Electromagnetic Induction

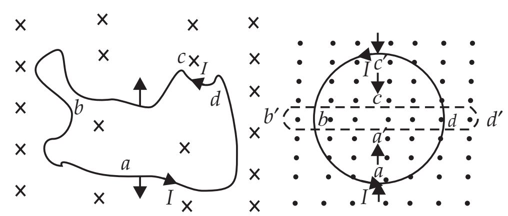
CHAPTER 6

NCERT FOCUS

ANSWERS

Topic 1

1.

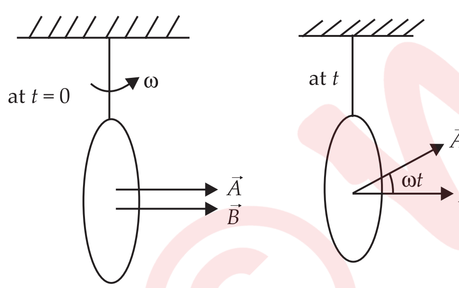


(a) (b)

(a) Due to change in shape, area increases and consequently, magnetic flux linked with it also increases. Using Lenz's law, an induced current is set up in the circular wire in the anticlockwise direction to produce opposing flux. So magnetic field due to it, is directed upward.

(b) Due to deformation of circular loop into a straight wire, its area decreases and consequently magnetic flux linked with it decreases. So an induced current is set up in the anticlockwise direction, hence magnetic field is upward.

2. If the circular coil rotates in the magnetic field B at an angular velocity ω, then instantaneous induced emf can be calculated.



(a) (b)

Instantaneous flux $\phi = BA\cos\omega t$

emf, $\varepsilon = -\dfrac{d\phi}{dt} = -BA\dfrac{d\cos(\omega t)}{dt}$

$\varepsilon = -NBA[-\omega\sin\omega t]$

$\varepsilon = NBA\omega\sin\omega t$

Max. emf for $\sin\omega t = 1$

$\varepsilon_{max} = NBA\omega = 3\times10^{-2}\times\pi(8\times10^{-2})^2\times50\times20$

$= 0.603$ Volt

Average emf over a complete cycle is zero.

Maximum current in the coil

$I_{max} = \dfrac{\varepsilon_{max}}{R} = \dfrac{0.603}{10} = 0.0603$ A

Average power lost $= \dfrac{1}{2}\varepsilon_{max}\times I_{max} = 0.018$ W

Source of the power is work done in rotating the coil.

3. Direction of induced current in all the situations shown above can be decided in the light of Lenz's law.

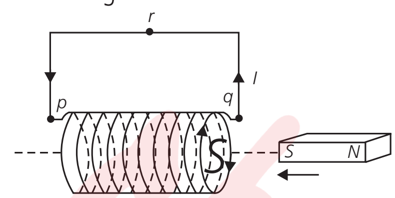


Fig. (a)

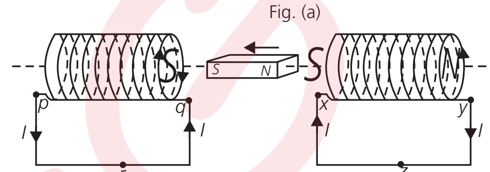
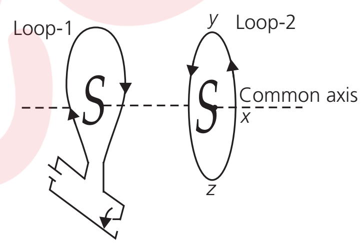
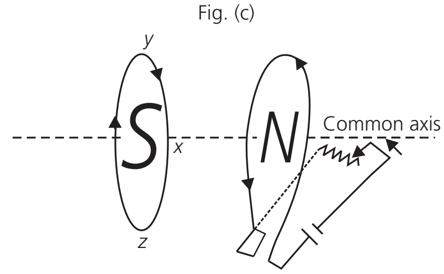


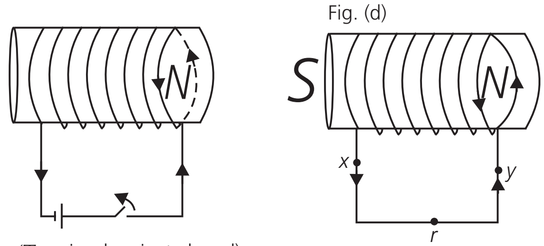
Fig. (b)



(Tapping key just closed)
Fig. (c)

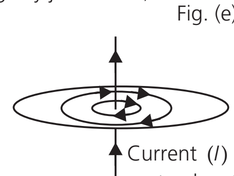


Rheostat setting being changed
Fig. (d)



(Tapping key just closed)

Fig. (e)



Current (*I*) decreasing at a steady rate
Fig. (f)

Fig. (a) : South pole is moving closer, so the current is clockwise in the end of solenoid closest to magnet.
Fig. (b) : Following Lenz's law, the current flow anticlockwise in the loop at the left and clockwise in the loop at the right.
Fig. (c) : Inner side of loop-1 become south pole whose strength increasing with increase in current. So the inner side of loop should also become south pole according to Lenz's law.
Fig. (d) : Current is decreasing with increase in rheostat, so North pole is getting weaker, the current in inner part of loop-1 will flow clockwise.
Fig. (e) : Induced current in the right coil is from X to Y.
Fig. (f) : No induced current since magnetic lines of force are in the plane of the loop.

4 According to Faraday's law of electromagnetic induction the induced emf is $\varepsilon = -\frac{d\phi}{dt}$

Thus a relation between electric field and rate of change of flux can be established.

$$\varepsilon = -\int \vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt}$$

$\vec{E}$ exist along circumference of radius 'a' due to change in magnetic flux.

$$E\int dl = -\frac{d}{dt}(\pi a^2 B), E\times 2\pi a = -\pi a^2\frac{dB}{dt}$$

$$E = -\frac{a}{2}\frac{dB}{dt} \quad \text{...(i)}$$

Linear charge density on rim is λ. So, total charge on rim $Q = \lambda 2\pi a$ ...(ii)

Electric Force on the charge

$$F = QE = -\pi a^2\lambda\frac{dB}{dt};\ m\frac{dv}{dt} = -\pi a^2\lambda\frac{dB}{dt}$$

In terms of angular velocity $v = R\omega$

$$m\frac{d}{dt}(R\omega) = -\pi a^2\lambda\frac{dB}{dt}$$

$$mR\,d\omega = -\pi a^2\lambda dB$$

$$d\omega = -\frac{\pi a^2\lambda}{mR}dB$$

Integrating both sides $\omega = -\frac{\pi a^2\lambda B}{mR}$

As direction of angular velocity is along axis.

$$\vec{\omega} = -\frac{\lambda a^2\pi}{mR}B\hat{k}$$

Topic 2

1. Here area is constant but the magnetic field is reducing at a constant rate.

$$\frac{dB}{dt} = -(0.02)\text{ T s}^{-1}$$

Area of the loop, $A = l\times b = 8\times 2\text{ cm}^2$
$= 16\text{ cm}^2 = 16\times 10^{-4}\text{ m}^2$

Induced emf in the loop

$$\varepsilon = -\frac{d\phi}{dt} = -A\frac{dB}{dt}$$

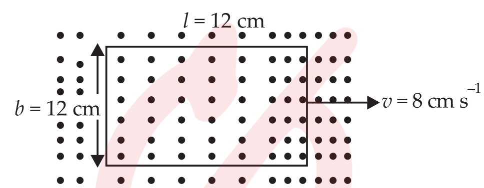
$\varepsilon = -16\times 10^{-4}[-0.02] = 32\times 10^{-6}$ Volt

Induced current in the closed loop

$$I = \varepsilon/R = \frac{32\times 10^{-6}}{1.6} = 20\ \mu\text{A}$$

Power loop as heat $P = I^2R$
$P = (20\times 10^{-6})^2\times 1.6 = 6.4\times 10^{-10}$ W
Source of the power is work done in changing magnetic field.

2.



Each side of square loop is 12 cm and magnetic field is decreasing along x direction.

$$\frac{dB}{dx} = -10^{-3}\text{ T cm}^{-1} = -0.1\text{ T m}^{-1}$$

Also the magnetic field is decreasing with time at constant rate

$$\frac{dB}{dt} = -10^{-3}\text{ Ts}^{-1}$$

Induced emf and rate of change of magnetic flux due to only time variation

$$\varepsilon_t = -\frac{d\phi}{dt} = -\frac{dBA}{dt} = -A\frac{dB}{dt}$$

$\varepsilon_t = -0.12\times 0.12[-10^{-3}] = 144\times 10^{-7}$ V

Induced emf and rate of change of magnetic flux due to change in position.

$$\varepsilon_x = -\frac{dBA}{dt} = -A\frac{dB}{dx}\times\frac{dx}{dt}$$

$$\varepsilon_x = -Av\frac{dB}{dx} = -0.12\times 0.12\times 0.08\times(0.1)$$

$= 1152\times 10^{-7}$ V

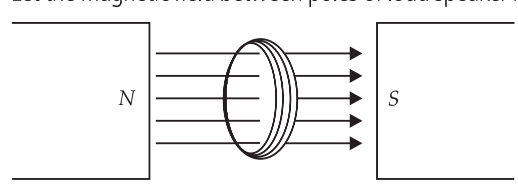
Both the induced emf have same sign and thus adds to provide net Induced emf in the loop

$\varepsilon_{net} = \varepsilon_t + \varepsilon_x = 1296\times 10^{-7}$ V

Induced current

$$I = \frac{\varepsilon_{net}}{R} = \frac{1296\times 10^{-7}}{4.5\times 10^{-3}} = 2.88\times 10^{-2}\text{ A}$$

3. Let the magnetic field between poles of loud speaker magnet is B.



Initial flux through the coil

$$\phi_i = NBA = 25B(2\times 10^{-4}) = 50\times 10^{-4}\times B\text{ Wb} \quad \text{...(i)}$$

Final flux through the coil is zero. Let coil is taken out in time 't'.

Magnitude of induced emf $\varepsilon = \dfrac{\Delta\phi}{\Delta t}$

$$\varepsilon = \frac{50\times10^{-4}B}{t} \quad \text{...(ii)}$$

Current in the coil

$$I = \frac{\varepsilon}{R} = \frac{50\times10^{-4}B}{0.5t} = \frac{10^{-2}B}{t} \quad \text{...(iii)}$$

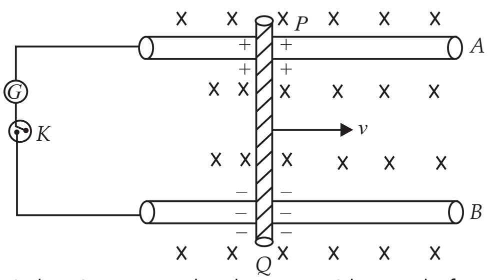
Total charge flowing in the coil

$q = it$

$$q = \frac{10^{-2}B}{t}\times t = 10^{-2}B \quad \text{or} \quad 7.5\times10^{-3} = 10^{-2}B$$

So, magnetic field between poles, $B = 0.75$ T

4. Here rails, rod and magnetic field are in three mutually perpendicular directions.



(a) Switch K is open and rod moves with speed of 12 cm s^{-1}.

Induced emf/motional emf

$\varepsilon = Bvl$

$\varepsilon = 0.5 \times 12 \times 10^{-2} \times 15 \times 10^{-2} = 9$ mV

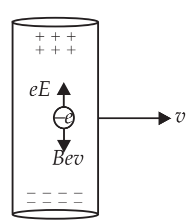
(b) When the K is open, upper end of the rod become positively charge, and lower end become negatively charged.

When the K is closed the charge flows in closed circuit but the excess charge is maintained by the flow of charge in the moving rod under magnetic force.

(c) In the state when K is open very soon a stage is reached when force due to electric field which is due to potential difference induced balances the magnetic force on electrons.

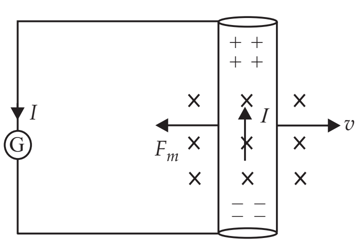
$eE = Bev$

$$e\frac{V}{l} = Bev$$



Motional emf $V = Bvl$.

(d) When the key is closed the current flows in a loop and the current carrying wire experience a retarding force in the magnetic field.



$F_m = iBl$

where, $I = \dfrac{Bvl}{R} = \dfrac{9\times10^{-3}}{9\times10^{-3}} = 1A$

$F_m = 1 \times 0.5 \times 15 \times 10^{-2} = 0.075$ N

(e) To keep the rod moving in closed circuit at constant speed the force required is $F = 0.075$ N.

So, power required

$P = \vec{F}\cdot\vec{v} = Fv\cos 0° = Fv$

$P = 0.075 \times 12 \times 10^{-2} = 9$ mW

when key K is open, no current flows and hence no retarding force, so no power is required to move at constant speed.

(f) Power lost in closed circuit due to flow of current

$P = i^2R = (1)^2 \times 9 \times 10^{-3} = 9$ mW

Power provided by external force to move the rod at constant speed is the source of this power lost.

(g) If $\vec{B}$ is parallel to rails, the induced/motional emf will be zero.

5. When the current changes through the solenoid, a change in magnetic field also take place within it.

Initial magnetic field in solenoid,

$$B_1 = \mu_0 n i_1 = 4\pi\times10^{-7}\times\frac{15}{10^{-2}}\times 2 = 120\pi\times10^{-5}\text{ T}$$

Final magnetic field, $B_2 = \mu_0 n i_2$

$$B_2 = 4\pi\times10^{-7}\times\frac{15}{10^{-2}}\times 4 = 240\pi\times10^{-5}\text{ T}$$

Initial flux through coil inside solenoid placed normal to axis.

$\phi_i = B_1 A = 120\pi \times 10^{-5} \times 2 \times 10^{-4}$

$\phi_i = 240\pi \times 10^{-9}$ Wb

Final flux, $\phi_f = B_2 A = 240\pi \times 10^{-5} \times 2 \times 10^{-4}$

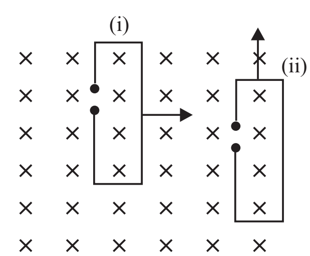
$\phi_f = 480\pi \times 10^{-9}$ Wb

$$\text{Induced emf, } \varepsilon = -\frac{(\phi_f - \phi_i)}{t} = -\frac{240\times10^{-9}\times3.14}{0.1} = -7.5\ \mu\text{V}$$

6. Here $A = 8 \times 2 = 16$ cm^2 $= 16 \times 10^{-4}$ m^2,

$B = 0.3$ T; $v = 1$ cm s^{-1} $= 10^{-2}$ m s^{-1}

Induced emf, $\varepsilon = ?$



(i) When velocity is normal to longer side,

$l = 8$ cm $= 8 \times 10^{-2}$ m

$\varepsilon = Blv = 0.3 \times 8 \times 10^{-2} \times 10^{-2} = 2.4 \times 10^{-4}$ V

$$\text{Time, } t = \frac{\text{distance moved}}{\text{velocity}} = \frac{2\times10^{-2}}{10^{-2}} = 2\text{ sec}$$

(ii) When velocity is normal to shorter side, $l = 2$ cm $= 2 \times 10^{-2}$ m

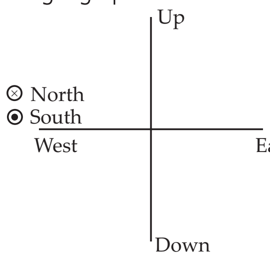
$\varepsilon = Blv = 0.3 \times 2 \times 10^{-2} \times 10^{-2} = 0.6 \times 10^{-4}$ V

$$\text{Time, } t = \frac{\text{distance moved}}{\text{velocity}} = \frac{8\times10^{-2}}{10^{-2}} = 8\text{ sec}$$

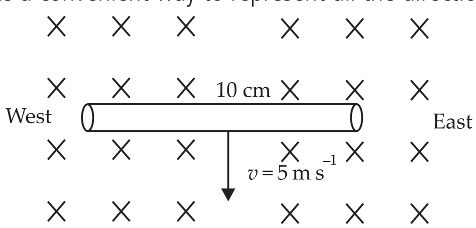
7. Constant and uniform magnetic field is parallel to axis of the wheel and thus normal to plane of the wheel.

Induced emf, $\varepsilon = \frac{B\omega l^2}{2}$; $\varepsilon = \frac{0.5 \times 400 \times 1}{2} = 100\text{ V}$

8. The direction of earth's magnetic field is in the direction of geographical south to geographical north



Let us take a convenient way to represent all the directions.

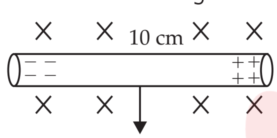


(a) Instantaneous emf $\varepsilon = Bvl$

$\varepsilon = 0.3 \times 10^{-4} \times 5 \times 10 = 15 \times 10^{-4}$ volt = 1.5 mV

(b) Direction of emf. will be west to east.

(c) West end of the wire will be charged at higher potential.

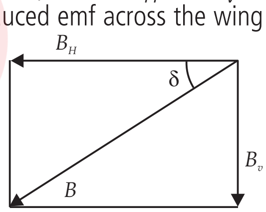


9. Let 'L' be the coefficient of self inductance, the back emf

$\varepsilon = -L\frac{dI}{dt}$; $200 = -L\frac{(i_f - i_i)}{t}$ or $200 = -L\frac{(0-5)}{0.1}$

$L = 4$ H.

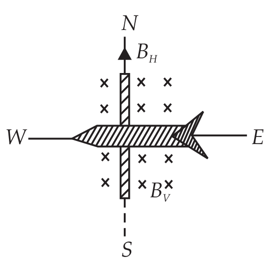
10. Earth magnetic field will have two components, B_H and B_V. It is vertical component which develop induced emf across the wing in N-S direction.

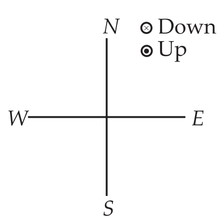


$B_V = B \sin \delta$

$B_V = 5 \times 10^{-4} \sin 30°$

$B_V = 2.5 \times 10^{-4}$ T





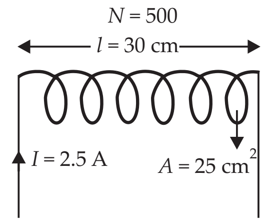
Induced emf $\varepsilon = B_V vl$ $\quad$ ($\because$ 1800 km h^{-1} = 500 m s^{-1})

$\varepsilon = 2.5 \times 10^{-4} \times 500 \times 25$

$\varepsilon = 3.125$ V

Topic 3

1. Magnetic field inside solenoid



$B = \frac{\mu_0 NI}{l}$

Flux linked with solenoid

$\phi_i = BAN = \frac{\mu_0 N^2 AI}{l}$

Initial flux,

$$\phi_i = \frac{4\pi \times 10^{-7} \times (500)^2 \times 25 \times 10^{-4} \times 2.5}{30 \times 10^{-2}}\text{ Wb}$$

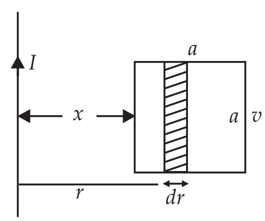
$\phi_i = 6.54 \times 10^{-3}$ Wb

Final flux, $\phi_f = 0$ [$I = 0$]

Average back emf

$$e_{av} = -\frac{(\phi_f - \phi_i)}{t} = -\left[\frac{0 - 6.54 \times 10^{-3}}{10^{-3}}\right] = 6.54\text{ V}$$

2. (a) As the magnetic field will be variable with distance from long straight wire, so the flux through square loop can be calculated by integration.



Let us assume a width 'dr' of the square loop at a distance 'r' from straight wire

$B = \frac{\mu_0}{4\pi}\frac{2I}{r}$; $\phi = B \cdot Adr = \frac{\mu_0}{4\pi}\frac{2I}{r}a\,dr$

Total flux associated with square loop

$$\phi = \int d\phi = \frac{\mu_0}{4\pi}2Ia\int_x^{x+a}\frac{dr}{r} \quad \text{or} \quad \phi = \frac{\mu_0}{4\pi}2Ia[\log_e r]_x^{x+a}$$

$$\phi = \frac{\mu_0}{4\pi}2Ia\left[\log_e \frac{x+a}{x}\right] \quad \text{or} \quad \phi = \frac{\mu_0 Ia}{2\pi}\log_e(1 + a/x)$$

(b) The square loop is moving right with a constant speed v, the instantaneous flux can be taken as

$$\phi = \frac{\mu_0 Ia}{2\pi}\log_e(1 + a/x)$$

Induced emf, $\varepsilon = -\frac{d\phi}{dt} = -\frac{d\phi}{dx}\frac{dx}{dt} = -v\frac{d\phi}{dx}$

$$\varepsilon = -\frac{\mu_0 Iav}{2\pi}\frac{d(\log_e(1 + a/x))}{dx}$$

$$\varepsilon = -\frac{\mu_0 Iav}{2\pi}\frac{1}{\left(1 + \frac{a}{x}\right)}\left[-a/x^2\right] \quad \text{or} \quad \varepsilon = \frac{\mu_0}{2\pi}\frac{a^2 v}{x(x+a)}I$$

or $\quad \varepsilon = 2 \times 10^{-7}\frac{[0.1]^2 \times 10 \times 50}{0.2[0.2 + 0.1]} = 1.67 \times 10^{-5}\text{ V}$

3. Let the current changes from 0 to 20 A in coil 1 and we are looking for change of flux linked with coil 2.

$\phi_2 = MI_1$ and $\Delta\phi_2 = M\Delta I_1$

$\Delta\phi_2 = 1.5[20 - 0]$ or $\Delta\phi_2 = 30$ Wb