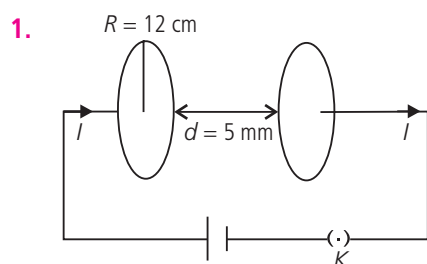


Electromagnetic Waves



ANSWERS

Topic 1



(a) Capacitance of the parallel plate capacitor

$$C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times \pi \times (12 \times 10^{-2})^2}{5 \times 10^{-3}}$$

$$C = 80.1 \times 10^{-12} \text{ F} = 80.1 \text{ pF}$$

We know $Q = CV$

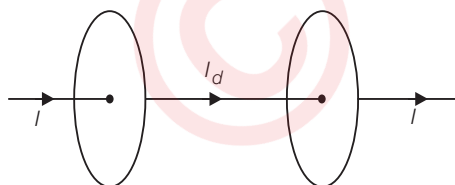
$$I = \frac{dQ}{dt} = C \frac{dV}{dt}$$

$$\text{So, } \frac{dV}{dt} = \frac{I}{C} = \frac{0.15}{80.1 \times 10^{-12}} = 1.875 \times 10^9 \text{ V s}^{-1}$$

(b) The magnitude of displacement current is equal to conduction current.

$$\therefore I_d = I = 0.15 \text{ A}$$

(c) Yes, Kirchhoff's first law is very much applicable to each plate of capacitor as $I_d = I$. So current is continuous and constant across each plate.



2. (a) Capacity of capacitor $C = 100 \text{ pF}$

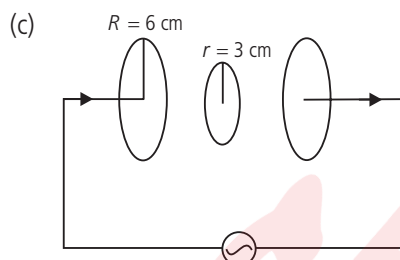
$$\text{Capacitive reactance } X_C = \frac{1}{\omega C} = \frac{1}{300 \times 100 \times 10^{-12}}$$

$$\therefore X_C = \frac{10^8}{3} \Omega$$

If I_{rms} is the rms value of conduction current

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{X_C} = 230 \times 3 \times 10^{-8} = 690 \times 10^{-8} = 6.9 \mu\text{A}$$

(b) Yes, the conduction current in wires always equal to displacement current within plates.



To find magnetic field B of a point 3 cm from the axis within plates, let us assume a loop of radius 3 cm with center on axis.

Now modified Ampere's Law.

$$\int \vec{B} \cdot d\vec{l} = \mu_0 \left[\frac{I}{\pi R^2} \pi r^2 \right]$$

$$B \times 2\pi r = \frac{\mu_0 I r^2}{R^2}$$

For amplitude of magnetic field, we require

$$I_0 = I_{\text{rms}} \sqrt{2} = 6.9 \sqrt{2} \mu\text{A}$$

$$\text{So, } B_0 = \frac{\mu_0}{2\pi} \times \frac{I_0 r}{R^2} = 2 \times 10^{-7} \times \frac{6.9 \sqrt{2} \times 10^{-6} \times 3 \times 10^{-2}}{(6 \times 10^{-2})^2}$$

$$B_0 = 1.63 \times 10^{-11} \text{ T}$$

Topic 2

1. Though they all have different wavelengths and frequencies, but they have same speed *i.e.*, speed of light $3 \times 10^8 \text{ m s}^{-1}$ in vacuum.

2. Maximum wavelength in the band is for lowest frequency

$$C = v_{\text{min}} \lambda_{\text{max}}$$

$$\lambda_{\text{max}} = \frac{C}{v_{\text{min}}} = \frac{3 \times 10^8}{7.5 \times 10^6} = 40 \text{ m}$$

Minimum wavelength in the band is for highest frequency

$$C = v_{\text{max}} \lambda_{\text{min}}$$

$$\lambda_{\text{min}} = \frac{C}{v_{\text{max}}} = \frac{3 \times 10^8}{12 \times 10^6} = 25 \text{ m}$$

So, the wavelength band for tuning is between 25 m to 40 m.

3. The frequency of electromagnetic wave produced will be same as the frequency of oscillation of charged particles *i.e.*, 10^9 Hz .

4. We know the relation, $\frac{E_0}{B_0} = c$

$$E_0 = B_0 c = 510 \times 10^{-9} \times 3 \times 10^8$$

Peak value of $E_0 = 1530 \times 10^{-1} = 153 \text{ N C}^{-1}$ electric field.

5. (a) By relation, $\frac{E_0}{B_0} = c$

Peak value of magnetic field

$$B_0 = \frac{E_0}{c} = \frac{120}{3 \times 10^8} = 40 \times 10^{-8} \text{ T}$$

Angular frequency, $\omega = 2\pi\nu = 100\pi \times 10^6 \text{ Hz}$.

Speed of light $c = \nu\lambda$

$$\text{Wavelength, } \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{50 \times 10^6}, \lambda = 6 \text{ m}$$

Propagation constant,

$$k = \frac{2\pi}{\lambda} = \frac{\pi}{3} \text{ m}^{-1} = 1.05 \text{ m}^{-1}$$

(b) Let the wave is propagating along x-direction, electric field \vec{E} is along y - direction and magnetic field \vec{B} along z-direction.

$$\vec{E}_y = E_0 \sin(kx - \omega t) \hat{j}$$

$$\vec{E}_y = 120 \sin\left(\frac{\pi}{3}x - 100\pi \times 10^6 t\right) \hat{j}$$

$$\vec{B}_z = B_0 \sin(kx - \omega t) \hat{k} = 40 \times 10^{-8} \sin\left(\frac{\pi}{3}x - \pi \times 10^8 t\right) \hat{k}$$

6. (a) Wavelength of electromagnetic wave is

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{2 \times 10^{10}} = 1.5 \times 10^{-2} \text{ m}$$

(b) Amplitude of magnetic field, $\frac{E_0}{B_0} = c$

$$B_0 = \frac{E_0}{c} = \frac{48}{3 \times 10^8} = 16 \times 10^{-8} \text{ T}$$

(c) Energy density as electric field,

$$u_E = \frac{1}{2} \epsilon_0 E^2$$

$$\text{Here, } \frac{E}{B} = c$$

$$\text{So, } u_E = \frac{1}{2} \epsilon_0 c^2 B^2$$

where speed of electromagnetic wave,

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\text{So } u_E = \frac{1}{2} \frac{\epsilon_0}{\mu_0 \epsilon_0} B^2$$

$$u_E = \frac{B^2}{2\mu_0} = u_B$$

[Energy density as magnetic field]

7. (a) Electromagnetic wave is propagating in $-y$ direction.

(b) Propagation constant, $k = \frac{2\pi}{\lambda} = 1.8$

$$\lambda = \frac{2\pi}{1.8} = \frac{\pi}{0.9} \text{ m} = 3.49 \text{ m}$$

(c) Angular frequency, $\omega = 2\pi\nu = 5.4 \times 10^8$

$$\nu = \frac{5.4}{2 \times 3.14} \times 10^8 = 86 \times 10^6 \text{ Hz} = 86 \text{ MHz}$$

(d) Amplitude of magnetic field part, $\frac{E_0}{B_0} = c$

$$\text{So, } B_0 = \frac{E_0}{c} = \frac{3.1}{3 \times 10^8} = 1.03 \times 10^{-8} \text{ T}$$

$$= 10.3 \times 10^{-9} \text{ T} = 10.3 \text{ nT}$$

(e) Expression for magnetic field part of wave

$$\vec{B}_z = B_0 \cos(ky + \omega t) \hat{k}$$

$$\vec{B}_z = 1.03 \times 10^{-8} \cos(1.8y + 5.4 \times 10^6 t) \hat{k}$$

8. The bulb as a point source, radiate light in all direction uniformly and it is given that only 5% of power is converted to visible radiations.

(a) Let us assume a sphere of radius 1 m.

$$\text{Surface area } A = 4\pi r^2 = 4\pi (1)^2 = 4\pi \text{ m}^2$$

Intensity on the sphere

$$I = \frac{\text{Power}}{\text{Area}} = \frac{100 \times 0.05}{4\pi} = 0.4 \text{ W m}^{-2}$$

(b) Let us assume a sphere of 10 m radius.

$$\text{Surface area } A = 4\pi r^2 = 4\pi (10)^2 = 400\pi \text{ m}^2$$

Intensity on the sphere

$$I = \frac{\text{Power}}{\text{Area}} = \frac{100 \times 0.05}{400\pi} = 0.004 \text{ W m}^{-2}$$

9. A black body at very high temperature produce a continuous spectrum. Using Wien's displacement law we can calculate the wavelength corresponding to maximum intensity of radiation emitted.

Required absolute temperature,

$$\lambda_m T = 0.29 \text{ cm K} \quad \text{or} \quad T = \frac{0.29 \times 10^{-2}}{\lambda_m} \text{ K}$$

Using above formula, the temperature of black body required for various wavelength is calculated.

	Types of radiation	Wavelength of radiation	Temperature required
1	Radio waves	1 m	$29 \times 10^{-4} \text{ K}$
2.	Micro waves	$3 \times 10^{-3} \text{ m}$	1 K
3.	Infrared waves	$3 \times 10^{-5} \text{ m}$	10^2 K
4.	Visible waves	$5 \times 10^{-7} \text{ m}$	$6 \times 10^3 \text{ K}$
5.	Ultraviolet rays	$3 \times 10^{-7} \text{ m}$	10^4 K

6.	X-rays	10^{-10} m	29×10^6 K
7.	γ -rays	10^{-12} m	29×10^8 K

To produce electromagnetic radiations of different wavelength, we need temperature ranges. To produce visible radiation of $\lambda = 5 \times 10^{-7}$ m, we need to have source at temperature of 6000 K. A source at lower temperature will produce this wavelength but not with maximum intensity.

10. (a) 21 cm wavelength corresponds to radiowaves, at nearly 1445 MHz.

(b) Electromagnetic wave of 10^{57} MHz also corresponds to radiowaves.

(c) Using Wien's Law the wavelength of the radiation mostly emitted at given temperature of heavenly bodies can be calculated $\lambda_m T = b = 0.29$ cm K

$$\text{or } \lambda_m = b/T = \frac{0.29}{2.7} = 0.11 \text{ cm}$$

This wavelength corresponds to microwave region of electromagnetic waves.

(d) This wavelength corresponds to visible region (yellow) of electromagnetic waves.

(e) The frequency of radiation can be calculated

$$E = h\nu$$

$$\nu = \frac{E}{h} = \frac{14.4 \times 10^3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} \text{ Hz}$$

$$= 3.49 \times 10^{18} \text{ Hz}$$

This frequency corresponds to X-ray region of electromagnetic waves.

11. (a) Electromagnetic waves in frequency range of shortwave band reflect from ionosphere where lower frequency radio waves i.e., medium waves are absorbed.

So, short waves are suitable for long distance radio broadcast.

(b) Very high frequency (VHF) and ultra high frequency (UHF) electromagnetic waves used in T.V. transmission after frequency modulation do not reflect from ionosphere and can only be transmitted straight from antenna. So, the range is highly limited. In order to send the transmission for long distances, satellite is used.

(c) Atmosphere absorb X-rays, while visible and radiowaves can penetrate through it. Hence optical telescope can work on ground but X-ray astronomical telescopes only work above atmosphere, hence installed on the satellite orbiting around earth.

(d) The thin ozone layer absorbs harmful ultraviolet radiations, γ -rays and cosmic radiations. All these high energy radiations can cause damage to the living cells.

Thus the thin ozone layer on top of the stratosphere is crucial for human survival.

(e) In the absence of atmosphere, the day temperature can rise to many fold and temperature at night will be much below 0°C . Although the average temperature will be reduced in the absence of green house effect.

(f) The thick clouds produced by global nuclear war will prevent solar radiation to reach down across the globe. This would cause severe 'nuclear winter'.

12. The photon of given energy are released during transition between energy levels in the atom and the emitted photon energy is equal to difference of energies of energy levels, among which the transition has taken place.

For example photon energy for

$\lambda = 1$ m can be calculated

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1} \text{ J}$$

$$E = \frac{19.88 \times 10^{-26}}{1.6 \times 10^{-19}} \text{ eV} = 12.425 \times 10^{-7} \text{ eV}$$

So, for emission of radio waves the energy difference between energy levels should be 12.425×10^{-7} eV.

Similarly, photon energy for other wavelengths can also be calculated

Types of radiation	Wavelength of radiation	Photon energy
1. Radio waves	1 m	12.425×10^{-7} eV
2. Micro waves	3×10^{-3} m	4.14×10^{-4} eV
3. Infrared waves	3×10^{-5} m	4.14×10^{-2} eV
4. Visible waves	5×10^{-7} m	2.485 eV
5. Ultraviolet rays	3×10^{-7} m	4.14 eV
6. X-rays	10^{-10} m	12.425×10^3 eV
7. γ -rays	10^{-12} m	12.425×10^5 eV

So, for emission of γ -rays the energy difference among energy levels should be of the order of MeV, where for visible radiation it should be of the order of eV.

13. In electromagnetic wave, the electric field vector \vec{E} and magnetic field vector \vec{B} show their variations perpendicular to the direction of propagation of wave as well as perpendicular to each other. As the electromagnetic wave is travelling along z-direction, hence \vec{E} and \vec{B} show their variation in x-y plane.

$$\text{Wavelength, } \lambda = \frac{c}{\nu} = \frac{3 \times 10^8 \text{ m/s}}{30 \times 10^6 \text{ s}^{-1}} = 10 \text{ m.}$$

