(10 + S_1) = (0.1 - 10^{-3})(S_2 + S_3)$

$10 + S_1 = 99(S_2 + S_3)$ ...(ii)

Similarly, for $I_3 = 1$A,

$I_g(G + S_1 + S_2) = (I_3 - I_g)S_3$

$10^{-3}(10 + S_1 + S_2) = 10^{-3}(1000 - 1)S_3$

$10 + S_1 + S_2 = 999\, S_3$ ...(iii)

Solving eqns (i), (ii) and (iii), we get

$S_1 = 1\ \Omega$, $S_2 = 0.1\ \Omega$, $S_3 = 0.01\ \Omega$

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