Alternating Current

CHAPTER 7

NCERT FOCUS

ANSWERS

Topic 1

1. (a) : The peak value of a.c. supply is given 300 V.

$E_0 = 300$ V

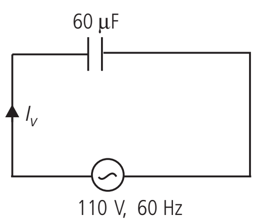
So, rms value of voltage

$$E_v = \frac{E_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 150\sqrt{2} \text{ V} = 212.1 \text{ V}$$

(b) Here $I_v = 10$ A

Thus, peak current $I_0 = I_v\sqrt{2} = 10\sqrt{2}$ A = 14.1 A

2.

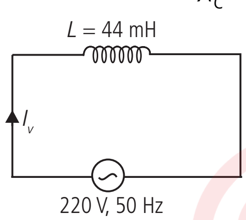


Capacitive reactance $X_C = \frac{1}{2\pi fC}$

$$X_C = \frac{1}{2\times\pi\times 60\times 60\times 10^{-6}} = 44.2\,\Omega$$

The rms current is, $I_V = \frac{E_V}{X_C} = \frac{110}{44.2} = 2.5$ A

3.



The rms current is

$$I_V = \frac{E_V}{X_L} = \frac{220}{2\pi\times 50\times 44\times 10^{-3}} = 15.9 \text{ A}$$

Topic 2

1. Resonant angular frequency in series LCR circuit

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times 32\times 10^{-6}}} = 125 \text{ rad/sec}$$

Quality factor $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32\times 10^{-6}}} = 25.$

2. Initial energy on capacitor

$$U_E = \frac{q_0^2}{2C} = \frac{\left(6\times 10^{-3}\right)^2}{2\times 30\times 10^{-6}} = 0.6 \text{ J}$$

Any time total energy in the circuit is constant, hence energy later is 0.6 J.

3. (a) Condition for resonance is when applied frequency matches with natural frequency.

Resonant frequency $\omega_r = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{5(80\times 10^{-6})}} = 50 \text{ rad s}^{-1}$$

(b) At resonance, impedance $Z = R$

as $X_L = X_C$

So, $Z = 40\ \Omega$

rms current, $I_v = \frac{E_V}{R} = \frac{230}{40} = 5.75$ A

Amplitude of current, $I_0 = I_v\sqrt{2} = 8.13$ A

(c) Potential drop across 'L'

$V_L = I_v X_L = 5.75 \times (\omega L)$

$V_L = 5.75 \times 50 \times 5 = 1437.5$ V

Potential drop across 'C'

$V_C = I_v \times X_C = 5.75 \times \frac{1}{\omega C}$

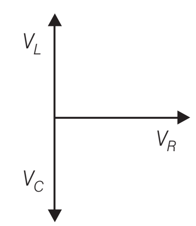
$$= 5.75\times\frac{1}{50\times 80\times 10^{-6}}$$

= 1437.5 V

Potential drop across R

$V_R = I_v R = 5.75 \times 40 = 230$ V

As $V_L - V_C = 0$, So $E_v = V_R = 230$ V.



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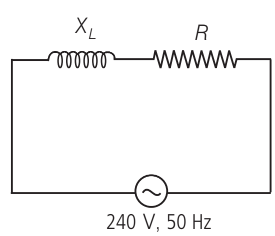


Inductive reactance

$X_L = 2\pi fL = 2\pi \times 50 \times 0.5 = 157\ \Omega$

Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{(100)^2 + (157)^2}$

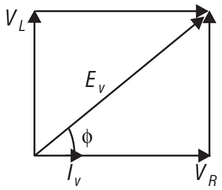
$= 186.14\ \Omega$

(a) Virtual current in the coil

$$I_v = \frac{E_V}{Z} = \frac{240}{186.14} = 1.29 \text{ A}$$

Maximum current, $I_0 = I_V\sqrt{2} = 1.82$ A

(b)



Phase lag, $\tan\phi = \frac{X_L}{R} = 1.57$

$\phi = \tan^{-1}(1.57) = 57.5°$ or $\phi = 0.32\,\pi$ radian

Time lag, $t = \phi/\omega = 3.2$ ms

5. At very high frequency, X_L increases to infinitely large, hence, circuit behaves as open circuit.

$X_L = 2\pi fL = 2\pi(10 \times 10^3) \times 0.5 = 31400\ \Omega$

(a) Current in the coil, $I_{rms} = \frac{\varepsilon_{rms}}{Z}$

Maximum current in the coil,

$$I_0 = \sqrt{2}\, I_{rms} = \sqrt{2} \times \frac{\varepsilon_{rms}}{Z}$$

$$= 1.414 \times \frac{240\ \text{V}}{3.14 \times 10^4\ \Omega}\ \text{A} = 1.10 \times 10^{-2}\ \text{A}$$

This current is extremely small. Thus, at high frequencies, the inductive reactance of an inductor is so large that it behaves as an open circuit.

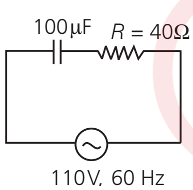
(b) As, $\tan\phi = \frac{X_L}{R} = \frac{3.14 \times 10^4}{100} = 314,\ \phi \approx 90°$

Clearly, time lag $= \frac{90°}{360°} \times \frac{1}{10^4}\ \text{s} = 25 \times 10^{-6}\ \text{s}$

In dc circuit (after steady state), $v = 0$ and as such $X_L = 0$.

In this case, the inductor behaves like a pure resistor as it has no inductive reactance.

6.



Capacitive reactance, $X_C = \frac{1}{2\pi fC}$

$$X_C = \frac{1}{2 \times \pi \times 60 \times 100 \times 10^{-6}} = 26.54\ \Omega$$

Impedance, $Z = \sqrt{R^2 + X_C^2}$

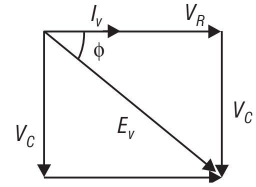
$$= \sqrt{(40)^2 + (26.54)^2} = 48\ \Omega$$

(a) Virtual current in the circuit

$I_v = \frac{E_V}{Z} = \frac{110}{48} = 2.29$ A

Maximum current $I_0 = I_v\sqrt{2} = 3.24$ A

(b)



$\tan\phi = \frac{V_C}{V_R} = \frac{1}{\omega CR}$

Phase lag $\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right) = \tan^{-1}\left(\frac{26.54}{40}\right)$

$\phi = 33.56° = 0.186\pi$ radian

Time lag $t = \phi/\omega = \frac{0.18\pi}{2\pi(60)} = 1.5$ ms

7. Given, $E_{rms} = 110$ V, $\upsilon = 12$ kHz $= 12 \times 10^3$ Hz

(a) $X_c = \frac{1}{2 \times 3.14 \times (12 \times 10^3) \times 10^{-4}} = 0.1326\ \Omega$

As, $R = 40\ \Omega$, $X_c <<< R$

$Z \approx R = 40\ \Omega$

$I_{rms} = \frac{\varepsilon_{rms}}{Z} = \frac{110}{40} = 2.75$ A

$I_0 = \sqrt{2}\, I_{rms} = 1.414(2.75) = 3.9$ A

This value of current is same as that without capacitor in the circuit. So, at high frequency, a capacitor offer negligible resistance (0.1326 Ω in this case), it behave like a conductor.

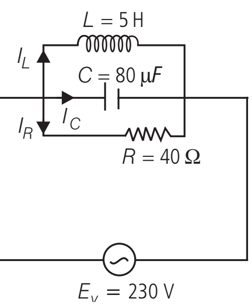
(b) As, $\tan\phi = \frac{X_C}{R} = \frac{0.1326\ \Omega}{40\ \Omega} = 0.0033,$

$\phi = 0.189° \approx 0°$

Time lag $= \frac{0.189°}{360°} \times \frac{1}{12 \times 10^3} = 43.8 \times 10^{-9}$ s

In dc circuit, after steady state, $v = 0$ and accordingly, $X_C = \infty$, *i.e.*, a capacitor amounts to an open circuit, *i.e.*, it is a perfect insulator of current.

8.



Resonating angular frequency

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50\ \text{rad s}^{-1}$$

∴ Resonance of L and C in parallel can be calculated

$$\frac{1}{X} = \frac{1}{X_L} = \frac{1}{X_C} = \frac{1}{\omega L} - \omega C$$

Impedance of R and X in parallel is given by

$$\frac{1}{Z}=\sqrt{\frac{1}{R^2}+\frac{1}{X^2}}$$

At resonating frequency of series LCR, $X_L = X_C$

So, $\frac{1}{X}=\frac{1}{X_L}-\frac{1}{X_C}=0$

Thus, impedances $Z = R$ and will be maximum. Hence, in parallel resonant circuit, current is minimum at resonant frequency.

Current through circuit elements

$$I_R=\frac{E_V}{R}=\frac{230}{40}=5.75\text{ A},\quad I_c=\frac{E_V}{X_L}=\frac{230}{\omega L}=\frac{230}{50\times5}=0.92\text{ A}$$

$$I_C=\frac{E_V}{X_L}=\frac{230}{(1/\omega C)}$$

$$= 230 \times 50 \times 80 \times 10^{-6} = 0.92\text{ A}$$

Since, I_L and I_C are opposite in phase, so net current,

$$I_v = I_R + I_L + I_C$$

$$I_v = 5.75 + 0.92\sqrt{2}\sin(\omega t - \pi/2) + 0.92\sqrt{2}\sin(\omega t + \pi/2)$$

$$I_v = 5.75 - 0.92\sqrt{2}\cos\omega t + 0.92\sqrt{2}\cos\omega t$$

$$I_v = 5.75\text{ A}$$

9. $\upsilon_r=\frac{1}{2\pi\sqrt{LC}}$

$$=\frac{1}{2\times3.14\times\sqrt{3\times27\times10^{-6}}}=\frac{1000}{6.28\times9}=17.7$$

resonant frequency, $\omega_r = 2\pi\upsilon_r = 2 \times 3.14 \times 17.7 = 111\text{ rad sec}^{-1}$

Quality factor in the given resonant circuit

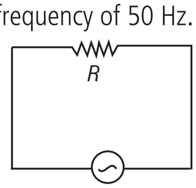
$$Q=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{1}{7.4}\sqrt{\frac{3}{27\times10^{-6}}}=45$$

we want to improve the quality factor to twice, without changing resonant frequency (without changing L and C).

$$Q'=2Q=90=\frac{1}{R'}\sqrt{\frac{L}{C}}\quad\text{or}\quad R'=\frac{1}{90}\sqrt{\frac{3}{27\times10^{-6}}}=3.7\,\Omega$$

Topic 3

1. (a) Here virtual a.c. voltage is 220 V at a frequency of 50 Hz. So, rms value of current



$$I_V=\frac{E_V}{R}=\frac{220}{100}=2.2\text{ A}$$

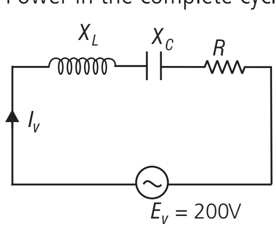
(b) Power in complete cycle

$$P = E_vI_v\cos\phi = E_vI_v\cos 0°$$

$$P = 2.2 \times 220 = 484\text{ W}$$

2. Power in the complete cycle $P = E_vI_v\cos(-\pi/2) = 0$.

3.



Average power transferred to the circuit in one complete cycle at resonance

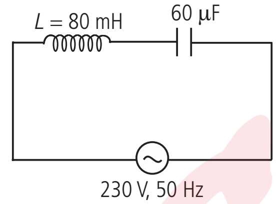
$$P = E_vI_v\cos\phi$$

$$P = E_v\frac{E_v}{Z}\cos\phi$$

At resonance $Z = R$, $\cos\phi = \cos 0° = 1$

$$P = 200 \times \frac{200}{20} = 2000\text{ W}$$

4.



(a) Inductive reactance, $X_L = 2\pi fL$

$$X_L = 2\pi(50)\, 80 \times 10^{-3} = 25.12\,\Omega$$

Capacitive reactance, $X_C=\frac{1}{2\pi fC}$

$$X_C=\frac{1}{2\times3.14\times50\times60\times10^{-6}}=53.05\,\Omega$$

Impedance $= X_C - X_L = 53.05 - 25.12 = 27.93\,\Omega$

rms value of current, $I_v=\frac{E_V}{Z}=\frac{230}{27.93}=8.235\text{ A}$

Peak value $I_0 = I_v\sqrt{2} = 11.644\text{ A}$

(b) Potential drop across L, $V_L = I_vX_L = 206.68\text{ V}$

Potential drop across C, $V_C = I_vX_C = 436.87\text{ V}$

(c) Average power transferred to inductor is zero, because of phase difference $\pi/2$.

$$P = E_vI_v\cos\phi$$

$$\phi = \pi/2,\quad \therefore\quad P = 0$$

(d) Average power transferred to capacitor is also zero, because of phase difference $\pi/2$.

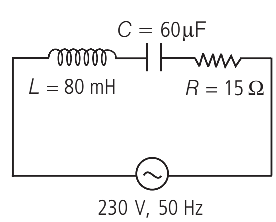
$$P = E_vI_v\cos\phi$$

$$\phi = \pi/2,\quad \therefore\quad P = 0$$

(c) Total power absorbed by the circuit

$$P_{Total} = P_L + P_C = 0$$

5. If the circuit has a resistance of 15 Ω, now it is LCR series resonant circuit.



Now the impedance,

$$Z=\sqrt{R^2+(X_L-X_C)^2}$$

$$Z=\sqrt{15^2+(27.93)^2}=31.7\,\Omega$$

Virtual current, $I_v = \frac{E_V}{Z} = \frac{230}{31.7} = 7.26$ A

Average power transferred to 'L',

$P_L = I_v E_v \cos\pi/2 = 0$

Average power transferred to 'C',

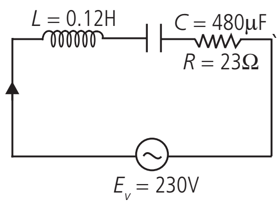
$P_C = E_v I_v \cos\pi/2 = 0$

Average power transferred to 'R',

$P_R = V_{R_V} I_v \cos 0°$

$P_R = (I_v R)\, I_v = I_V^2 R = (7.26)^2 \times 15 = 791$ W

6.



(a) At resonant frequency, the current amplitude is maximum.

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.12\times480\times10^{-9}}} = 663\,\text{Hz}$$

$$I_v = \frac{E_V}{R},\ I_0 = I_v\sqrt{2} = \frac{E_V\sqrt{2}}{R} = \frac{230\sqrt{2}}{23} = 14.14\ \text{A}$$

(b) Maximum power loss at resonant frequency,

$P = E_v I_v \cos\phi$

$$P = E_v \frac{E_V}{R}\cos 0° = \frac{E_V^2}{R} = \frac{(230)^2}{23} = 2300\ \text{W}$$

(c) Let at an angular frequency, the source power is half the power at resonant frequency.

$P = E_v I_v \cos\phi$

$$\frac{1}{2}\left[\frac{E_V^2}{R}\right] = \frac{E_V E_V}{Z}\frac{R}{Z}$$

$Z^2 = 2R^2$

$R^2 + (X_L - X_C)^2 = 2R^2$

$X_L - X_C = R$

$$\omega_L - \frac{1}{\omega C} = \text{R} \quad \text{or} \quad \omega^2 - \frac{1}{LC} = \frac{R}{L}\omega$$

where resonant angular frequency

$$\omega_r = \frac{1}{LC} = \frac{1}{0.12\times480\times10^{-9}}$$

$$\text{so, } \omega^2 - \omega_r^2 = \pm\frac{R}{L}\omega$$

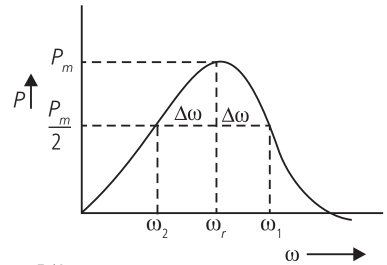
two quadratic equations can be formed

$$\omega^2 - \frac{R}{L}\omega - \omega_r^2 = 0 \ \text{ and } \ \omega^2 + \frac{R}{L}\omega - \omega_r^2 = 0$$

On solving, we get

$$\omega_1 = \frac{R}{2L} + \left[\omega_r^2 + R^2/4L^2\right]^{1/2} = \omega_r + \Delta\omega \text{ and}$$

$$\omega_2 = -\frac{R}{2L} + \left[\omega^2 + \frac{R^2}{4L^2}\right]^{1/2} = \omega_r - \Delta\omega$$



Now, $\omega_1 - \omega_2 = R/L$

$[\omega_r + \Delta\omega] - [\omega_r - \Delta\omega] = R/L$ or $\Delta\omega = R/L$

$\Delta\omega = \frac{R}{2L}$ bandwidth of angular frequency

So, band width of frequency

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{R}{4\pi L} = \frac{23}{4\times3.14\times0.12}$$

$\Delta f = 15.26$ Hz

Hence the two frequencies for half power

$f_2 = f_r - \Delta f$ and $f_1 = f_r + \Delta f$

$f_2 = 663 - 15.26 = 647.74$ Hz

and $f_1 = 663 + 15.26 = 678.26$ Hz

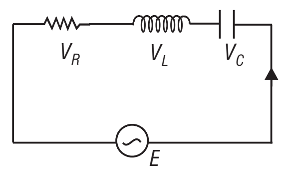
At these frequencies the current amplitude is

$$I = \frac{I_0}{\sqrt{2}} = 10\ \text{A}$$

(d) Q-factor, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$Q = \frac{1}{23}\sqrt{\frac{0.12}{480\times10^{-9}}} = 21.7$$

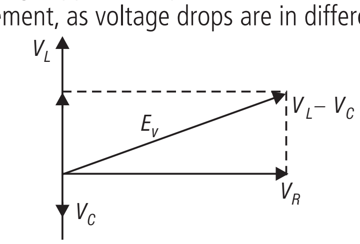
7. (a) It is true that applied instantaneous voltage is equal to algebraic sum of instantaneous potential drop across each circuit element is series.



$E = V_R + V_L + V_C$

$$E_0\sin\omega t = \frac{E_0}{R}\sin\omega t + \frac{E_0}{X_L}\sin(\omega t - \pi/2) + \frac{E_0}{X_C}\sin(\omega t + \pi/2)$$

But the rms voltage applied is equal to vector sum of potential drop across each element, as voltage drops are in different phases.



$$E_v = \sqrt{(V_L - V_C)^2 + V_R^2}$$

(b) At the time of broken circuit of the induction coil, the induced high voltage charges the capacitor. This avoid sparking in the circuit.

(c) Inductive reactance, $X_L = 2\pi fL$

For a.c., $X_L \propto f$

For d.c., $f = 0, X_L = 0$

Capacitive reactance, $X_C = \dfrac{1}{2\pi fC}$

For a.c., $X_C \propto \dfrac{1}{f}$

For d.c., $f = 0, X_C = \infty$

So, superimpose applied voltage will have all d.c. potential drop across X_C and will have most of a.c potential drop across X_L.

(d) Inductor offer no hinderance to d.c. $X_L = 0$, so insertion of iron core does not effect the d.c. current or brightness of lamp connected.

But it definitely effect a.c. current as insertion of iron core increases 'L',

$$L = \mu_m nl$$

thus increases X_L $(2\pi fL)$. a.c. current in the circuit reduces $I_V = \dfrac{E_V}{X_L}$ and brightness of the bulb also reduces.

(e) A fluorescent tube is connected directly across a 220 V source, it would draw large current which may damage the filaments of the tube. So, a choke coil which behaves as L-R circuit reduces the current to appropriate value, and that also with a lesser power loss.

$$I_V = \frac{E_V}{\sqrt{R^2 + X_L^2}}, \quad P = E_V I_V \cos\phi$$

An ordinary resistor used to control the current would have maximum power wastage as heat.

$$I_V = \frac{E_V}{R}, \quad P_{max} = E_V I_V$$

8. Here $E_p = 2300$ V, $N_p = 4000$ turns, $E_s = 230$ V, $N_s = ?$

We know in a transformer, $\dfrac{E_S}{E_p} = \dfrac{N_S}{N_p}$

$$N_s = \frac{E_s N_p}{E_p} = \frac{230 \times 4000}{2300} = 400 \text{ turns}$$

9. Workdone by liquid pressure = pressure × volume shifted power of flowing water

Hydro-power = $\dfrac{\text{Work}}{\text{time}}$ = pressure × $\dfrac{\text{volume}}{\text{time}}$

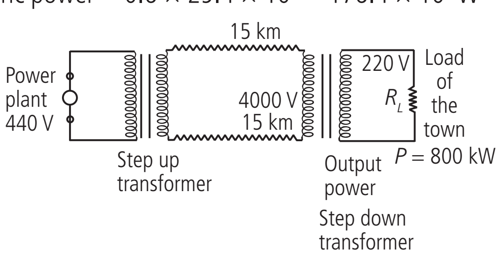
Hydro-power = $h\rho g \times (V/t)$

$= 300 \times 10^3 \times 9.8 \times 100 = 29.4 \times 10^7$ Watt

Efficiency of turbine, $\eta = \dfrac{\text{Eletric power}}{\text{Hydro-power}}$; $0.6 = \dfrac{\text{Eletric power}}{29.4 \times 10^7}$

Electric power $= 0.6 \times 29.4 \times 10^7 = 176.4 \times 10^6$ W = 176.4 MW

10.



Line resistance = length of two wire line × resistance per unit length

Line resistance (R) = 2 × 15 km × 0.5 $\dfrac{\Omega}{\text{km}}$ = 15 Ω

Virtual a.c. in the line, $P = E_V I_V$

$800 \times 10^3 = 4000\, I_V$ or $I_v = 200$ A

(a) Line power loss, $P_{loss} = I_v^2 R = (200)^2 \times 15$
$= 600$ kW

(b) Assuming no power loss due to leakage, total power need to be supply by the power plant

$P_{total} = P_{loss} + P_{output} = 600$ kW + 800 kW = 1400 kW

(c) Potential drop in the line,

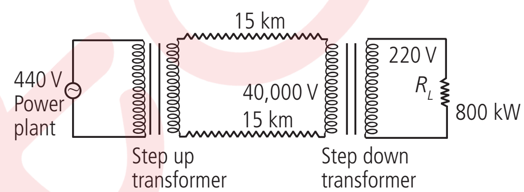
$V = I_v R = 200 \times 15 = 3000$ V

So, the voltage output of step-up transformer at the plant should be

4000 + 3000 = 7000 V.

Hence at the plant the step-up transformer should be 440 – 7000 V.

11.



Virtual a.c. in the time

$$I_V = \frac{P_{output}}{E_V} = \frac{800 \times 10^3}{40{,}000} = 20 \text{ A}$$

(a) Line power loss, $P_{loss} = I_v^2 R = (20)^2 \times 15$
$= 6$ kW

(b) Power supplied by the plant

$P_{Total} = P_{Loss} + P_{output} = 6$ kW + 800 kW
$= 806$ kW

(c) Voltage drop in the line,

$V = I_v R = 20 \times 15 = 300$ V.

Voltage output of step-up transformer at power station

= 40,000 + 300 = 40,300 V

So, the step up transformer at the power plant is 220 V - 40,300 V.

Power loss in earlier arrangement,

$$P_1 = \frac{600 \times 10^3}{1400 \times 10^3} \times 100 = 43\%$$

Power loss in this arrangement,

$$P_2 = \frac{6 \times 10^3}{806 \times 10^3} \times 100 = 0.74\%$$

So, by supply of electricity at higher voltage, 40,000 V instead by 4000 V the power loss is reduced greatly that is why the electric power is always transmitted at very high voltage.