Ray Optics and Optical Instruments

CHAPTER 9

EXAM DRILL

ANSWERS

1. **(d) :** Dispersion is the phenomenon of splitting of light into its component colours

2. **(b)**

3. **(d) :** TIR occur when incidence angle $(\sin i) \geq \frac{1}{n_{21}}$ where, n_{21} is refractive index of medium 2 w.r.t medium 1.

4. **(a) :** $P = P_1 + P_2 = 2 - 1 = 1$ D

$f = \frac{1}{1} = 1\,m = 100$ cm, so converging.

5. **(b) :** During reflection, frequency and wavelength do not change. 5000 Å

6. **(d) :** Image formed by concave mirror is real or virtual depending on the position of object.

7. **(c) :** Given, $\frac{\mu_g}{\mu_w} = \frac{9}{8}$ and $v_g = 2 \times 10^8$ m/s

Now $v_g = \mu_g = v_w \mu_w$; $v_w = 2.25 \times 10^8$ m/s

8. **(b) :** For biconcave lens, $\frac{1}{F} = (\mu - 1)\left[\frac{1}{-R} - \frac{1}{R}\right]$

$= \frac{-2(\mu - 1)}{R}$

For plano-concave lens,

$\frac{1}{F'} = (\mu - 1)\left[\frac{1}{-R} - \frac{1}{\infty}\right] = \frac{-(\mu - 1)}{R} \Rightarrow F' = 2F \Rightarrow P' = \frac{P}{2}$

9. **(d)**

10. **(a) :** We know, $\mu = \dfrac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$

$A = 60°$

For equilateral triangle,

now $\dfrac{\sqrt{3} = \sin\left(\frac{60 + \delta_m}{2}\right)}{\sin\frac{60}{2}} \Rightarrow \delta_m = 60°$

11. **(b) :** After refraction at two parallel faces of a glass slab, a ray of light emerges in a direction parallel to the direction of incidence of white light on the slab. As rays of all colours emerge in the same direction (of incidence of white light), hence there is no dispersion, but only lateral displacement.

12. **(a) :** Goggles have zero power.

The focal length is given by $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. For goggle lenses, both sides are curved the same way. R_1 and R_2 are positive. If they are the same, $\frac{1}{f} = 0$ *i.e.* power is zero.

13. **(b) :** $\angle i = 0$. $\therefore$ $\angle r = 0$

Also, $\sin C = 1/\mu$,

Therefore, critical angle (C) for TIR is smaller when light travels from glass to water than when it travels from glass to air.

14. **(d) :** Light travels faster in air than in glass, because glass is denser than air.

15. **(i) (a) :** Here, $f_1 = 20$ cm, $F = 80$ cm

As, $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$ $\therefore$ $\frac{1}{f_2} = \frac{1}{F} - \frac{1}{f_1} = \frac{1}{80} - \frac{1}{20} = \frac{3}{80}$; $f_2 = -26.7$ cm

(ii) **(a) :** $P_2 = P - P_1 = \frac{100}{F} - \frac{100}{f_1}$

$\therefore$ $P_2 = \frac{100}{80} - \frac{100}{20} = 1.25 - 5 = -3.75$ D

(iii) **(a) :** Here $P = P_1 + P_2 = \frac{100}{F} = \frac{100}{80} = 1.25$ D

$P_3 = \frac{100}{f_3} = \frac{100}{-20} = -5$ D

$\therefore$ Power of the combination of three lenses is

$P' = P_1 + P_2 + P_3 = 1.25 - 5 = -3.75$ D

(iv) **(d) :** $F' = \frac{100}{P'} = \frac{100}{-3.75} = -26.7$ cm

16. $\theta = 0°$, $n = \frac{360°}{0°} - 1 = \infty$

17. $\delta_{blue} > \delta_{orange}$

when the incident blue light is replaced with orange light the angle of minimum deviation of a glass decreases.

18. $\mu = \dfrac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$

OR

$f = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\mu_{Violet} > \mu_{Red}$ $\therefore$ Power of lens will be increased

19. Here, ${}^a\mu_w = \frac{4}{3}$ and ${}^a\mu_g = \frac{3}{2}$

Refractive index of glass w.r.t. water

$${}^{w}\mu_g = \frac{{}^{a}\mu_g}{{}^{a}\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$$

Now, in water, $i = 30°$, $r = ?$

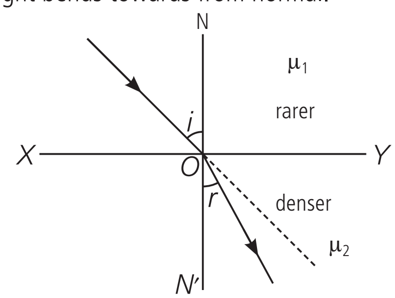
$$\frac{\sin i}{\sin r} = {}^{w}\mu_g = \frac{9}{8};\ \sin r = \frac{8}{9}\sin i = \frac{8}{9}\sin 30° = \frac{4}{9} = 0.444$$

$r = \sin^{-1}(0.4444) = 26°\ 23'$

20. When light goes from a denser to a rarer medium, then from the snell's law.

If $\mu_1 < \mu_2 \Rightarrow \sin i > \sin r$ or $i > r$

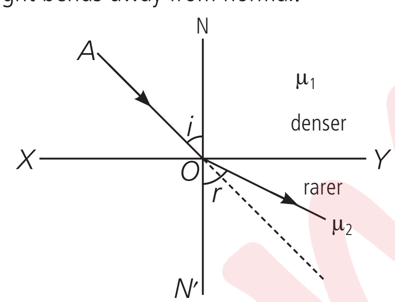
$\therefore$ Ray of light bends towards from normal.



$i > r$

If $\mu_1 > \mu_2 \Rightarrow \sin r > \sin i$ or $r > i$

$\therefore$ Ray of light bends away from normal.



$i < r$

OR

When refracted ray is parallel to the base of the prism, deviation is minimum.

$\therefore$ $r = A/2 = 60/2 = 30°$

From $\mu = \dfrac{\sin i}{\sin r}$; $\sin i = \mu \sin r = \sqrt{3}\sin 30° = \dfrac{\sqrt{3}}{2}$ $\quad\therefore i = 60°$

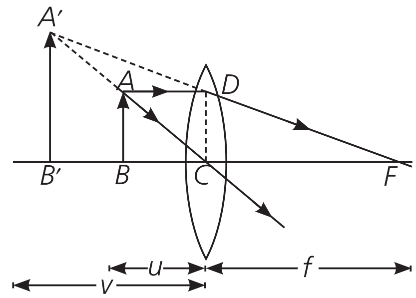
21. Here, $A = 60°, i = \frac{3}{4}A = \frac{3}{4}\times 60° = 45°$

In the position of minimum deviation,

$$r = \frac{A}{2} = 30°,\ \mu = \frac{\sin i}{\sin r} = \frac{\sin 45°}{\sin 30°} = \frac{\frac{1}{\sqrt{2}}}{1/2} = \sqrt{2}$$

$$\mu = \frac{c}{v}, v = \frac{c}{\mu} = \frac{3\times 10^8}{\sqrt{2}} = 2.12\times 10^8 \text{ m/s}$$

22. Image formed is virtual, when the object AB is held close to the lens, between C and F, the image $A'B'$ formed by convex lens is virtual, erect and magnified, as shown in Figure.



As $\Delta A'B'C$ and ΔABC are similar,

$$\therefore \quad \frac{A'B'}{AB} = \frac{CB'}{CB} \quad \text{...(iii)}$$

Again as $\Delta A'B'F$ and ΔCDF are similar

$$\therefore \quad \frac{A'B'}{CD} = \frac{B'F}{CF}$$

But $CD = AB$

$$\therefore \quad \frac{A'B'}{AB} = \frac{B'F}{CF} \quad \text{...(iv)}$$

From (iii) and (iv), $\dfrac{CB'}{CB} = \dfrac{B'F}{CF} = \dfrac{CB' + CF}{CF}$

Using New Cartesian Sign Convention, let $CB = -u$, $CB' = -v$, $CF = +f$

$$\therefore \quad \frac{-v}{-u} = \frac{-v+f}{f}$$

$$uv - uf = -vf \Rightarrow uv = uf - vf$$

Divide both sides by uvf

$$\frac{uv}{uvf} = \frac{uf}{uvf} - \frac{vf}{uvf} \quad \text{or} \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

23. From lens formula, $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$

Now for a convex lens, $f > 0$ and for an object on the left of the lens, $u < 0$

$\because$ $u = -2f$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{-2f}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{2f} \Rightarrow v = 2f$$

i.e., the real image is formed at $2f$.

24. (a) Essential conditions for total internal reflection :

(i) Light should travel from a denser medium to a rarer medium.

(ii) Angle of incidence in denser medium should be greater than the critical angle for the pair of media in contact.

(b) ${}^{a}\mu_b = \dfrac{1}{\sin C}$, where a and b are the rarer and denser media respectively and C is the critical angle for the given pair of optical media.

OR

Here, $R = -20$ cm, $f = R/2 = -10$ cm

$m = -2$ (image is real)

u = object distance, v = image distance

$m = -\frac{v}{u} \Rightarrow v = 2u$

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{2u} + \frac{1}{u} = \frac{1}{-10} \Rightarrow \frac{3}{2u} = \frac{1}{-10}$ $\therefore$ $u = -15$ cm

Hence, $v = 2u = -30$ cm

(b) For convex mirror : $f > 0$, $u < 0$

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

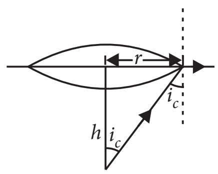
$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{f} - \frac{1}{(-u)} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u} \Rightarrow v = \frac{f \times u}{f + u}$$

$\therefore$ $v > 0$

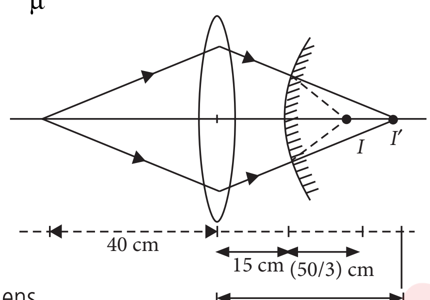
This implies that image of object placed in front of a convex mirror is always formed behind the mirror which is virtual in nature.

25. Radius, $r = h \tan i_c = h\frac{\sin i_c}{\cos i_c}$

$$r = h\frac{1/\mu}{\sqrt{1 - \frac{1}{\mu^2}}} = \frac{h}{\sqrt{\mu^2 - 1}}$$



26.



For the lens

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$u = -40$ cm, $f = +20$ cm. This gives $v = +40$ cm

This image acts as a (virtual) object for the convex mirror.

$\therefore$ $u = (+40 - 15)$ cm $= 25$ cm

Also $f = +\frac{20}{2}$ cm $= +10$ cm

From $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

We get $v = \frac{50}{3}$ cm $\simeq 16.67$ cm

The final image is, therefore formed at a distance of 16.67 cm $\left(= \frac{50}{3} \text{ cm}\right)$ to the right of the convex mirror.

(at a distance of 31.67 cm $\left(= \frac{95}{3} \text{ cm}\right)$ to the right of the convex lens.

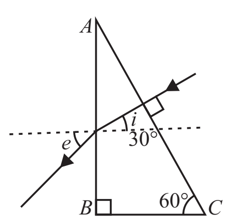
27. Given $n_g = \sqrt{3}$

$i = 0$

At the interface AC,

By Snell's law $\frac{\sin i}{\sin r} = \frac{n_a}{n_g}$

But $\sin i = \sin 0° = 0$, hence $r = 0$



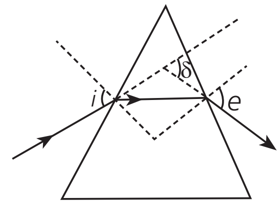
At the interface AB, $i = 30°$

Applying Snell's law

$$\frac{\sin 30°}{\sin e} = \frac{n_a}{n_g} = \frac{1}{\sqrt{3}} \Rightarrow \sin e = \sqrt{3}\sin 30° \Rightarrow e = 60°$$

OR

The incident ray is deviated through $\delta = 62°\ 48'$ when angle $i = 40°6'$. From the principle of reversibility of light, it is clear from the figure that the emergent ray (for which angle e = 82°42′) is also deviated through the same angle δ. Now,



$\delta = (i + e) - A$

or $A = (i + e) - \delta = 40°6' + 82°42' - 62°48'$ or $A = 60°$

which is the refractive angle of the prism.

For minimum deviation, $i = e$

Hence, $\delta_{min} = 2i - A$

or $i = \left(\frac{\delta_{min} + A}{2}\right) = \frac{(51° + 60°)}{2} = 55°30'$

Which is the angle of incidence at minimum deviation? The refractive index of the material of the prism is given by

$$\mu = \frac{\sin\frac{(d_{min} + A)}{2}}{\sin\frac{A}{2}} \quad \text{or} \quad \mu = \frac{\sin\left(\frac{51° + 60°}{2}\right)}{\sin\frac{60°}{2}} \quad \text{or} \quad \mu = 1.648$$

28. (a) Cause of dispersion: Each and every colour has its own characteristics wavelength (λ).

According to Cauchy's formula, the refractive index (μ) of a material depends on wavelength (λ) of light falling on its as

$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ...$, where A, B, C .... are constant so, μ is different for different colours / wavelengths

$$\delta = (\mu - 1)A.$$

$\lambda_{violet} < \lambda_{red}$, $\mu_{violet} > \mu_{red} \Rightarrow \delta_{violet} > \delta_{red}$.

that is why violet colour is at the lower end of the spectrum and red colour is at the upper end.

(b) In figure, refracted ray LM suffers total internal reflection on face AC' and grazes along the face AC'.

$\therefore$ $\angle LMN' = C$, critical angle

As $\sin C = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$ $\therefore$ $C = 45°$

$\angle AML = 90° - 45° = 45°$

$\therefore$ $\angle ALM = 180° - 60° - 45° = 75°$

$r = 90° - 75° = 15°$

From $\mu = \frac{\sin i}{\sin r}$

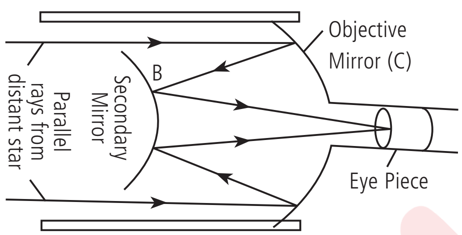
$\sin i = \text{m} \sin r = \sqrt{2} \sin 15° = 1.414 \times 0.2588 = 0.3659$

$i = \sin{-1}\ (0.3659) = 21.46°$

29. Reflecting type telescope : To improve the performance of refracting type astronomical telescope, the objective lens is replaced by a concave parabolic mirror of large aperture which is free from all alterations.

The image formed is much brighter and the reflecting type telescope has much higher resolving power compared to the refracting type telescope, Such a telescope is known as Cassegrainian telescope.

A reflecting type telescope was designed initially be Newton for observing distant stars. The model has been modified from time to time. In figure cassegrain type telescope which was designed by laurent Cassegrain is shown. C is a parabolic concave reflector of about 200 inch aperture with a narrow hole at the centre. Parallel rays from a distant star entering the telescope in a direction parallel to principal axis of the mirror tend to collect at the focus of the mirror. But these reflected rays encounter a secondary convex mirror B before meeting at the focus. The convex mirror reflects them onto the eye piece. The final image is seen through the eye piece. The hole in the centre of the objective gives an advantage that the astronomer looks through the telescope in the direction of the star.



Note that the final image as seen through the eye piece is inverted with respect to the object. But for astronomical purposes, it does not matter, as the objects are usually spherical.

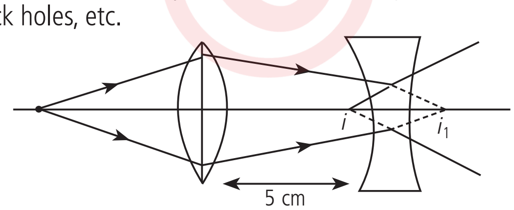
In normal adjustment, magnifying power of a reflecting type telescope is given by

$$m = \frac{f_0}{f_e} = \frac{(R/2)}{f_e}$$

where R is radius of curvature of concave reflector.

The telescope is used to observe the ice caps on the surface on Mars, volcanic activity on surface of the Jupiter satellite, stars and black holes, etc.

30.



For refraction at the convex lens, we have

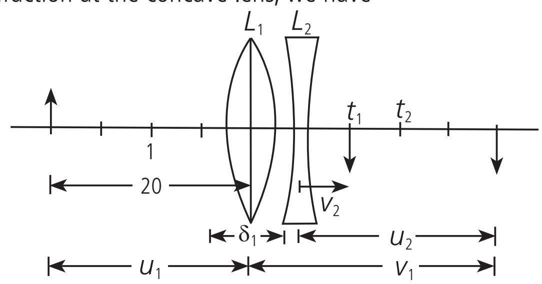
$u = -20$ cm; $f_1 = 10$ cm; $v = v_1$

Using lens formula, we have

$$\frac{1}{v_1} - \frac{1}{(-20)} = \frac{1}{10} \Rightarrow v_1 = 0 + 20 \text{ cm}$$

The convex lens produces converging rays trying to meet at I_1, 20 cm from the convex lens, *i.e.*, 15 cm behind the concave lens. I_1 will serve as a virtual object for the concave lens.

For refraction at the concave lens, we have



For concave lens,

$u = 20 - 5 = 15$ cm, $f = -10$ cm

As per sign convention

$u = -15$, $f = -10$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}; \ \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = -\frac{1}{30} - \frac{1}{15} = \frac{-3-2}{30} = \frac{-6}{30}$$

$$\frac{1}{v} = -\frac{1}{5} \quad \text{or} \quad v = -5 \text{ cm}$$

i.e., This image is in the side of object 5 cm right to concave and 10 cm (5 + 5) from convex

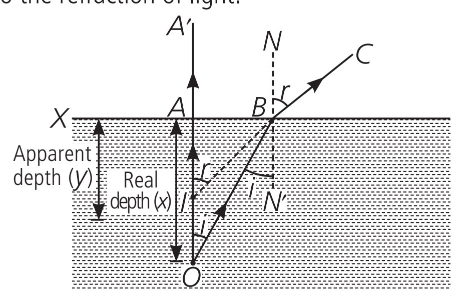
$u = +15$ cm, $f = -10$ cm

Using lens formula, we have

$$\frac{1}{v} - \frac{1}{15} = -\frac{1}{10} \quad \text{or} \quad v = -30 \text{ cm}$$

Hence, the final image is virtual and is located at 30 cm to the left of the concave lens.

31. We observe that the water tank full of water appears shallow, it is due to the refraction of light.



Let O is a point object at an actual depth OA below the free surface of water XY in a tank as shown in figure.

A ray of light incident on XY, normally along OA passes straight along OAA'. Another ray of light from O incident at $\angle i$ on XY, along OB deviates away from normal. It is refracted at $\angle r$ along BC. On producing back, BC meets OA at I. Therefore, I is virtual image of O, *i.e.*, when seen through water, O appears at I. Therefore, apparent depth = IA, which is less than the real depth OA.

$\angle AOB = \angle OBN' = i$ (alternate angles)

$\angle AIB = \angle NBC = r$ (corresponding angles)

In ΔOAB, $\sin i = \frac{AB}{OB}$; In ΔIAB, $\sin r = \frac{AB}{IB}$

As light is travelling from denser medium (water) to rarer medium (air)

$$\therefore \quad {}^a\mu_w = \frac{\sin r}{\sin i} = \frac{AB}{IB} \times \frac{OB}{AB} = \frac{OB}{IB}$$

When angles are small, B is close to A

$$\therefore \quad {}^a\mu_w = \frac{OA}{IA} \Rightarrow {}^a\mu_w = \frac{\text{real depth}}{\text{apparent depth}} = \frac{x}{y}$$

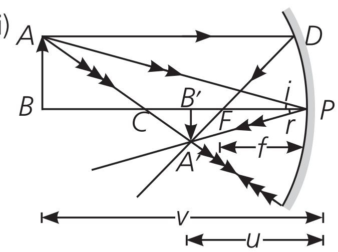
This relation is valid only for normal incidence of light.

Normal shift in the position of the object

$$= AO - AI = AO\left(1-\frac{AI}{AO}\right) = d\left(1-\frac{1}{\mu}\right)$$

where $d = AO$ = real depth and μ is absolute refractive index of the medium.

32. (i)

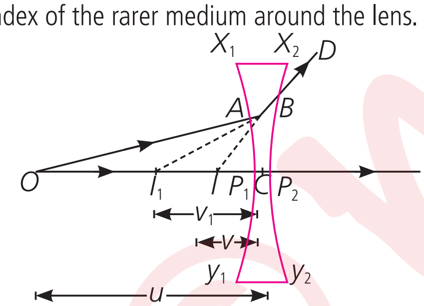


(ii) When the half portion of mirror is painted by black, the image is dull i.e., the intensity of image decreases.

33. Lens maker's formula for concave lens :

A concave lens is made up of two concave spherical refracting surfaces. The final image is formed after two refractions. One at each surface as shown in Figure

P_1, P_2 are the poles ; C_1, C_2 are the centres of curvature of the two surfaces of concave lens (not shown) with optical centre at C. Let μ_2 be the refractive index of the material of the lens and μ_1 be the refractive index of the rarer medium around the lens.



Consider a point object O lying on the principal axis of the lens. A ray of light starting from O and incident normally on the surface $X_1P_1Y_1$ along OP_1 passes straight. Another ray incident on $X_1P_1Y_1$ along OA is refracted along AB. If the lens material were continuous and there is no boundary or second surface $X_2P_2Y_2$ of the lens, the refracted ray AB would go straight and appear to come from I_1. Therefore, I_1 would have been a virtual image of O formed after refraction at $X_1P_1Y_1$.

If $CI_1 = P_1I_1 = v_1$; $CO = P_1O = u$, and R_1 = radius of curvature of surface $X_1P_1Y_1$, then as refraction occurs from rarer to denser medium,

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v_1} = \frac{\mu_2-\mu_1}{R_1} \quad \text{...(i)}$$

Actually, the lens material is not continuous. Therefore, the refracted ray AB suffers further refraction at B and emerges along BD. It appears to come from I. Therefore, I is final virtual image of point object O, formed after refraction through the concave lens.

For refraction at the second surface $X_2P_2Y_2$, we can assume I_1 as an object whose image is formed at I.

$\therefore \quad u = CI_1 = P_2I_1 = v_1$ and $P_2I = CI = v$

Let R_2 be the radius of curvature of second surface of the lens.

As refraction is now taking place from a denser to a rarer medium, therefore,

$$-\frac{\mu_2}{v_1} + \frac{\mu_1}{v} = \frac{\mu_1-\mu_2}{R_2} = \frac{\mu_2-\mu_1}{-R_2} \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$-\frac{\mu_1}{u} + \frac{\mu_1}{v} = (\mu_2-\mu_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$m_{\min} = \frac{D}{f_e} \text{ or } \frac{D}{u_{\min}} = \frac{25}{5}; \; m_{\min} = \frac{D}{f_e} \text{ or } \frac{D}{u_{\min}} = \frac{25}{5}$$

Put $\dfrac{\mu_2}{\mu_1} = \mu$ = refractive index of material of the lens with respect to the surrounding medium.

$$\frac{1}{v} - \frac{1}{u} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

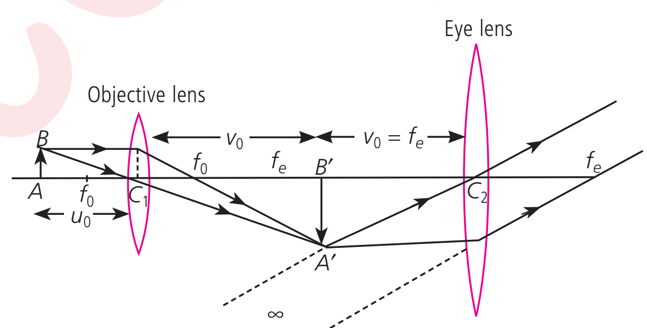
When object on the left of the lens is at infinity, image is formed at the principal focus of the lens.

i.e. when $u = \infty$, $v = f$ = focal length of the lens. $\therefore$ from (ii),

$$\frac{1}{f} - \frac{1}{\infty} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \text{ or } \frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

OR

When image formed at infinity : The ray diagram is shown below:



As is known magnifying power of objective lens, $m_0 = \dfrac{v_0}{u_0}$

When final image is at infinity, magnifying power of eye lens, $m_e = \dfrac{d}{f_e}$

$\therefore$ Magnifying power of compound microscope is

$$m = m_0 \times m_e = \frac{v_0}{u_0} \times \frac{d}{f_e}$$

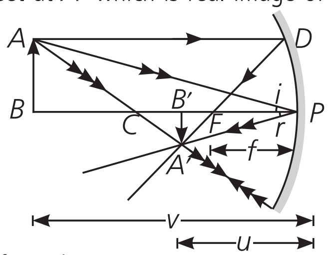
When object is very close to principal focus of the objective and image formed is very close to eye lens, $u_0 = f_0$ and $v_0 = L$ = Length of microscopic tube.

$$m = -\frac{L}{f_0} \times \frac{d}{f_0}$$

This is said to be in normal adjustment.

34. (a) For concave mirror (Real Image is formed) : When an object is held in front of a concave mirror beyond the principal focus F of the mirror, the image formed is real.

An object AB is held perpendicular to the principal axis of the mirror beyond C. A ray of light starting from A and incident on the mirror along AD parallel to the principal axis, gets reflected from the mirror and passes through F. Another ray of light incident along AP is reflected along PA' such that $\angle APB = \angle i = \angle BPA' = \angle r$. The two reflected rays actually meet at A' which is real image of the point A. A third ray starting from A and incident on the mirror along AC falls normally on the mirror and retraces its path, meeting the two reflected rays at A'. From A' draw $A'B'$ perpendicular on the principal axis. Thus, $A'B'$ is real inverted image of AB formed by reflection from the concave mirror.



As ΔABC and $\Delta A'B'C$ are similar

$$\therefore \quad \frac{AB}{A'B'} = \frac{CB}{CB'} \qquad \text{...(i)}$$

Again as $\Delta\, ABP$ and $\Delta A'B'P$ are similar

$$\therefore \quad \frac{AB}{A'B'} = \frac{PB}{PB'} \qquad \text{...(ii)}$$

From equation (i) and (ii), $\dfrac{CB}{CB'} = \dfrac{PB}{PB'}$ ...(iii)

Measuring all distances from P, we have

$CB = PB - PC$ and $CB' = PC - PB'$

$\therefore$ From equation (iii), $\dfrac{PB - PC}{PC - PB'} = \dfrac{PB}{PB'}$ ...(iv)

Using new cartesian sign Conventions, $PB = -u$ = distance of object

$PC = -R \Rightarrow PB' = -v$ = distance of image

we get from (iv), $\dfrac{-u + R}{-R + v} = \dfrac{-u}{-v}$

or $+uR - uv = uv - vR$ or $uR + vR = 2uv$

Dividing both sides by $u\ v\ R$,

we get $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{2}{R}$

As $R = 2f$

$$\therefore \quad \frac{1}{v} + \frac{1}{u} = \frac{2}{R} = \frac{2}{2f} = \frac{1}{f}$$

(b) As clear from figure end B lies at centre of curvature C of the mirror ($\because PC = 20$ cm). Therefore, its image is formed at B' itself (*i.e.*, at centre of curvature C only).

For end A, $u = -(20 + 10) = -30$ cm, $f = -10$ cm.

As, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$$\therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} + \frac{1}{30} = \frac{-3+1}{30}$$

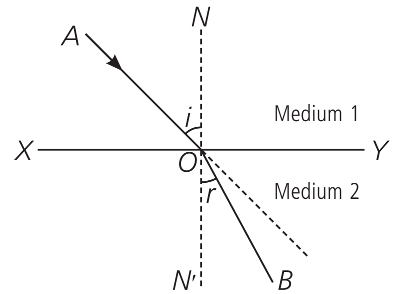
$\therefore$ $v = -15$ cm.

$\therefore$ $PA' = 15$ cm

Size of image $= A'B' = 20 - 15 = 5$ cm

OR

(a) Here, AO is incident ray, OB is refracted ray and NON' is normal to the surface. XY is the interface separating two media.



There are two laws of refraction :

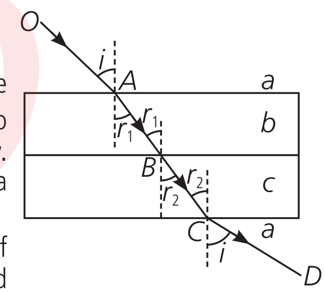
1. The incident ray, refracted ray and normal to the interface all lie in the same plane.

2. The ratio of the sine of the angle of incidence and sine of angle of refraction is constant for a given pair of media.

$$\Rightarrow \quad \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = {}^1\mu_2 = \text{constant}$$

where, ${}^1\mu_2$ represents the refractive index of medium 2 with respect to medium 1. This is called Snell's law.

(b) Refraction of light through a compound slab.



Consider a compound plate made of transparent media b and c bounded by parallel faces as shown in figure, let it be held in a medium a (say, air). A ray of light is incident on the compound plate along OA at $\angle i$. It is refracted along AB at $\angle r_1$. According to Snell's law

$${}^a\mu_b = \frac{\sin i}{\sin r_1} \qquad \text{...(i)}$$

The refracted ray AB is incident on the face separating media b and c at $\angle r_1$. It is refracted along BC at $\angle r_2$.

$$\therefore \quad {}^b\mu_c = \frac{\sin r_1}{\sin r_2} \qquad \text{...(ii)}$$

Finally, the refracted ray BC falls at $\angle r_2$ on boundary of media c and a. It emerges along CD. As the various interfaces of media are parallel, the angle of emergence will be equal to angle of incidence i ; *i.e.* $CD \parallel OA$.

$$\therefore \quad {}^c\mu_a = \frac{\sin r_2}{\sin i} \qquad \text{...(iii)}$$

Multiply (i), (ii) and (iii)

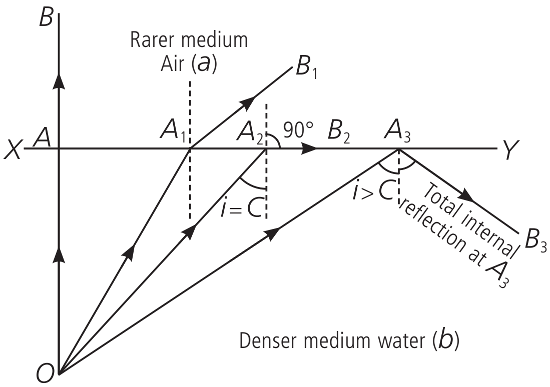
$${}^a\mu_b \times {}^b\mu_c \times {}^c\mu_a = \frac{\sin i}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin I} = 1$$

$$\therefore \quad {}^a\mu_b \times {}^b\mu_c = \frac{1}{{}^c\mu_a} = {}^a\mu_c$$

Note that in passing through a glass plate obliquely, a ray of light neither converges nor diverges, but is displaced parallel to itself *i.e.* it undergoes lateral displacement.

35. Total internal reflection is the phenomenon of reflection of light into a denser medium from an interface of the denser medium and a rarer medium.

Let an interface XY separates the rarer medium (a) from a denser medium (b). Let a is air and b is water. Let O is a point object in the denser medium. Let a ray of light starting from point O and incident normally along OA_1 on XY passes straight without deviation. Another ray OA_1 deviates away from normal and refracted along A_1B_1. Here angle of refraction is more than angle of incidence. Now for a particular value of angle of incidence C, the angle of refraction is 90°, for ray OA_2 which refracts as A_2B_2. When the angle of incidence $i > C$, as for ray OA_3, the refracted ray is along A_3B_3 as it is reflected from, interface XY. This phenomenon is called total internal reflection.



There are two necessary conditions for total internal reflection.

(i) Light should travel from a denser medium to a rarer medium.

(ii) Angle of incidence in denser medium should be greater than the critical angle for the pair of media in contact. We may define

Critical angle for a pair of media in contact as the angle of incidence in the denser medium corresponding to which angle of refraction in the rarer medium is 90°.

It is represented by C and its value depends on the nature of media in contact.

Hence, we conclude that when a ray of light travelling from an optically denser medium to an optically rarer medium is incident on the interface at an angle greater than the critical angle for the pair of media in contact, the ray is totally reflected back into denser medium.

(b) Relation between Refractive Index and Critical angle

When $i = C$, $r = 90°$

Applying Snell's law at A_2, $\mu_b \sin C = \mu_a \sin 90° = \mu_a \times 1$

$$\frac{\mu_b}{\mu_a} = \frac{1}{\sin C}$$

$$\Rightarrow \ {}^a\mu_b = \frac{1}{\sin C}$$

If λ_0 is wavelength of light in vacuum, λ_m is wavelength of light in medium, then

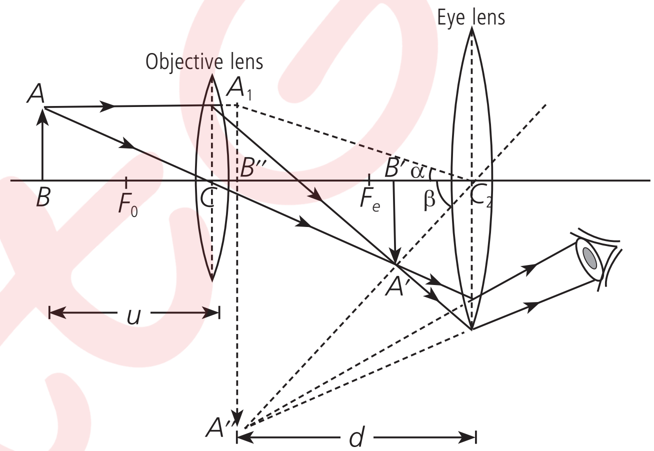
$${}^a\mu_b = \mu = \frac{\lambda_0}{\lambda_m} \quad \Rightarrow \quad \sin C = \frac{1}{\mu} = \frac{\lambda_m}{\lambda_0}$$

OR

Compound microscope : It is used to observe highly magnified images of tiny objects

Construction : It consists of two lenses which are converging in nature. One is called objective lens which is of very small focal length and towards the object. Another lens is called eye piece which has large focal length then objective lens and towards the eye. The both lenses held coaxially at the free ends of two tubes. The distance between the lenses can by adjusted by rack and pinon system. The ray diagram shown given below. Here two positions of image are formed.

(i) When image is formed at least distance of distinct vision : AB is a tiny object held perpendicular to the common principal axis, in front of the objective lens beyond its principal focus F_0. A real, inverted and enlarged image $A'B'$ of this object is formed by the objective lens. Now $A'B'$ acts as an object for the eye lens, whose position is so adjusted that $A'B'$ lies between optical centre C_2 of eye lens and its principal focus F_e. A virtual and magnified image $A'' B''$ is formed by the eye lens. This image is erect w.r.t. $A'B'$ but inverted w.r.t. AB. the final image $A'' B''$ is seen by the eye held close to eye lens. The adjustments are so made that $A''B''$ is at the least distance of distinct vision (d) from the eye, *i.e.*, $C_2 B'' = d$.



Magnifying power or Angular Magnification of a compound microscope is defined as the ratio of the angle subtended at the eye by the final image to the angle subtended at the eye by the object, when both the final image and the object are situated at the least distance of distinct vision from the eye.

In figure $C_2 B'' = d$, Imagine the object AB to be shifted to A_1B'' so that it is at a distance d from the eye. If $\angle A'' C_2B'' = \beta$ and $\angle A_1C_2B'' = \alpha$, then by definition,

$$\text{Magnifying power, } m = \frac{\beta}{\alpha}$$

For small angles expressed in radian, $\alpha \approx \tan \alpha$ and $\beta \approx \tan \beta$

$$m = \frac{\tan\beta}{\tan\alpha}$$

In $\Delta A''B'' C_2$, $\tan\beta = \dfrac{A'' B''}{C_2B''}$

In $\Delta A_1B'' C_2$, $\tan\alpha = \dfrac{A_1 B''}{C_2B''} = \dfrac{AB}{C_2B''}$

we get $m = \dfrac{A''B''}{C_2B''} \times \dfrac{C_2B''}{AB} = \dfrac{A''B''}{AB} = \dfrac{A''B''}{A'B'} \times \dfrac{A'B'}{AB}$

$$m = m_e \times m_0$$

where $m_e = \dfrac{A''B''}{A'B'}$, magnification produced by eye lens, and $m_0 = \frac{A'B'}{AB}$, magnification produced by objective lens.

Form me : $m_e = \frac{A''B''}{A'B'} = \frac{v_e}{u_e}$

Now, $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \Rightarrow 1 - \frac{v_e}{u_e} = \frac{v_e}{f_e}$

$$1 - m_e = \frac{v_e}{f_e} \Rightarrow v_e = -d,\ m_e = 1 + \frac{d}{f_e}$$

where d is C_2B'' = least distance of distinct vision, f_e is focal length of eye lens. And

$$m_0 = \frac{A'B'}{AB} = \frac{\text{distance of image } A'B' \text{ from } C_1}{\text{distance of object } AB \text{ from } C_1} = \frac{C_1B'}{C_1B} = \frac{v_0}{-u_0}$$

$$m = \frac{v_0}{-u_0}\left(1 + \frac{d}{f_e}\right) = \frac{v_0}{|u_0|}\left(1 + \frac{d}{f_e}\right)$$

As the object AB lies very close to F_0, the focus of objective lens, therefore, $u_0 = C_1B = C_1F_0 = f_0$ = focal length of objective lens.

As $A'B'$ is formed very close to eye lens whose focal length is also short, therefore, $v_0 = C_1B' = C_1C_2 = L$ = Length of microscope tube.

we get $m = \frac{L}{-f_0}\left(1 + \frac{d}{f_e}\right) = \frac{1}{|f_0|}\left(1 + \frac{d}{f_e}\right)$

we know that $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$

Multiplying both sides by v_0, we get

$1 - \frac{v_0}{u_0} = \frac{v_0}{f_0}$ or $\frac{v_0}{u_0} = \left(1 - \frac{v_0}{f_0}\right)$

we get, $m = \left(1 - \frac{v_0}{f_0}\right)\left(1 + \frac{d}{f_e}\right)$