CHAPTER 7

EXAM DRILL

Alternating Current

ANSWERS

1. **(c) :** Average value of AC for complete cycle is zero. Hence, AC cannot be measured by DC ammeter.

2. **(a) :** The 220 V AC line voltage that we receive in our homes is the rms value.

3. **(c) :** $X_C = \frac{1}{\omega C} = \frac{1}{(2\pi\upsilon)C}$

$\therefore \frac{X_{C_2}}{X_{C_1}} = \frac{\upsilon_1 C_1}{\upsilon_2 C_2} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4} \quad \therefore 2X_{C_2} = \frac{X_{C_1}}{4} = \frac{X}{4}$

4. **(d) :** At resonance, voltage across inductor = voltage across capacitor *i.e.*, $V_L = V_C$.

5. **(d) :** At Resonance $X_L = X_C$.

6. **(b) :** In a pure inductor circuit, the average power is zero.

7. **(c) :** Here, $X_L = 1\ \Omega$, $R = 2\ \Omega$, $V_{rms} = 6$ V

Impedance of the circuit

$Z = \sqrt{X_L^2 + R^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{5}\ \Omega$

$I_{rms} = \frac{V_{rms}}{Z} = \frac{6}{\sqrt{5}}$ A

Power dissipated

$P = V_{rms} I_{rms} \cos\phi = V_{rms} I_{rms} \frac{R}{Z}$

$= 6 \times \frac{6}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{72}{5} = 14.4$ W

8. **(a) :** Secondary coil has more turns.

9. **(d) :** $I_{rms} = \frac{I_0}{\sqrt{2}}$

10. **(c) :** For better tuning of an *LCR* circuit used for communication the circuit should possess high quality factor of resonance.

i.e., $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ should be high.

For it *R* should be low, *L* should be high and *C* should be low, therefore combination in option (c) is correct.

11. **(b) :** Like direct current, an alternating current also produces magnetic field. But the magnitude and direction of the field goes on changing continuously with time.

12. **(b) :** Both assertion and reason are true but reason is not the correct explanation of assertion.

We can use a capacitor of suitable capacitance as a choke coil, because average power consumed in an ideal capacitor is zero. Therefore, like a choke coil, a condenser can reduce A.C. without power dissipation.

13. **(i) (c) :** As $\frac{E_s}{E_p} = \frac{n_s}{n_p} \Rightarrow E_s = E_p \cdot \frac{n_s}{n_p}$

$= \frac{120 \times 50}{2000} = 3$ V

(ii) (d) : $I_s = \frac{E_s}{R} \Rightarrow I_s = \frac{3}{0.6} = 5$ A

(iii) (a) : As $\frac{I_p}{I_s} = \frac{E_s}{E_p}$

$\Rightarrow I_p = \frac{E_s}{E_p} \times I_s = \frac{3}{120} \times 5 = 0.125$ A

14. The quality factor (*Q*) of resonance in series *LCR* circuit is defined as the ratio of voltage drop across inductor (or capacitor) to the applied voltage,

i.e., $Q = \frac{V_L}{V_R} = \frac{I_0 X_L}{I_0 R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$

It is an indicator of sharpness of the resonance.

Quality factor has no unit.

15. 220 V a.c. has a peak value of $220\sqrt{2}$ = 311 V, while 220 V d.c. has a peak value of 220 V only. So a.c. of some voltage is more dangerous than d.c.

16. In resonance condition, the impedance of the LCR circuit becomes minimum and so current in the circuit rises to a maximum value.

OR

As inductive reactance, $X_L = 2\pi fL$, for d.c., $f = 0$, so, $X_L = 0$ and for a.c. it has some finite value because *f* has some value. Hence an inductor offers an easy path to d.c. and a resistive path to a.c.

17. Yes, we can use capacitor instead of a choke coil for reducing current in an a.c. circuit because average power dissipated per cycle in an ideal capacitor is also zero.

18. The current in an a.c., circuit is wattless if the average power consumed in the circuit is zero.

19. (a) The frequency of d.c. is zero.

(b) It is that value of a direct current which produces the same heating effect in a given resistor as it produces by the given alternating current when passed for the same time.

$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.7071$, where I_0 is the peak value of a.c.

20. (iii) Admittance– The inverse of impedance is called the admittance and is denoted by Y. Its unit is $(\text{ohm})^{-1}$ or Ω^{-1} or mho.

$$Y = \frac{1}{Z}$$

(iv) Susceptance– The reciprocal of reactance is called susceptance and it denoted by S.

$$S = \frac{1}{X}$$

It is of two types :

(a) Inductive susceptance, $S_L = \frac{1}{X_L} = \frac{1}{2\pi fL}$

(b) Capacitive susceptance, $S_C = \frac{1}{X_C} = \omega C = 2\pi fC$

21. (a) Maximum value of power factor is 1 while minimum value of power factor is 0.

(b) At resonance condition, $Z = R$

$\therefore$ Power factor, $\cos\phi = \frac{R}{Z} = 1$

OR

(a) Average power over full cycle of the ac voltage source connected across an ideal inductor is zero.

(b) $P_{av} = V_{rms} I_{rms} \cos\phi$

$$= 200 \times 1.5 \times \frac{R}{Z} = 200 \times 1.5 \times \frac{2}{4} = 150 \text{ W}$$

22. When an iron core is inserted in the choke coil the self inductance L increases.

Therefore, the inductive reactance, $X_L = \omega L$ increases, and it decreases the current in the circuit and the bulb glows dimmer.

23. (a) The eddy currents set-up in the iron core heat up a transformer under operation.

(b) To minimise the energy losses due to eddy current. The core of the transformer is laminated.

24. (a) As $X_L = 2\pi fL$

or $L = \frac{X_L}{2\pi f}$

or $L = \frac{1}{2\pi} \times \text{slope of the graph}$

$$= \frac{1}{2\pi} \times \frac{(4-0)}{(400-0)} = \frac{1}{200\pi} = 1.59 \times 10^{-3} \text{ H}$$

(b) $\because Z = \sqrt{R^2 + X_L^2}$ or $Z = \sqrt{(4)^2 + (2)^2} = 4.47\ \Omega$

OR

Given : $C = 50\ \mu\text{F} = 50 \times 10^{-6}\text{F}$

$\varepsilon_{rms} = 100$ V

$f = 50$ Hz

$\therefore X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 50 \times 50 \times 10^{-6}}$

or $X_C = \frac{10^6}{15700} = 63.69\ \Omega$

$\therefore I_{rms} = \frac{\varepsilon_{rms}}{X_C} = \frac{100}{63.69} = 1.57$ A

25. Two uses of transformer are following :

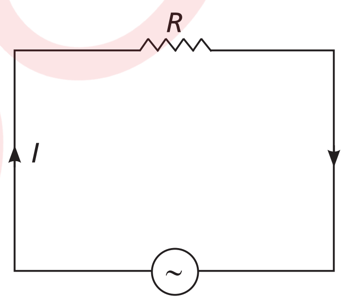
(i) In voltage regulation for TV, refrigerators computers, air conditioners etc.

(ii) It is used in electrical power distribution.

26. Let us consider a resistor of resistance R is connected to a source of alternating emf which is given by

$\varepsilon = \varepsilon_0 \sin \omega t$ ....(i)

At any instant, I be the current in the circuit, so the potential drop across the resistance R will be IR.



According to Kirchhoff's loop rule,

$\varepsilon_0 \sin \omega t = IR$

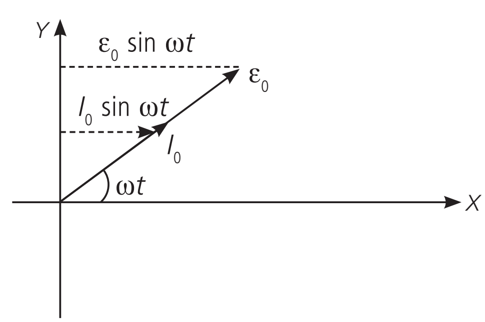
or $I = \frac{\varepsilon_0 \sin \omega t}{R}$

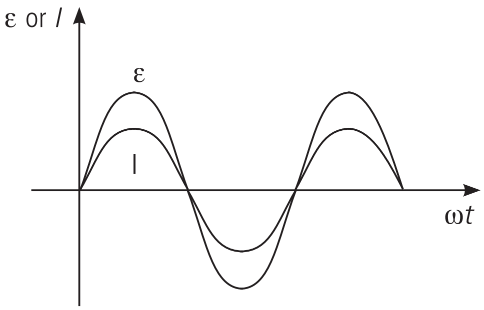
or $I = I_0 \sin \omega t$ ...(ii)

where, $I_0 = \frac{\varepsilon_0}{R} =$ the maximum or peak value of alternating current.

From equations (i) and (ii), we can conclude that in a purely resistive circuit the current I and emf ε are functions of $\sin \omega t$ and both are in same phase.

The phasor diagram for purely resistive a.c. circuit as shown in figure corresponding to ε or I versus ωt graph.





27. Resistance. It is the opposition offered by a pure resistor to the flow of current in a circuit.

It depends on the nature of the material of the conductor and does not depend upon the frequency of alternating current. Its unit is Ω.

Reactance : The non-resistive opposition to the flow of current is called reactance. If may be inductive or capacitive.

Inductive reactance, $X_L = \omega L = 2\pi f L$

Capacitive reactance, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

Its SI unit is also ohm (Ω).

Impedance : It is the effective resistance of an a.c. circuit containing any two or all the three elements, resistor (R), inductor (L) and capacitor (C). It plays same role in a.c. circuit as resistance plays in d.c. circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Its S.I. unit is also ohm (Ω).

OR

The impedance of a series LCR circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

When the circuit is brought into resonance condition, i.e,

$$X_L = X_C \quad \text{or} \quad \omega L = \frac{1}{\omega C}$$

then, $Z = R$

Thus, the impedance Z of the circuit is minimum at resonance and hence the current through the circuit is maximum.

If capacitance is increased, the capacitive reactance decreases and so impedance increases, due to this current decreases.

28. Some advantages of a.c. over d.c. are following :

(i) A.C. machines are simple and do not require much attention during their use.

(ii) The generation of a.c. is more economical than d.c.

(iii) All a.c. can be easily converted into d.c. by using rectifiers.

Some disadvantages of a.c. over d.c. are following :

(i) Peak value of a.c. is high, so dangerous to work with a.c.

(ii) A.C. is transmitted more from the surface of the conductor than from the inside. So, several fine insulated wires are required for transmission of a.c.

29. (i) Power loss = $I^2R = (40)^2R = 1600\ R$

(ii) Power loss = $I^2R = (0.2)^2R = 0.04R$

Thus the power loss in case (ii) is much less than that in case (i). Therefore case (ii) is more economical to supply 15 kW power.

30. Here, $L = 80$ mH $= 80 \times 10^{-3}$ H.

$C = 60\ \mu$F $= 60 \times 10^{-6}$ F, $\varepsilon_{rms} = 230$ V, $f = 50$ Hz

(a) Clearly, $X_L = \omega L = 2\pi f L = (2 \times \pi \times 50)(80 \times 10^{-3})\ \Omega$
$= 25.14\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 50\ (60 \times 10^{-6})}\Omega = \frac{1000}{6\pi}\Omega = 53.04\ \Omega$$

Thus, $Z = X_C - X_L = (53.04 - 25.14)\ \Omega$

$= (53.04 - 25.14)\ \Omega = 27.9\ \Omega$

and $I_{rms} = \dfrac{\varepsilon_{rms}}{Z} = \dfrac{230\ \text{V}}{27.9\ \Omega} = 8.24$ A

Current amplitude, $I_0 = \sqrt{2}\, I_{rms} = 1.414\ (8.24\ \text{A}) = 11.65$ A

(b) rms value of potential difference across L, *i.e.*,

$V^L_{rms} = I_{rms} \times X_L = (8.24\ \text{A})\ (25.14\ \Omega) = 207$ V

rms value of potential difference across C, *i.e.*,

$V^C_{rms} = I_{rms} \times X_C = 8.24 \times 53.04 = 437$ V

(c) Average power transferred to the inductor

$= \varepsilon_{rms} I_{rms} \cos(\pi/2) = 0$

(d) Average power transferred to the capacitor

$= \varepsilon_{rms} I_{rms} \cos(\pi/2) = 0$

(e) Total power absorbed by the circuit = 0

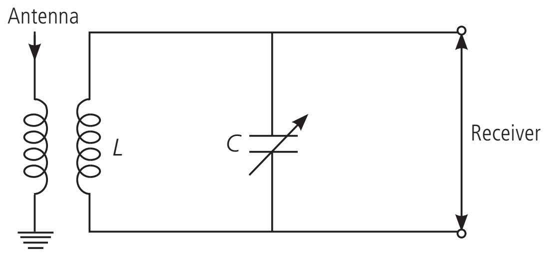
$= \varepsilon_{rms} I_{rms} \cos(\pi/2) = 0$

$V^C_{rms} - V^L_{rms} = 437\ \text{V} - 207\ \text{V} = 230$ V = applied rms voltage. This is due to the reason that the voltages across L and C get subtracted because they are 180° out of phase.

31. The tuning circuit of a radio or television is an example of LCR resonance circuit. Different frequency signals are transmitted by different stations, which are catch by antenna and according to these frequencies, number of voltages appear across the series LCR circuit. In this process maximum current flows through the circuit which has frequency same as resonance frequency, *i.e.*,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

By changing the value of the adjustable capacitor, the signals can be tuned in.



32. Given : $C = 4\ \mu F = 4 \times 10^{-6}$ F

$R = 50\ \Omega$

$L = 4$H

$\varepsilon_0 = 100$ V

$$\therefore \quad f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\times\sqrt{4\times4\times10^{-6}}}$$

$$\text{or} \quad f_r = \frac{1}{2\times3.14\times4\times10^{-3}} = \frac{1000}{25.12} = 39.80\,\text{Hz}$$

(i) $X_L = X_C$ at resonance condition

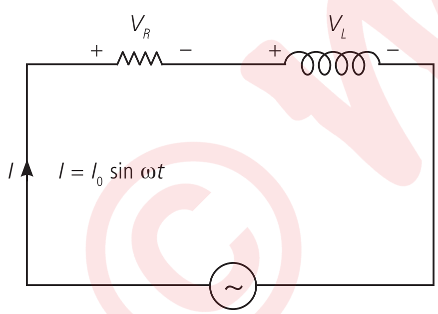
So, $X_L = 2\pi f_r L = 2 \times 3.14 \times 39.80 \times 4 = 999.77$

$\simeq 1000\ \Omega$

(ii) At resonance condition, $Z = R = 50\ \Omega$

(iii) $I_0 = \frac{\varepsilon_0}{Z} = \frac{100}{50} = 2\text{A}$

33. A.C. circuit containing resistance and inductance in series (*LR* series circuit)



Let us consider a resistor *R* and inductance *L* connected in series to a source of alternating emf.

At any instant when current *I* flows through the circuit, then across the resistor some potential difference occurs is V_R and similarly across the inductor, potential difference is V_L.

So, $V_R(t) = (V_0)_R \sin \omega t$; as in pure resistor circuit, potential and current are in same phase.

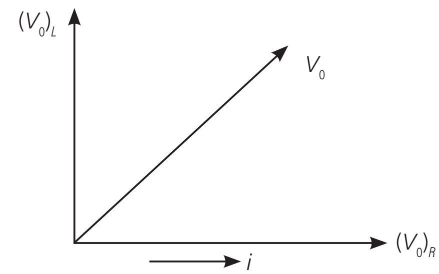
Similarly, $V_L(t) = (V_0)_L \sin\left(\omega t + \frac{\pi}{2}\right)$; as in purely inductive circuit, voltage leads to the current by 90°.

The phasor diagram of these as shown in figure.

As $(V_0)_L$ and $(V_0)_R$ are the phasor, so we add them vectorially.

So, by the law of vector addition.

$$(V_0)^2 = (V_0)_R^2 + (V_0)_L^2$$



$$\text{or} \quad V_0 = \sqrt{(V_0)_R^2 + (V_0)_L^2}$$

As in pure resistive circuit $I_0 = \frac{V_0}{R}$ and in pure inductive circuit, $I_0 = \frac{V_0}{X_L} = \frac{V_0}{\omega L}$

So, that the above equation can be written as,

$$V_0 = \sqrt{(I_0R)^2 + (I_0X_L)^2}$$

$$\text{or} \quad V_0 = \sqrt{I_0^2 R^2 + I_0^2 X_L^2}$$

$$\text{or} \quad V_0 = \left(\sqrt{R^2 + X_L^2}\right) I_0$$

Where $\sqrt{R^2 + X_L^2}$ is the effective resistance of the series L-R circuit which opposes the flow of a.c. through if and it is called impedance which is denoted by *Z*.

So, $V_0 = I_0 Z$

$$\therefore \quad Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

OR

A series *LCR* circuit is said to be in the resonance condition when the current through it has its maximum value.

The current amplitude I_0 for a series *LCR* circuit is given by

$$I_0 = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\text{or} \quad I_0 = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \left[\text{As } X_L = \omega L \text{ and } X_C = \frac{1}{\omega C}\right]$$

When $\omega \to 0$; $I_0 = 0$ and when $\omega \to \infty$; $I_0 = 0$

When $\left(\omega L - \frac{1}{\omega C}\right) = 0$ or $\omega = \frac{1}{\sqrt{LC}}$

then, $I_0 = \frac{V_0}{R}$ = maximum value and the impedance,

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = R,$$ *i.e.*, impedance is minimum,

so the circuit is purely resistive. The current and voltage are in same phase and the current in the circuit is maximum.

This condition in the LCR circuit is called resonance condition.

The frequency at which the current amplitude I_0 attains a peak value is called natural or resonant frequency of the LCR circuit. It is denoted by f_r, which is given by,

$$f_r = \frac{\omega_r}{2\pi}$$

$$\therefore \quad f_r = \frac{1}{2\pi\sqrt{LC}} \qquad \left[\because \omega = \frac{1}{\sqrt{LC}}\right]$$

34. (a) It is that value of D.C. which would produce some heat in a given resistance in a given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}} = \sqrt{\frac{\int_0^T (I_0 \sin \omega t)^2 dt}{\int_0^T dt}}$$

$$\text{or} \quad I_{rms} = \sqrt{\frac{I_0^2}{T}\int_0^T \left[\frac{(1-\cos 2\,\omega t)}{2}\right]dt}$$

$$\text{or} \quad I_{rms} = \frac{I_0}{\sqrt{2}}$$

or $I_{rms} = 0.707\, I_0 = 70.7\%$ of I_0

Similarly, for alternating emf,

$$\varepsilon_{rms} = \frac{\varepsilon_0}{\sqrt{2}}$$

or $\varepsilon_{rms} = 0.707\varepsilon_0 = 70.7\%$ of ε_0

(b) Given : $\varepsilon = 200 \sin 220\, t$

$\therefore$ $\varepsilon_0 = 200$ V

and $\omega = 220$

or $2\pi f = 220$

$$\text{or} \quad f = \frac{220}{2\pi} = \frac{220}{2 \times 3.14} = 35.03\,\text{Hz}$$

or $f \simeq 35\,\text{Hz}$

OR

Expression for Q-factor

At ω_r, $Z = R$ while

at ω_1 and ω_2, $Z = \sqrt{2}R$

$$i.e., \quad Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$$

$$\text{or} \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\text{or} \quad \left(\omega L - \frac{1}{\omega C}\right) = \pm R$$

So we can write,

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \quad \text{and} \qquad \ldots\text{(i)}$$

$$\omega_2 L - \frac{1}{\omega_2 C} = -R \qquad \ldots\text{(ii)}$$

By adding equation (i) and (ii), we get

$$(\omega_1 + \omega_2)L - \frac{1}{C}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) = 0$$

$$\text{or} \quad \omega_1\,\omega_2 = \frac{1}{LC}$$

By subtracting equations (i) and (ii), we get

$$(\omega_2 - \omega_1)L + \frac{1}{C}\left(\frac{1}{\omega_2} - \frac{1}{\omega_1}\right) = 2R$$

$$\text{or} \quad \omega_2 - \omega_1 = \frac{R}{L}$$

$$\therefore \quad Q = \frac{\omega_r}{2\Delta\omega} = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r L}{R}$$

$$\text{or} \quad Q = \frac{\omega_r\, L.I._{rms}}{R.\,I_{rms}}$$

$$\text{or} \quad Q = \frac{\text{Voltage drop across inductor } L \text{ or capacitor } C}{\text{applied voltage}}$$

$$\text{As} \quad \omega_r = \frac{1}{\sqrt{LC}}$$

$$\text{or} \quad (\omega_r)^2 = \frac{1}{LC} \quad \text{or} \quad \omega_r L = \frac{1}{\omega_r C}$$

$$\therefore \quad Q = \frac{1}{\omega_r CR} = \frac{\sqrt{LC}}{CR} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \left[\because \omega_r = \frac{1}{\sqrt{LC}}\right]$$

35. Principle : It works on the principle of mutual induction, *i.e.*, when a charging or varying current is passed through one of the two inductively coupled coils, an induced emf is set up in the other coil.

Let us consider N_1 and N_2 be the numbers of turns in the primary and secondary coils respectively. Let us assume the situation when there is not load is present in the secondary coil.

Here, induced emf in the primary coil is given by,

$$\varepsilon_1 = -N_1 \frac{d\phi}{dt}$$

Similarly, induced emf in the secondary coil is

$$\varepsilon_2 = -N_2 \frac{d\phi}{dt} \quad \therefore \quad \frac{\varepsilon_2}{\varepsilon_1} = \frac{N_2}{N_1}$$

Let us consider ε be the emf applied to the primary coil. According to len's law ε_1 opposes ε in the primary coil.

$\therefore$ Net emf in the primary coil $= \varepsilon - \varepsilon_1$

$\therefore \quad \varepsilon - \varepsilon_1 = RI_1$

or $\quad \varepsilon = \varepsilon_1$; as resistance R is very small.

So, that we can say that ε_1 is the input emf and ε_2 is the output emf.

The ratio N_2/N_1 is the ratio of number of turns of secondary coil to number of turns of primary coil is also called turn ratio or transformation ratio.

In a step up transformer, $N_2 > N_1$ therefore, $\varepsilon_2 > \varepsilon_1$. A

Also in a step down transformer, $N_2 < N_1$ therefore, $\varepsilon_2 < \varepsilon_1$.

As in a ideal transformer there is no loss of power, input power is equal to output power, *i.e.*,

$$\varepsilon_1 I1 = \varepsilon_2 I_2$$

Where, I_1 and I_2 are the currents in the primary and secondary coils respectively.

$$\therefore \quad \frac{\varepsilon_2}{\varepsilon_1} = \frac{I_2}{I_1} = \frac{N_2}{N_1}$$

OR

(i) $$Li\frac{di}{dt} + Ri^2 + \frac{qi}{C} = vi \qquad \text{...(i)}$$

Since, $Li\frac{di}{dt} = \frac{d}{dt}\frac{1}{2}Li^2$ and $\frac{qi}{C} = \frac{d}{dt}\left(\frac{q^2}{2C}\right)$

Putting in equation (i)

$$\frac{d}{dt}\left(\frac{1}{2}Li^2\right) + Ri^2 + \frac{d}{dt}\left(\frac{q^2}{2c}\right) = vi \qquad \text{...(ii)}$$

(ii) $\frac{d}{dt}\left(\frac{1}{2}Li^2\right) \rightarrow$ rate of change of energy stored in an inductor

$Ri^2 \rightarrow$ rate of joule heating loss

$\frac{d}{dt}\left(\frac{q^2}{2C}\right) \rightarrow$ rate of change of energy stored in capacitor.

$vi \rightarrow$ rate of energy supply by source.

(iii) Equation (ii) is in the form of energy conservation statement.

(iv) $$\int_0^T \frac{d}{dt}\left(\frac{1}{2}Li^2 + \frac{q^2}{2C}\right)dt + \int_0^T Ri^2 dt = \int_0^T vi\, dt$$

$$= 0 + (+\text{ve}) = \int_0^T vi\, dt$$

$\therefore \int_0^T vi\, dt > 0$, phase difference between v and i is acute.