Ray Optics and Optical Instruments

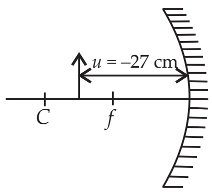
CHAPTER 9

NCERT FOCUS **ANSWERS**

Topic 1

1. The object is kept between f and C. So the image should be real, inverted and beyond C. To locate the sharp image, screen should be placed at the position of image.

The focal length, $f = \frac{-R}{2} = -18$ cm



Object distance, $u = -27$ cm

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

or $\frac{1}{v} + \frac{1}{-27} = \frac{1}{-18}$ or $\frac{1}{v} = -\frac{1}{18} + \frac{1}{27}$

$\frac{1}{v} = \frac{-3+2}{54} \Rightarrow \frac{1}{v} = -\frac{1}{54}$; $v = -54$ cm

Size of image can be calculated by magnification

$m = -\frac{v}{u} = \frac{-I}{O}$ or $\frac{-I}{O} = -\frac{v}{u}$

$\frac{I}{+2.5} = -\frac{-54}{-27} \Rightarrow I = -5$ cm

So, the image is inverted and magnified.

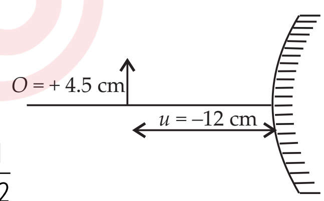
Thus in order to locate the sharp image, the screen should be kept 54 cm in front of concave mirror and image on the screen will be observed real, inverted and magnified. If the candle is moved closer to the mirror, the real image will move away from the mirror, hence screen has to be shifted away from the mirror to locate the sharp image.

2. A convex mirror always form virtual image, which is erect and small in size of an object kept in front of it.

Focal length of convex mirror $f = +15$ cm

Object distance $u = -12$ cm

Using mirror formula



$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} + \frac{1}{-12} = \frac{1}{15}$ or $\frac{1}{v} = \frac{1}{15} + \frac{1}{12}$

or $\frac{1}{v} = \frac{4+5}{60}$; $v = +\frac{60}{9}$ cm $= +6.66$ cm

Therefore, image is virtual, formed at 6.67 cm at the back of the mirror.

Magnification $m = -\frac{v}{u}$

$m = -\frac{\frac{60}{9}}{-12} \Rightarrow m = \frac{5}{9} = +0.55$

Size of image $h_2 = m \times h_1 = \frac{5}{9} \times h_1 = \frac{5}{9} \times 4.5$; $h_2 = 2.5$ cm

It shows the image is erect, small in size and virtual.

When the needle is moved farther from mirror the image also move towards focus and decreasing in size.

As u approaches ∞, v approaches focus but never beyond f.

3. (a) We know for a concave mirror $f < 0$ [negative] and $u < 0$ [negative]

$2f < u < f$

$\therefore \frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$ or $\frac{-1}{2f} < \frac{-1}{u} < \frac{-1}{f}$

or $\frac{1}{f} - \frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < \frac{1}{f} - \frac{1}{f}$

or $\frac{1}{2f} < \frac{1}{v} < 0$ $\left\{\because \frac{1}{f} - \frac{1}{u} = \frac{1}{v}\right\}$

Which implies that $v < 0$ to form image on the left.

Also $2f > v$ {$\because$ $2f$ and v are –ve}

$|2f| < |v|$

So, the real image is formed beyond $2f$.

(b) For a convex mirror, $f > 0$, always positive and object distance $u < 0$, always negative.

Now mirror formula, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ or $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

This implies that $\frac{1}{v} > 0$, or $v > 0$.

So, whatever be the value of u, a convex mirror always forms a virtual image.

(c) In convex mirror focal length is positive hence $f > 0$ and for an object distance from mirror with negative sign ($u < 0$)

So, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

The results $\frac{1}{v} > \frac{1}{f}$ or $v < f$ (both positive) hence the image is located between pole and focus of the mirror.

Also magnification $m = -\frac{v}{u} = -\frac{+v}{-u}$

$m < [1]$ (positive)

So, the image is virtual and diminished.

(d) In concave mirror, $f < 0$ for object placed between focus and pole of concave mirror

$f < u < 0$ (both negative)

$\frac{1}{f} > \frac{1}{u}$

Now mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$\frac{1}{v} > 0$ or $v > 0$ (positive)

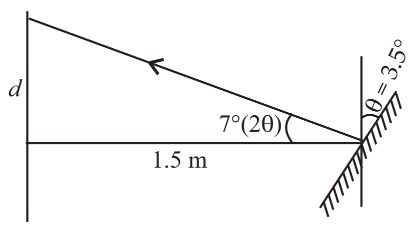
hence the image is virtual.

Also magnification $m = \frac{-v}{u}$

here $\frac{1}{v} < \frac{1}{|u|}$ or $v > |u|$

So, $m > 1$ hence the image is enlarged.

4. If the mirror deflect by 3.5°, the reflected light deflect by 7°, deflection of the spot d can be calculated



$\tan 7° = \frac{d}{1.5}$

$d = 1.5 \tan 7° = 1.5\,[0.1228]$ m or $d = 0.1842$ m = 18.42 cm.

Topic 2

1. We know the relation

$$\text{Apparent depth} = \frac{\text{Real depth}}{{}^a\mu_w}$$

$$9.4\text{ cm} = \frac{12.5\text{ cm}}{{}^a\mu_w} \quad \text{or} \quad {}^a\mu_w = \frac{12.5}{9.4} = 133$$

Now if the water is replaced by other liquid, the apparent depth will change and microscope will have to be further moved to focus the image.

With new liquid

$$\text{Apparent depth} = \frac{\text{Real depth}}{{}^a\mu_l}$$

$$\text{Apparent depth} = \frac{12.5}{1.63} = 7.67\text{ cm}$$

Now the microscope will have to shift from its initial position to focus on the needle again which is at 7.67 cm depth.

Shift distance = 9.4 – 7.67 = 1.73 cm.

2. (a) Applying Snell's law for the refraction from air to glass.

Refractive index of glass with respect to air

$${}^a\mu_g = \frac{\sin 60°}{\sin 35°} = \frac{0.8660}{0.5736} = 1.51$$

(b) Now Snell's law for the refraction from air to water

$${}^a\mu_w = \frac{\sin 60°}{\sin 47°} = \frac{0.8660}{0.6560} = 1.32$$

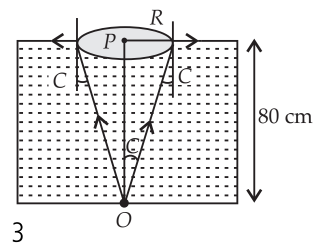
(c) Now the light beam is incident at an angle 45° from water to glass

$${}^w\mu_g = \frac{\sin 45°}{\sin r} \Rightarrow \frac{1.51}{1.32} = \frac{\sin 45°}{\sin r} = \frac{0.7071}{\sin r}$$

$$\sin r = \frac{1.32 \times 0.7071}{1.51} = 0.6181 \quad \therefore \quad r \cong 38.2°$$

3. As shown in the figure all those light rays which are incident on the surface at angle of incidence more than critical angle, does total internal reflection and are reflected back in water only. All those light rays which are incident below critical angle emerges out of surface bending away from normal. All those light beams which are incident at critical angle grazes the surface of water.

We know



$$\sin C = \frac{1}{{}^a\mu_w}$$

$$C = \sin^{-1}\left(\frac{1}{{}^a\mu_w}\right)$$

$$C = \sin^{-1}\left(\frac{1}{1.33}\right) \Rightarrow \sin C = \frac{1}{1.33} = \frac{3}{4}$$

$$\tan C = \frac{R}{OP}\text{(radius)} \qquad [\because\ (0.80)^2 = 0.6400]$$

$R = \tan C \times OP = \tan C\,(0.80)$

Area = $\pi R^2 = \pi \times \tan^2 C\,(0.64)$

$A = \pi(0.64) + \tan^2 C$

$$= \pi(0.64) \times \frac{\sin^2 C}{\cos^2 C} = \pi(0.64) \times \frac{\sin^2 C}{1 - \sin^2 C}$$

$$= \pi \times 0.64 \times \frac{9/16}{7/16} = \pi(0.64) \times \frac{9}{16} \times \frac{16}{7}$$

$$= \frac{22}{7} \times 0.64 \times \frac{9}{7} = 2.6\text{ m}^2$$

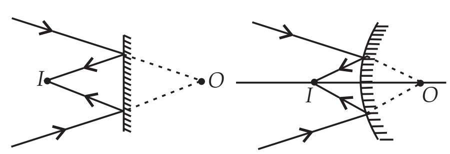
4. The shift in the image by the thick glass slab can be calculated. Here, shift only depend upon thickness of glass slab and refractive index of glass.

Shift = Real thickness – Apparent of thickness

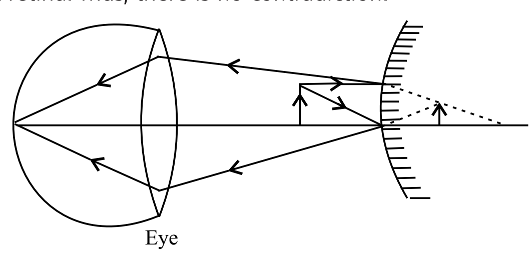
$$\text{Shift} = t_g\left[1 - \frac{1}{{}^a\mu_g}\right] = 15\left[1 - \frac{1}{1.5}\right] = 15 \times \frac{0.5}{1.5} = 5\text{ cm}$$

The answer does not depend on the location of the slab.

5. (a) In this situation when rays are convergent behind the mirror, both plane mirror and convex mirror can form real images of virtual objects.

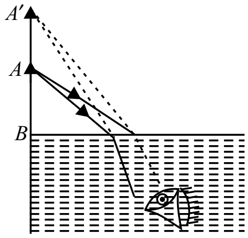


(b) Here, the retina is working as a screen, where the rays are converging, but this screen is not at the position of formed virtual image, in fact the reflected divergent rays are converged by the eye lens at retina. Thus, there is no contradiction.



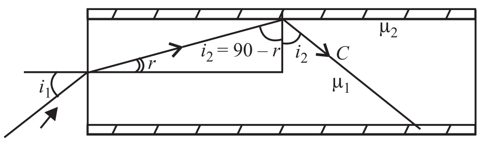
Eye

(c) An observer in denser medium will observe the fisherman taller than actual height, due to refraction from rare to denser medium.



(d) Apparent depth decreases if viewed obliquely as compared to when observed near normally.

6. (a) Let us first derive the condition for total internal reflection.



Critical angle for the interface of medium 1 and medium 2.

$$\sin C = \frac{1}{{}^1\mu_2} = \frac{\mu_1}{\mu_2} = \frac{1.44}{1.68} = 0.8571$$

So, critical angle $C = 59°$

Condition for total internal reflection from core to cladding

$$i_2 > 59° \text{ or } r \leq \frac{\pi}{2} - 59° \quad \text{or} \quad r \leq 31°$$

Now, for refraction at first surface air to core.

Snell's law, $\dfrac{\sin i_1}{\sin r} = {}^a\mu_1$

$\sin i_1 = {}^a\mu_1 \sin r = 1.68 \sin 31°$ or $i_1 \approx 60°$

Thus all incident rays which makes angle of incidence between 0° and 60° will suffer total internal reflection in the optical fibre.

(b) When there is no outer covering critical angle from core to surface.

$$\sin C = \frac{1}{{}^1\mu_a} = \frac{\mu_a}{\mu_1}$$

$$\sin C = \frac{1}{1.68} \Rightarrow C = \sin^{-1}\left(\frac{1}{1.68}\right) = 36.5°$$

So, condition for total internal reflection from core to surface

$$i_2 > 36.5° \text{ or } r < \frac{\pi}{2} - 36.5° \text{ or } r < 53.5°$$

Let us find range of incident angle at first surface air to core.

$${}^a\mu_1 = \frac{\sin i_1}{\sin r}$$

$\sin i_1 = 1.68 \times \sin 53.5° = 1.68 \times 0.8039$

$\sin i_1 = 1.35$ or $i \approx 90°$.

Thus all incident rays at first surface 0° to 90° will suffer total internal reflection inside core.

Topic 3

1. Both faces should be of same radius of curvature

$|R_1| = |R_2| = R$

So, lens makers formula

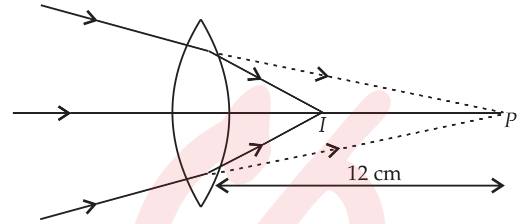
$$\frac{1}{f} = ({}^a\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Rightarrow \frac{1}{f} = (1.55 - 1)\left[\frac{1}{R} - \frac{1}{-R}\right]; \ \frac{1}{f} = 0.55\left[\frac{2}{R}\right]$$

$$\Rightarrow \frac{1}{20} = \frac{1.10}{R} \quad \Rightarrow R = 20 \times 1.1 = 22 \text{ cm}$$

So, the radius of curvature should be 22 cm for each face of lens.

2. (a) The convex lens is placed in the path of convergent beam.

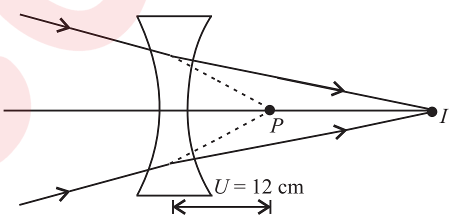


Using lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$$\frac{1}{v} - \frac{1}{+12} = \frac{1}{+20} \text{ or } \frac{1}{v} = \frac{1}{20} + \frac{1}{12} = \frac{3+5}{60}; \ v = \frac{60}{8} = +7.5 \text{ cm}$$

The image I is formed by further converging beams at a distance of 7.5 cm from lens.

(b) A concave lens is placed in the path of convergent beam, the concave lens further diverge the light.



Using lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

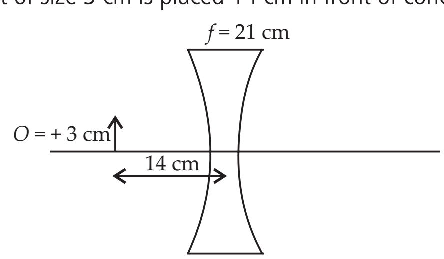
$$\frac{1}{v} - \frac{1}{+12} = \frac{1}{-16}$$

$$\text{or } \frac{1}{v} = -\frac{1}{16} + \frac{1}{12} = \frac{-3+4}{48} = \frac{1}{48}$$

$v = +\,48$ cm

The image I is formed by diverged rays at 48 cm away from concave lens.

3. Object of size 3 cm is placed 14 cm in front of concave lens.



Lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}; \ \dfrac{1}{v} - \dfrac{1}{-14} = \dfrac{1}{-21}$

$$\frac{1}{v}=-\frac{1}{21}-\frac{1}{14}=\frac{-2-3}{42}\ ;\ v=-\frac{42}{5}=-8.4\text{ cm}$$

Formula for magnification

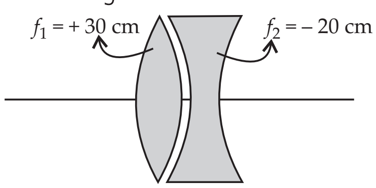
$$m=\frac{I}{O}=+\frac{v}{u}\quad\text{or}\quad\frac{I}{O}=\frac{v}{u}\quad\text{or}\quad\frac{I}{+3}=\frac{-8.4}{-14}$$

$\Rightarrow \quad I = +\ 1.8$ cm

So, the image is virtual, erect, of the size 1.8 cm and is located 8.4 cm from the lens on the same side as object.

As the object is moved away from the lens, the virtual image moves towards the focus of the lens but never beyond it. The image also reduces in size as shift towards focus.

4. Equivalent focal length of the combination



$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$

or $\dfrac{1}{f_{eq}}=\dfrac{1}{30}+\dfrac{1}{-20}=\dfrac{2-3}{60}$

or $f_{eq} = -\ 60$ cm

Hence, system will behave as a diverging lens of focal length 60 cm.

5. (a) (i) Let a parallel beam of light incident first on convex lens, refraction at convex lens

$$\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_1}\Rightarrow\frac{1}{v_1}-\frac{1}{\infty}=\frac{1}{30}\ ;\ v_1=30\text{ cm}$$

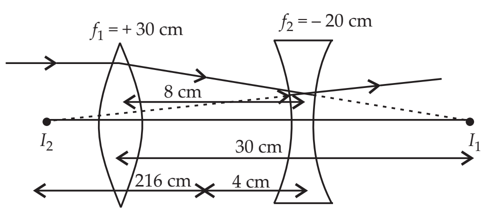
So, a virtual object for the concave lens at +22 cm.

Now refraction at concave lens

$$\frac{1}{v_2}-\frac{1}{u_2}=\frac{1}{f_2}$$

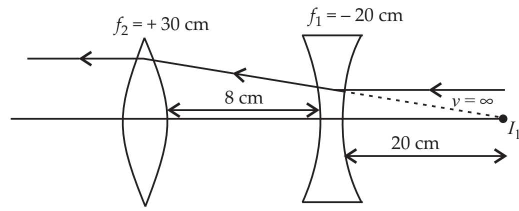
or $\dfrac{1}{v_2}-\dfrac{1}{+22}=\dfrac{1}{-20}$

$$\frac{1}{v_2}=-\frac{1}{20}+\frac{1}{22}=-\frac{1}{220}\quad\text{or}\quad v_2=-\ 220\text{ cm}$$



The parallel beam of light appears to diverge from a point 216 cm from the center of the two lens system.

(ii) Now let a parallel beam of light incident first on concave lens.



Refraction at concave lens

$$\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_1}$$

or $\dfrac{1}{v_1}-\dfrac{1}{\infty}=\dfrac{1}{-20}$ or $v_1 = -\ 20$ cm

The image I_1 will act as real object for convex lens at 28 cm.

$$\frac{1}{v_2}-\frac{1}{u_2}=\frac{1}{f_2}$$

or $\dfrac{1}{v_2}-\dfrac{1}{-28}=\dfrac{1}{30}$

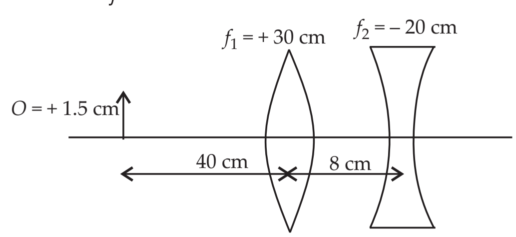
$$\frac{1}{v_2}=\frac{1}{30}-\frac{1}{28}=-\frac{1}{420}\quad\text{or}\quad v_2=-\ 420\text{ cm}$$

Thus the parallel incident beam appears to diverge from a point 420 – 4 = 416 cm on the left of the center of the two lens system.

Hence, the answer depend upon which side of the lens system the parallel beam is made incident.

Therefore the effective focal length is different in two situations.

(b) Now an object of 1.5 cm size is kept 40 cm in front of convex lens in the same system of lenses.



Refraction by the first lens

$$\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_1}\quad\text{or}\quad\frac{1}{v_1}-\frac{1}{-40}=\frac{1}{30}$$

$$\frac{1}{v_1}=\frac{1}{30}-\frac{1}{40}=\frac{1}{120}\quad\text{or}\quad v_1=120\text{ cm}$$

Magnification by first lens

$$m_1=\frac{I}{O}=\frac{v}{u}\quad\text{or}\quad m_1=\frac{I_1}{1.5}=\frac{120}{-40}$$

$m_1 = -\ 3$ and $I_1 = -\ 4.5$ cm

Refraction by the second lens

$$\frac{1}{v_2}-\frac{1}{u_2}=\frac{1}{f_2}\quad\text{or}\quad\frac{1}{v_2}-\frac{1}{+112}=\frac{1}{-20}$$

$$\frac{1}{v_2}=-\frac{1}{20}+\frac{1}{112}$$

or $v_2=-\dfrac{112\times 20}{92}$ cm

Magnification by concave lens

$$m_2 = \frac{v_2}{u_2} = \frac{-112 \times 20}{+112\,(92)} = -\frac{20}{92}$$

Total magnification by two lenses

$$m = m_1 \times m_2 \quad \text{or} \quad m = -3 \times \left(-\frac{20}{92}\right) = 0.652$$

Size of image finally obtained

$$m = \frac{I_2}{O_1}$$

or $0.652 = \dfrac{I_2}{1.5}$; $I_2 = 1.5 \times 0.652 = 0.98$ cm.

6. Let the object be placed x m in front of lens the distance of image from the lens is $(3 - x)$ m.

Applying lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{(3-x)} - \frac{1}{-x} = \frac{1}{f} \quad \text{or} \quad \frac{1}{(3-x)} + \frac{1}{x} = \frac{1}{f}$$

$$\frac{x + 3 - x}{x\,(3-x)} = \frac{1}{f} \quad \text{or} \quad 3f = 3x - x^2$$

so, $x^2 - 3x + 3f = 0$

Now $x = \dfrac{+3 \pm \sqrt{9 - 4 \times (3f)}}{2}$

or $x = \dfrac{+3 \pm \sqrt{9 - 12f}}{2}$

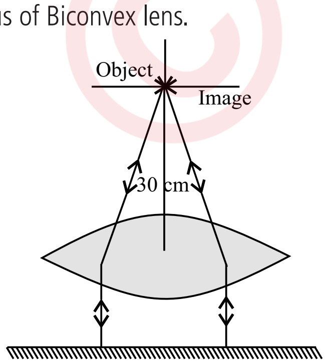
Condition for image to be obtained on the screen, *i.e.*, real image.

$9 - 12f \geq 0$ or $9 \geq 12f$ or $f \leq 0.75$ m.

So, maximum focal length is 0.75 m.

7. Let us first consider the situation when there is no liquid between lens and plane mirror and the image is formed at 30 cm *i.e.*, at the position of object.

As the image is formed on the object position itself, the object must be placed at focus of Biconvex lens.



$f_o = 30$ cm

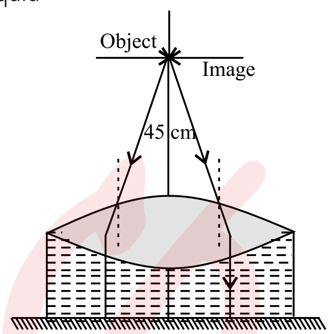
Radius of curvature of convex lens can be calculated

$$\frac{1}{f} = ({}^{a}\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $\dfrac{1}{30} = \left(\dfrac{3}{2} - 1\right)\left(\dfrac{1}{R} - \dfrac{1}{-R}\right)$ or $\dfrac{1}{30} = \dfrac{1}{2}\left(\dfrac{2}{R}\right) \Rightarrow R = 30$ cm

Now a liquid is filled between lens and plane mirror and the image is formed at position of object at 45 cm.

The image is formed on the position of object itself, the object must be placed at focus of equivalent lens of Biconvex of glass and Plano convex lens of liquid



$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{...(i)}$$

Equivalent total length $f_{eq} = 45$ cm

Focal length of Biconvex lens $f_1 = 30$ cm

Focal length of plano convex lens

$$\frac{1}{f_2} = [\mu - 1]\left[\frac{1}{-R} - \frac{1}{\infty}\right] \quad \text{or} \quad \frac{1}{f_2} = [\mu - 1]\left[\frac{-1}{30}\right];\ f_2 = \frac{-30}{\mu - 1}$$

Now equation (i),

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{45} = \frac{1}{30} - \left(\frac{\mu - 1}{30}\right)$$

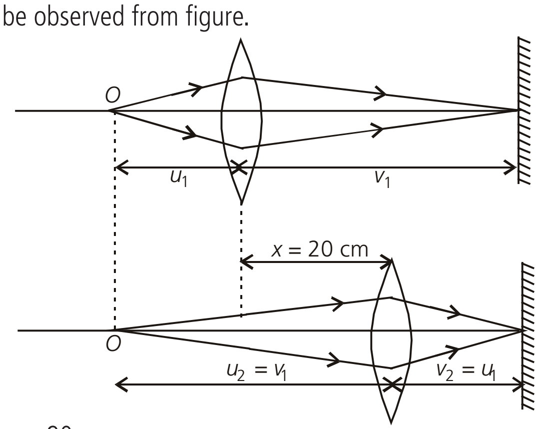
$$\frac{\mu - 1}{30} = \frac{1}{90} \quad \text{or} \quad \mu - 1 = \frac{1}{3},\ \mu = \frac{1}{3} + 1 = \frac{4}{3}$$

$${}^{l}\mu_g = \frac{4}{3} \Rightarrow \frac{{}^{a}\mu_g}{{}^{a}\mu_l} = \frac{4}{3} \Rightarrow {}^{a}\mu_l = \frac{3}{4} \times {}^{a}\mu_g = \frac{3}{4} \times 1.5 = \frac{4.5}{4} = 1.25$$

8. The image of the object can be located on the screen for two positions of convex lens such that u and v are exchanged.

The separation between two positions of the lens is $x = 20$ cm.

It can be observed from figure.



$u_1 + v_1 = 90$ ...(i)

$v_1 - u_1 = 20$ ...(ii)

Solving equation (i) and equation (ii)

$v_1 = 55$ cm, $u_1 = 35$ cm

Now lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{55} - \frac{1}{-35} = \frac{1}{f}$$

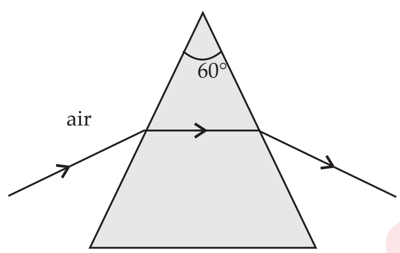
or $$\frac{1}{55} + \frac{1}{35} = \frac{1}{f}$$

$$\frac{35+55}{55 \times 35} = \frac{1}{f}$$

or $$f = \frac{55 \times 35}{90} = +21.38 \text{ cm}$$

Topic 4

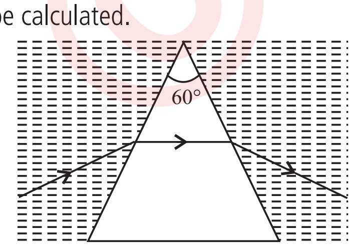
1. When the light beam is incident from air on to the glass prism, the angle of minimum deviation is 40°.
Refractive index of glass w.r.t. air.



$$^a\mu_g = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$$

$$^a\mu_g = \frac{\sin\frac{60°+40°}{2}}{\sin\frac{60°}{2}} = \frac{\sin 50°}{\sin 30°} = \frac{0.766}{0.50} = 1.532$$

Now the prism is placed in water, new angle of minimum deviation can be calculated.



$$^w\mu_g = \frac{\sin\frac{A+\delta'_m}{2}}{\sin\frac{A}{2}}$$

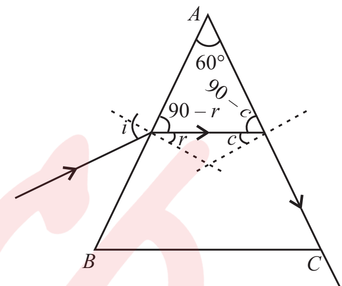
$$\frac{^a\mu_g}{^a\mu_w} = \frac{\sin\frac{60+\delta'_m}{2}}{\sin 30°}$$

$$\sin\left(30° + \frac{\delta'_m}{2}\right) = \frac{1}{2}\left[\frac{1.532}{1.33}\right] = 0.5759$$

$$30° + \frac{\delta'_m}{2} = \sin^{-1}(0.5759) \quad \text{or} \quad 30° + \frac{\delta'_m}{2} = 35°10'$$

New angle of deviation $\delta'_m = 10°20'$

2. The beam should be incident at critical angle or more than critical angle, for total internal reflection at second surface of the prism.



Let us first find critical angle for air glass interface.

We know

$$\sin C = \frac{1}{^a\mu_g}\,;\; C = \sin^{-1}\left(\frac{1}{^a\mu_g}\right) = \sin^{-1}\left(\frac{1}{1.524}\right)$$

Critical angle $C = 41°$

Now we can calculate 'r', as $60° + (90° - r) + (90° - C) = 180°$

or, $r = 19°$

Using Snell's law, required angle of incidence i at first surface can be calculated.

$$^a\mu_g = \frac{\sin i}{\sin r}$$

or $$1.524 = \frac{\sin i}{\sin 19°}$$

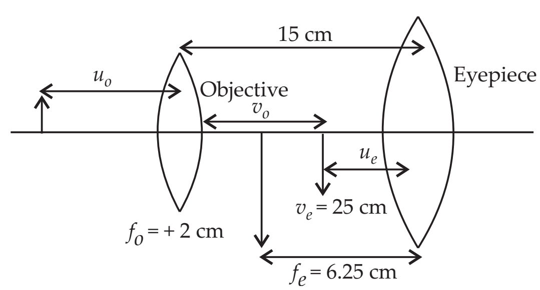
$\sin i = 1.524 (\sin 19°)$

or $i = \sin^{-1}(0.4962)$

$\Rightarrow i \cong 29.75°$

Topic 5

1. (a) We want the final image at least distance of distinct vision. Let the object in front of objective is at distance u_0.



Let us first find the u_e, the object distance for eye piece.

Here $v_e = -25$, $f_e = 6.25$, $u_e = ?$

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}\,;\; \frac{1}{-25} - \frac{1}{u_e} = \frac{1}{6.25} \quad \text{or} \quad -\frac{1}{u_e} = \frac{1}{6.25} + \frac{1}{25} = \frac{4+1}{25}$$

$u_e = -5$ cm

So, image distance of objective lens $v_o = 15 - u_e$

$v_o = 15 - 5 = 10$ cm

Now we can get required position of object in point of objective.

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o} \quad \text{or} \quad \frac{1}{+10} - \frac{1}{u_o} = \frac{1}{2}$$

$$\frac{1}{u_o} = \frac{1}{10} - \frac{1}{2} = \frac{1-5}{10} \quad \Rightarrow \quad u_o = -2.5 \text{ cm}$$

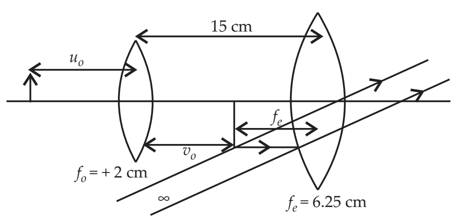
So, the object should be 2.5 cm in front of objective lens.

Magnifying power (most strained eye)

$$m = -\frac{v_o}{u_o}\left[1 + \frac{D}{f_e}\right]$$

$$\text{or } m = -\frac{10}{-2.5}\left[1 + \frac{25}{6.25}\right] = 4\,[5] = 20$$

(b) We want the final image at infinity. Let us again assume the object in front of objective at distance u_o.



$\because \quad v_e = \infty$

$$\therefore \quad \frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

The object distance u_e for the eyepiece should be equal to $f_e = 6.25$ cm to obtain final image at ∞.

So, image distance of objective lens

$v_o = 15 - f_e = 15 - 6.25 = 8.75$ cm

Now, lens formula, $\dfrac{1}{v_o} - \dfrac{1}{u_o} = \dfrac{1}{f_o}$

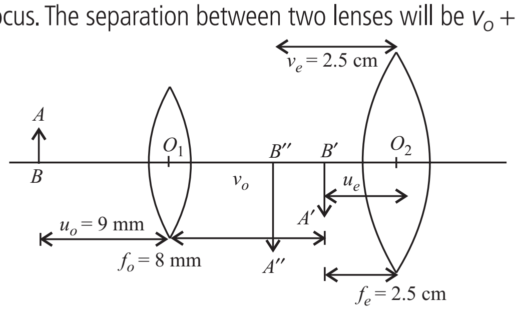
$$\frac{1}{8.75} - \frac{1}{u_o} = \frac{1}{2} \quad \text{or} \quad \frac{1}{u_o} = \frac{1}{8.75} - \frac{1}{2} = \frac{2-8.75}{17.5}$$

$$u_o = -\frac{17.5}{6.75}\text{ cm} = -2.59\text{ cm}$$

Magnifying power (most relaxed eye)

$$m = -\frac{v_o}{u_o}\left[\frac{D}{f_e}\right] \Rightarrow m = -\frac{8.75}{-2.59}\left[\frac{25}{6.25}\right] = 13.5$$

2. The image is formed at least distance of distinct vision for sharp focus. The separation between two lenses will be $v_o + |u_e|$



Let us find first v_o the image distance for objective lens.

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o} \quad \text{or} \quad \frac{1}{v_0} - \frac{1}{-9} = \frac{1}{8} \quad \text{or} \quad \frac{1}{v_o} = \frac{1}{8} - \frac{1}{9} = \frac{1}{72}$$

$\Rightarrow \quad v_o = 72$ mm $= 7.2$ cm

Also we can find object distance for eyepiece u_e, as we know

$v_e = D = 25$ cm $= 250$ mm

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e};\ \frac{1}{-250} - \frac{1}{u_e} = \frac{1}{25} \quad -\frac{1}{u_e} = \frac{1}{25} + \frac{1}{250} = \frac{10+1}{250}$$

$$u_e = -\frac{250}{11}\text{ mm} = -22.7\text{ mm} = -2.27\text{ cm}$$

Separation between lenses

$L = v_o + |u_e| = 72 + 22.7$ or $L = 94.7$ mm

$= 9.47$ cm

Magnifying power $m = -\dfrac{v_o}{u_o}\left[1 + \dfrac{D}{f_e}\right]$

$$m = -\frac{72}{-9}\left[1 + \frac{250}{25}\right] = 8\,[11] = 88$$

3. Given, $f_0 = 144$ cm, $f_e = 6$ cm

$m = ?,\ L = ?$

Magnifying power $= \dfrac{f_0}{f_e} = \dfrac{144}{6} = 24$

Separation between objective and eyepiece

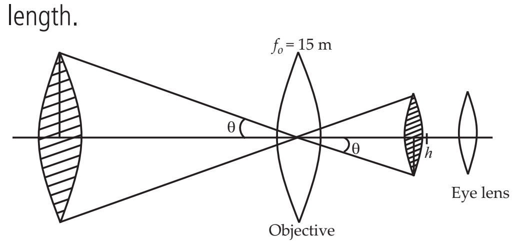
$(L) = f_0 + f_e = 144 + 6 = 150$ cm.

4. (a) $f_o = 15$ m and $f_e = 1.0$ cm

angular magnification by the telescope in normal adjustment

$$m = -\frac{f_o}{f_e} = \frac{1500\text{ cm}}{1.0\text{ cm}} = -1500$$

(b) The image of the moon by the objective at lens is formed on its focus only as the moon is nearly infinite distance as compared to focal length.



Height of object

i.e., Radius of moon $R_m = \dfrac{3.48}{2} \times 10^6$ m $= 1.74 \times 10^6$ m

Distance of object

i.e., Radius of lunar orbit, $R_o = 3.8 \times 10^8$ cm

Distance of image for objective lens *i.e.*, focal length of objective lens

$f_o = 15$ m

Radius of image of moon by objective lens can be calculated.

$$\tan\theta = -\frac{R_m}{R_o} = \frac{h}{f_o}$$

$$h = \frac{R_m \times f_o}{R_o} = \frac{1.74 \times 10^6 \times 15}{3.8 \times 10^8} = 6.87 \times 10^{-2} \text{ m}$$

Diameter of the image of moon

$D_I = 2h = 13.74 \times 10^{-2}$ m = 13.74 cm

5. (a) In normal adjustment magnifying power

$$m = -\frac{f_o}{f_e} = \frac{140}{5} = 28$$

(b) For the image at least distance of distinct vision

Magnifying power $m = -\frac{f_o}{f_e}\left[1 + \frac{f_e}{D}\right]$

$$m = 28\left[\frac{30}{25}\right] = 33.6$$

6. (a) The separation between objective lens and the eyepiece can be calculated in both the conditions of most relaxed eye and most strained eye.

Most relaxed eye

$L = f_o + f_e = 140 + 5 = 145$ cm

Most strained eye

object distance 'ue' for eye lens

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

or $\frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5}$

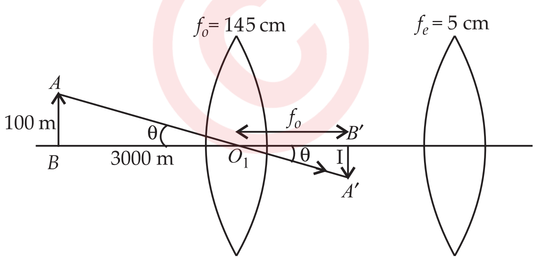
$$-\frac{1}{u_e} = \frac{1}{5} + \frac{1}{25} = \frac{6}{25}$$

or $u_e = -\frac{25}{6}$ cm $= -4.16$ cm

Separation between lenses

$L = f_o + |u_e| = 145 + 4.16 = 149.16$ cm

(b)



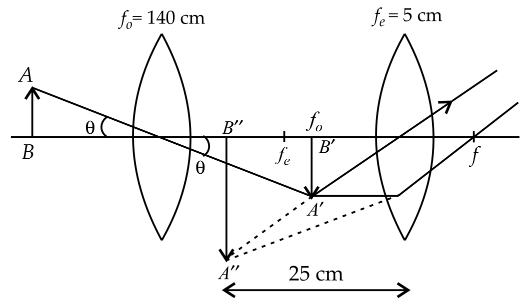
We can calculate height of image by objective lens

$$\tan\theta = \frac{A'B'}{B'O_1} = \frac{AB}{BO_1}$$

height of image $A'B' = B'O_1 \times \frac{AB}{BO_1}$

$$A'B' = 140 \times \frac{100}{3000} = 4.7 \text{ cm}$$

(c) Now we want to find the height of final image $A''B''$ assuming it to be formed at 25 cm.



Magnification by the eyepiece is

$$m_e = \left(1 + \frac{D}{f_e}\right) = \left(1 + \frac{25}{5}\right) = 6$$

Now height of final image, $m_e = \frac{A''B''}{A'B'}$

$A''B'' = m_e \times A'B' = 6 \times 4.7 = 28.2$ cm

7. Image formed by concave mirror acts as a virtual object for convex mirror.

Here parallel rays coming from infinity will focus at 110 mm an axis away from concave mirror.

Distance of virtual object for convex mirror $= 110 - 20 = 90$ mm

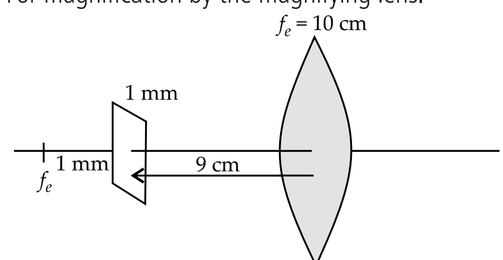
For convex mirror

$u = -90$ mm, $f = -70$ mm $\Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow v = -315$ mm

Hence image is formed at 315 mm from convex mirror.

8. (a) For magnification by the magnifying lens.



Let us use lens formula

$u = -9$ cm, $f = +10$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{v} - \frac{1}{-9} = \frac{1}{10}$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{9} = -\frac{1}{90}$$

Image position $v = -90$ cm

Magnification $m = \frac{I}{O} = \frac{v}{u}$ or $m = \frac{-90}{-9} = 10$

For area 1 mm^2, consider the height of object 1 mm, so height of image.

$$\frac{I}{O} = \frac{v}{u}, \quad \frac{I}{1\,mm} = 10$$

$I = 10$ mm

Area of image $A = 10 \times 10$ mm^2 = 100 mm^2 = 1 cm^2

(b) Angular magnification,

$$m = \frac{D}{u} = \frac{25}{9} = 2.78$$

(c) No, the linear magnification by a lens and magnifying power (angular magnification) of magnifying glass have different values.

The linear magnification is calculated using $m = \frac{v}{u}$, whereas angular magnification is $m = \frac{D}{u} = \frac{\beta}{\alpha}$, the ratio of angle subtended by image of object at eye lens 'β' to the angle subtended by object assumed to be at least distance at eye lens 'α'.

The linear magnification and angular magnification in microscope have similar magnitude when image is at least distance of distinct vision *i.e.*, 25 cm.

9. (a) For maximum magnifying power the image should be at least distance of distinct vision *i.e.*, 25 cm.

$\frac{1}{v} - \frac{1}{u} = \frac{1}{r}$ or $\frac{1}{-25} - \frac{1}{u} = \frac{1}{10}$ or $\frac{-1}{u}\frac{1}{10} + \frac{1}{25} = \frac{7}{50}$

$u = -\frac{50}{7}\text{ cm} = -7.14\text{ cm}$

(b) Linear magnification in the situation of maximum magnifying power.

$m = \frac{I}{O} = \frac{v}{u}$, $m = \frac{v}{u} = \frac{-25}{\frac{-50}{7}} = 3.5$

(c) Maximum magnifying power in the same situation

$m = \frac{D}{u_{min}}$ or $\left(1 + \frac{D}{f_e}\right)$

$m_{max} = \frac{25}{\frac{50}{7}}$ or $\left[1 + \frac{25}{10}\right]$

$m_{max} = 3.5$

So, it can be observed that in the situation when image is least distance of distinct vision the angular magnification and linear magnification have similar values.

10. Now we want the area of square shaped virtual image as 6.25 mm^2.

So, each side of image is $I = \sqrt{6.25} = 2.5\text{ mm}$

(Linear magnification)

For the given magnifying lens of focal length 10 cm we can calculate the required position of object.

Magnification $m = \frac{I}{O} = \frac{v}{u}$

$\frac{I}{O} = \frac{v}{u} \Rightarrow \frac{2.5\text{ mm}}{1\text{ mm}} = \frac{v}{u}$

So, $v = 2.5\,u$

Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{2.5\,u} - \frac{1}{u} = \frac{1}{+10}$

$u = -6$ cm and $v = -2.5 \times u = -15$ cm.

Thus the required virtual image is closer than normal near point.

Thus the eye cannot observe the image distinctly.

11. (a) In magnifying glass the object is placed closer than 25 cm, which produces image at 25 cm. This closer object has larger angular size than the same object at 25 cm. In this way although the angle subtended by virtual image and object is same at eye but angular magnification is achieved.

(b) On moving the eye backward away from lens the angular magnification decreases slightly, as both the angle subtended by the image at eye 'α' and by the object at eye 'β' decreases. Although the decrease in angle subtended by object α is relatively smaller.

(c) If we decrease focal length, the lens has to be thick with smaller radius of curvature. In a thick lens both the spherical aberrations and chromatic aberrations become pronounced. Further, grinding for small focal length is not easy. Practically we can not get magnifying power more than 3 with a simple convex lens.

(d) Magnifying power of a compound microscope is given by

$$m = -\frac{v_o}{u_o}\left[1 + \frac{D}{f_e}\right]$$

With approximations $m = -\frac{L}{f_o}\left[1 + \frac{D}{f_e}\right]$ where 'L' is separation between lenses.

Clearly, lesser is f_o and f_e higher is the magnitude of m.

(e) If we place our eye too close to the eyepiece, we shall not collect much of the light and also reduce our field of view. When we position our eye slightly away and the area of the pupil of our eye is greater, our eye will collect all the light refracted by the objective, and a clear image is observed by the eye.

12. Here we want the distance between given objective and eye lens for the required magnification of 30.

Let the final image is formed at least distance of distinct vision for eyepiece.

$m_e = \left(1 + \frac{D}{f_e}\right) = \frac{D}{u_e}$ or $m_e = 1 + \frac{25}{5} = \frac{25}{u_e}$

$m_e = 6$ and $u_e = \frac{25}{6}$ cm

Now magnification by objective lens

$m = m_o \times m_e$

$m_o = \frac{m}{m_e} = \frac{30}{6} = 5$

Also $m_o = \frac{v_o}{u_o} \Rightarrow m_0 = \frac{v_o}{u_o} = -5$

($\because$ real image is formed by objective.)

So, the relation $v_o = -5u_o$

Now lens formula for objective lens

$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$ or $\frac{1}{-5u_o} - \frac{1}{u_o} = \frac{1}{1.25}$

$\frac{1+5}{5u_o} = \frac{1}{1.25}$ or $5u_o = 7.5$ or $u_o = -1.5$ cm

Also $v_o = -5u_o = 7.5$ cm

So, required distance between objective and eyepiece

$$L = v_o + |u_e| = 7.5 + \frac{25}{6} = 11.67\text{ cm}$$