EXAM DRILL

Electromagnetic Waves

CHAPTER 8

ANSWERS

1. (d) : Displacement current exists as long as conduction current continues to flow in the wires of electrical setup.

2. (c) : EM waves are source of an accelerated charge.

3. (a) : Since, em waves are sinusoidal waves of electric and magnetic field which are perpendicular to each other and to propagation of waves.

4. (b) : The amplitude of magnetic field and electric field for an electromagnetic wave propagating in vacuum are related as

$$E_0 = B_0 c$$

where c is the speed of light in vacuum.

$$\therefore \quad \frac{B_0}{E_0} = \frac{1}{c}$$

5. (b) : X-rays are used for the investigation of structure of solids.

6. (c) : EM waves speed changes when propagates through different media having different optical density.

7. (b) : We know $E_0 = cB_0$. Here, $E_0 = 5\ \text{Vm}^{-1}$

Then, $B_0 = \frac{E_0}{c} \Rightarrow \frac{5}{3\times10^8} = 1.67\times10^{-8}$ T

8. (c) : As $\vec{E}$ and $\vec{B}$ are perpendicular to each other, their dot product must be zero.

$$\vec{E}\cdot\vec{B} = 0$$

Also, the wave is propagating along x-axis *i.e.* cross product of $\vec{E}\times\vec{B}$ is along $\hat{i}$.

Out of given options, only $(-\hat{j}+\hat{k}), (-\hat{j}-\hat{k})$ follow the above conditions.

9. (a) : From equation $E = h\upsilon$, the frequency of Red is less than Blue. Therefore, more energy exhibits more momentum from $P = \sqrt{2mE}$.

10. (c) : Infrared radiation have low frequency and high wavelength. These are sourced from rotational and vibrational transitions of molecule and is result of hot body radiation.

11. (c) : Cones in the retina of the human eye are most sensitive for radiation of wavelength $\lambda = 5600 \times 10^{-10}$ m. Therefore, its frequency

$$\upsilon = \frac{c}{\lambda} = \frac{3\times10^8}{5600\times10^{-10}} = 5.36\times10^{14}\ \text{Hz.}$$

Hence, eye is most sensitive for the light of frequency 5.36×10^{14} Hz.

12. (d) : The small ozone layer present on the top of the stratosphere absorbs ultraviolet radiations, X-rays, γ-rays from the sun along with the cosmic rays. These rays are dangerous and cause genetic damage to the living cells. The ozone layer prevent them from reaching the earth's surface, which are very harmful for plants, animals and human beings and thus helps in the survival of the life. The environmental damage has decreased the amount of ozone in the atmosphere, which increases the change of ultraviolet radiation reaching earth.

13. (b) : Electromagnetic waves transport linear momentum as well as energy. When electromagnetic waves strike a surface, a pressure is exerted on the surface. If the intensity of wave is I, the radiation pressure P (force per unit area) exerted on the perfectly absorbing surface is $P = I/c$.

14. (i) (a)

(ii) (a) : Greenhouse effect is due to infrared rays.

(iii) (a) : Ozone layer absorbs the harmful ultraviolet radiations coming from the sun.

(iv) (a) : Ozone layer lies in stratosphere.

(v) (b) : The atmosphere of earth is richest in infrared radiation.

15. EM Waves are combination of oscillating electric and magnetic field which are orthogonal to each other and to the direction of propagation of wave.

16. Frequency range of visible light is $4 \times 10^{14} - 8 \times 10^{14}$ Hz.

OR

Order of decreasing wavelengths Radio waves > microwaves > Infrared > X-rays > γ-rays.

17. Using equation, $c = \lambda\upsilon$ or $\lambda = \frac{c}{\upsilon} = \frac{3\times10^8}{3\times10^{12}} = 10^{-4}$ m.

18. Infrared radiation is used to keep plants warm due to its heating effect properties.

19. The two properties common are :

(i) They all travel with the speed of light.

(ii) They are composition of oscillating electric and magnetic fields.

20. Electromagnetic waves transport momentum means that electromagnetic waves carry momentum from one place to another as they travel through medium or space.

21. There is no ac voltage applied on a capacitor, when it is connected across a battery so no current flows in a capacitor. When a capacitor is charging, current flows towards the positive plate (as positive charge is added to that plate) and away from the negative plate. When the capacitor is discharging, current flows away from the positive and towards the negative plate, in the opposite direction.

OR

Yes, ammeter will show a momentary deflection. The momentary deflection is due to the flow of electrons in the circuit during the charging process. During the charging process the electric field between the capacitor plates is increasing and hence a displacement current flows in the gap. Hence we can say that there is a continuity of current in the circuit.

Current inside the capacitor,

$$I_d = \varepsilon_0 \frac{d\phi}{dt}$$

22. When an electromagnetic wave is propagating along the x-axis then, electric field vector oscillates in y-axis and magnetic field vector oscillates in z-axis.

23. Infrared radiations (IR) radiations are result of rotational and vibrational effect of molecule therefore, they have heating properties. The next range of spectrum is microwaves (longer wavelength) and previous range of spectrum is visible spectrum (shorter wavelength).

24. The energy density of em-wave $= \frac{1}{2}\epsilon_0 E_0^2$

$$\Rightarrow \quad u_E = \frac{1}{2}(8.85 \times 10^{-12}\ \text{C}^2/\text{Nm}^2)(50\ \text{NC}^{-1})^2$$

$= 1.1 \times 10^{-8}\ \text{J m}^{-3}$

The volume of the cylinder is $A \times l$.

$V = 10\ \text{mm}^2 \times 50\ \text{m} = 5 \times 10^{-4}\ \text{m}^3$

The energy contained in this volume

$= 1.1 \times 10^{-8}\ \text{J m}^{-3} \times 5 \times 10^{-4}\ \text{m}^3 = 5.5 \times 10^{-12}\ \text{J}$.

OR

The intensity of a plane electromagnetic wave is

$$I = u_A v_g \times c = \frac{1}{2}\epsilon_0 E_0^2 \cdot c = \frac{1}{2}\epsilon_0 (cB_0)^2 \cdot c$$

or $\quad I = \frac{1}{2}\epsilon_0 B_0^2 \times c^3 \Rightarrow B_0 = \sqrt{\frac{2I}{\epsilon_0 c^3}}$

$$= \left(\frac{2 \times 1380\ \text{Wm}^{-3}}{[8.85 \times 10^{-12}\text{C}^2/\text{Nm}^2] \times (3 \times 10^8 \text{ms}^{-1})^3}\right)^{1/2}$$

$\Rightarrow \quad B_0 = 3.4 \times 10^{-6}\ \text{T}$.

25. Conduction current is the actual current whereas, displacement current is apparent current which is result of time-dependent electric field. On the other hand, conduction current flows in external circuit through wires whereas, displacement current is produced b/w the region of capacitor plates.

26. (a) γ rays (b) Microwaves

(i) X-rays are used as a diagnostic tool in medicine.

(ii) Microwaves : These are used in Radar system for aircraft navigation.

(iii) Infra-red rays : These are used to treat muscular pain.

(iv) Gamma rays : These are used for the treatment of cancer.

27. We are given that;

$E_0 = 48$ V/m, $\upsilon = 2.0 \times 10^{10}$ Hz and $c = 3 \times 10^8\ \text{m s}^{-1}$

(a) Wavelength of the wave,

$$\lambda = \frac{c}{\upsilon} = \frac{3 \times 10^8\ \text{m/s}}{2.0 \times 10^{10}\ \text{s}^{-1}} = 1.5 \times 10^{-2}\ \text{m}$$

(b) Amplitude of the oscillating magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{48\ \text{V/m}}{3 \times 10^8\ \text{m/s}} = 1.6 \times 10^{-7}\ \text{T}$$

(c) Total average energy density,

$$u_{av} = \frac{1}{2}\epsilon_0 E_0^2$$

$$= \frac{1}{2}(8.85 \times 10^{-12})(48)^2\ \text{J/m}^3 = 1.0 \times 10^{-8}\ \text{J/m}^3$$

OR

Since the light wave is moving along the Z-direction, if the electric field is taken to be along the X-direction, the magnetic field would be along the Y-direction. Let E_0 be the peak value of the electric field along X-direction.

Clearly $E_0 = 1$ V/m

Total average energy density (due to both electric and magnetic fields)

$$= \frac{1}{2}\epsilon_0 E_0^2 = \frac{1}{2}(8.85 \times 10^{-12})(1)^2\text{T} = 4.42 \times 10^{-12}\ \text{J/m}^3$$

Since the energy is shared equally by the electric and magnetic fields,

average energy density of the electric field

$$= \frac{1}{2}(4.42 \times 10^{-12}\ \text{J/m}^3) = 2.21 \times 10^{-12}\ \text{J/m}$$

average energy density of the magnetic field

$$= \frac{1}{2}(4.42 \times 10^{-12}\ \text{J/m}^3) = 2.21 \times 10^{-12}\ \text{J/m}^3$$

28. (a) $\lambda = \frac{c}{\upsilon} = \frac{3 \times 10^8\ \text{m/s}}{3 \times 10^{10}\ \text{s}^{-1}} = 10^{-2}\ \text{m}$

(b) $E_0 = cB_0 = (3 \times 10^8\ \text{m/s})(10^{-7}\ \text{T}) = 30$ V/m

Since $\vec{B}$ acts along Y-direction, $\vec{E}$ acts along Z-axis. Expression representing the oscillating electric field is

$E_z = E_0 \sin(kx - \omega t)$

$$= E_0 \sin\left[\left(\frac{2\pi}{\lambda}\right)x - (2\pi\upsilon)t\right] = E_0 \sin 2\pi\left[\frac{x}{\lambda} - \upsilon t\right]$$

$$= 30 \sin 2\pi\left[\frac{x}{10^{-2}} - 3\times10^{10}t\right] \text{V/m}$$

or $E_z = 30 \sin 2\pi\,[100x - 3 \times 10^{10}\, t]$ V/m

29. Comparing the given equation with the standard equation : $B_y = B_0 \sin(\omega t + kx)$, we get

$B_0 = 8 \times 10^{-6}$ T, $\omega = 2 \times 10^{11}$, $k = 300\,\pi$

(a) As $k = \frac{2\pi}{\lambda}, \lambda = \frac{2\pi}{k} = \frac{2\pi}{300\pi}\text{m} = \frac{1}{150}\text{m} = 0.67$ cm

(b) $E_0 = cB_0 = (3 \times 10^8)(8 \times 10^{-6})$ V/m = 2400 V/m

The given electromagnetic wave propagates along negative X-axis $(-\hat{i})$. If $\vec{B}$ is along $\hat{j}$, $\vec{E}$ is along $\hat{k}$ *i.e.*, Z-axis as $\hat{k}\times\hat{j} = -\hat{i}$.

Hence, the expression for the oscillating electric field

$E_z = 2400 \sin[2 \times 10^{11}\, t + 300\,\pi x]$ V/m

30. (a) The half of the light that is absorbed exerts radiation pressure of $\frac{1}{2}(I/c)$ and half that is reflected exerts a pressure $\frac{1}{2}(2I/c) = I/c$.

The total radiation pressure on the surface that is

$$P_r = \frac{1}{2}(I/c) + (I/c) = \frac{3}{2}(I/c) = 1.5\frac{I}{c} = \frac{(1.5)(1\text{ kW/m}^2)}{3\times10^8\text{ m/s}}$$

$= 5 \times 10^{-6}$ Pa

(b) $\frac{P_r}{P_0} = \frac{5\times10^{-6}\text{Pa}}{1\times10^5\text{Pa}} = 5\times10^{-11}$

The radiation pressure due to sunlight at the Earth surface is thus negligibly small compared with the atmosphere pressure.

31. Since the efficiency of the bulb is 2.5%, effective power of the bulb, $P = \left(\frac{2.5}{100}\right)100\text{ W} = 2.5\text{ W}$

Wave intensity at a distance r from the bulb,

$$I = \frac{P}{4\pi r^2} = \frac{2.5\text{ W}}{4\times3.14\times(3\text{ m})^2} = 0.022\text{ W/m}^2$$

($4\pi r^2$ is the area of a sphere of radius r, centred on the source)

Also, as $I = \frac{1}{2}c\in_0 E_0^2$,

$$E_0 = \sqrt{\frac{2I}{c\in_0}} = \sqrt{\frac{2\times0.022}{(3\times10^8)(8.85\times10^{-12})}}\text{V/m} = 4.07\text{ V/m}$$

Clearly, $B_0 = \frac{E_0}{c} = \frac{4.07\text{ V/m}}{3\times10^8\text{ m/s}} = 1.36\times10^{-8}\text{T}$

32. In an electromagnetic wave, both E and B fields vary sinusoidally in space and time. The average energy density u of an e.m. wave can be obtained by replacing E and B by their rms value

$$U = \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0}B_{rms}^2 \text{ or } U = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0}B_0^2$$

$$\left[\because\; E_{rms} = \frac{E_0}{\sqrt{2}}, B_{rms} = \frac{B_0}{\sqrt{2}}\right]$$

Moreover, $E_0 = cB_0$ and $c^2 = \frac{1}{\mu_0\varepsilon_0}$, therefore

$$U_E = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{4}\varepsilon_0 (cB_0)^2$$

$$U_E = \frac{1}{4}\varepsilon_0 \cdot \frac{B_0^2}{\mu_0\varepsilon_0} = \frac{1}{4\mu_0}B_0^2 = U_B$$

33. (a) Consider a plane perpendicular to the direction of propagation of the wave. An electric charge, on the plane will be set in motion by the electric and magnetic fields of *e.m.* wave, incident on this plane. This illustrates that *e.m.* waves carry energy and momentum.

(b) Microwaves are produced by special vacuum tube like the klystron, magnetron and Gunn diode.

The frequency of microwaves is selected to match the resonant frequency of water molecules, so that energy is transformed efficiently to the kinetic energy of the molecules.

(c) Uses of infra-red waves :

(i) They are used in night vision devices during warfare. This is because they can pass through haze, fog and mist.

(ii) Infra-red waves are used in remote switches of household electrical appliances.

OR

In microwave oven, the frequency of the microwaves is selected to match the resonant frequency of water molecules so that energy from the waves get transferred efficiently to the kinetic energy of the molecules. This kinetic energy raises the temperature of any food containing water.

Microwaves are short wavelength radio waves, with frequency of order of GHz. Due to short wavelength, they have high penetrating power with respect to atmosphere and less diffraction in the atmospheric layers. So, these waves are suitable for the radar systems used in aircraft navigation.

34. (a) The waves is propagating along negative y-direction of its direction is $-\hat{j}$.

(b) Comparing the given equation with the standard equation.

$$\vec{E} = E_0 \cos\left[2\pi\left(\frac{y}{\lambda} + \nu t\right)\right]$$

We get, $\frac{2\pi}{\lambda} = 1.8$

$\therefore$ Wavelength, $\lambda = \frac{2\pi}{1.8} \times \frac{2\times3.14}{1.8} = 3.5$ m.

(c) Also, $2\pi\nu = 5.4 \times 10^6$

$$\therefore\ \nu = \frac{5.4\times10^6}{2\pi} = \frac{5.4\times10^8}{2\times3.14} = 85.9\times10^6\ \text{Hz.}$$

$\simeq 86$ MHz

OR

(a) Intensity equals power per unit area carried by the wave.

$\Rightarrow$ Power per unit area $= \dfrac{P}{\pi r^2}$

$$= \frac{4\,\text{W}}{3.14\times(0.2\times10^{-3}\,\text{m})^2}$$

$= 32 \times 10^6$ W/m^2

(b) Intensity $= \dfrac{E_0^2}{2\mu_0 c}$ $\Rightarrow$ $E_{max} = E_0 = \sqrt{(2\mu_0 c)(I)}$

$$= \sqrt{(8\pi\times10^{-7})(3\times10^8)(32\times10^6)}\ \text{SI units}$$

$= 1.6 \times 10^5$ V/m

(c) $B_{max} = B_0 = \dfrac{E_0}{c} = \dfrac{1.6\times10^5\ \text{V/m}}{3\times10^8\,\text{m/s}} = 5.3 \times 10^{-4}$ T

(d) $\Delta P = \dfrac{2U}{c}$ where ΔP is the momentum delivered on the sphere.

$$F = \frac{\Delta P}{\Delta t} = 2\left(\frac{U}{\Delta t}\right)\cdot\frac{1}{c}$$

Now, $\dfrac{U}{\Delta t} = I.A = I(\pi r^2) \Rightarrow F = \dfrac{2\cdot I\cdot\pi r^2}{c}$

Now the force F must support the sphere mass mg.

$$\Rightarrow\quad m = \frac{2I\pi r^2}{gc}$$

$$= \frac{2\left(3.2\times10^7\ \frac{\text{W}}{\text{m}^2}\right)(3.14\times1\times10^{-10}\,\text{m}^2)}{(9.8\ \text{m/s}^2)(3\times10^8\ \text{m/s})}$$

$= 6.8 \times 10^{-12}$ kg.

35. Electric field intensity on a surface due to the incident radiation is

$$E = \frac{U}{At} = \frac{P}{A} \qquad \left(\because\ \frac{U}{t} = P\right)$$

As, $E \propto P$ (for the given area of the surface)

$$\therefore\quad \frac{E'}{E} = \frac{P'}{P} = \frac{50}{100} = \frac{1}{2}$$

$$E' = \frac{E}{2}$$

OR

The bulb as a point source, radiate light in all direction uniformly and it is given that only 10% of power is converted to visible radiations.

(a) Let us assume a sphere of radius 5 m.

Surface area $A = 4\pi r^2 = 4\pi\,(5)^2 = 100\pi$ m^2

Intensity on the sphere

$$I = \frac{\text{Power}}{\text{Area}} = \frac{200\times0.1}{100\pi} = 0.06\ \text{W/m}^2$$

(b) Let us assume a sphere of 20 m radius.

Surface area $A = 4\pi r^2 = 4\pi(20)^2 = 1600\ \pi$m^2

Intensity on the sphere

$$I = \frac{\text{Power}}{\text{Area}} = \frac{200\times0.1}{1600\ \pi}$$

$= 0.004$ W/m^2 $= 0.004$ W/m^2