Mid Term

SOLUTIONS

1. **(d) :** We have, $\tan^2\theta\ (\text{cosec}^2\theta - 1)$

$= \tan^2\theta \cot^2\theta$ $\quad [\because 1 + \cot^2\theta = \text{cosec}^2\theta]$

$$= \tan^2\theta \times \frac{1}{\tan^2\theta} = 1$$

2. **(c) :** Mid-point of the line segment joining the points (7, 7) and (3, 5) = $\left(\frac{7+3}{2}, \frac{7+5}{2}\right)$ *i.e.*, (5, 6).

Distance of (5, 6) from (1, 3)

$$= \sqrt{(5-1)^2 + (6-3)^2} = \sqrt{25} = 5 \text{ units}$$

3. **(c) :** Since, $DE \parallel BC$ [Given]

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ [By B.P.T.]

$\Rightarrow \quad \frac{1}{DB} = \frac{2}{6} = \frac{1}{3} \Rightarrow DB = 3 \text{ cm}$

4. Let α, β be the roots of the equation $3x^2 + (2k + 1)x - (k + 5) = 0$

$\therefore \quad \alpha + \beta = \frac{-(2k+1)}{3}$ and $\alpha\beta = \frac{-(k+5)}{3}$

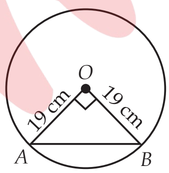
Now, $\alpha + \beta = \alpha\beta$ [Given]

$\Rightarrow \quad -(2k + 1) = -(k + 5) \Rightarrow k = 4$

5. Let AB be the given chord.

Then, $AB^2 = OA^2 + OB^2$

[By Pythagoras theorem]



$\Rightarrow \quad AB^2 = 19^2 + 19^2$

$\Rightarrow \quad AB^2 = 2(19)^2$

$\Rightarrow \quad AB = 19\sqrt{2}$ cm

6. If three terms are in A.P., then $13 - (2p + 1) = (5p - 3) - 13$

$\Rightarrow \quad 13 - 2p - 1 = 5p - 3 - 13 \Rightarrow 28 = 7p \Rightarrow p = 4$

7. Let $P(x, y)$ be the required point. Then,

$$x = \frac{3\times(-4)+2\times 6}{3+2} \text{ and } y = \frac{3\times 5+2\times 3}{3+2}$$

$\Rightarrow \quad x = 0$ and $y = \frac{21}{5}$

So, the coordinates of P are (0, 21/5).

8. Since, $\frac{278}{2^3 m}$ has a terminating decimal expansion

$\therefore \quad 2^3 \times m$ should be of the type $2^p \times 5^q$, where p and q are whole numbers.

$\Rightarrow \quad 2^3 \times m = 2^p \times 5^q \Rightarrow m = 5^q$

Since, $2 < m < 9$ [Given]

$\therefore \quad 2 < 5^q < 9$

$\Rightarrow \quad m = 5$ and $q = 1$

9. Since, the points $(k, 3)$, $(6, -2)$ and $(-3, 4)$ are collinear. Hence, the area of the triangle formed by these points will be zero.

$\therefore \quad \frac{1}{2}[k(-2-4) + 6(4-3) - 3(3+2)] = 0$

$\Rightarrow \quad -6k + 6 - 15 = 0 \Rightarrow -6k - 9 = 0 \Rightarrow k = -\frac{3}{2}$

10. L.H.S. = $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta$

$$= \frac{(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)}{\sin\theta+\cos\theta} + \sin\theta\cos\theta$$

$[\because (a^3 + b^3) = (a + b)(a^2 + b^2 - ab)]$

$= 1 - \sin\theta\cos\theta + \sin\theta\cos\theta = 1$ = R.H.S.

$\therefore$ L.H.S. = R.H.S.

11. Since, $AQ \parallel PR$ [Given]

$\therefore \quad \angle 1 = \angle 4$ and $\angle 2 = \angle 3$

Also $\angle 1 = \angle 2$ $\quad [\because PA = PQ]$

$\therefore \quad \angle 3 = \angle 4$

In ΔPQR and ΔPBR,

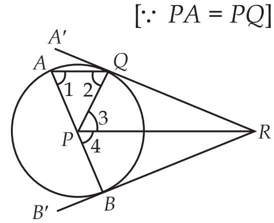
$PR = PR$ [Common]

$PQ = PB$ [Radii]

$\angle 3 = \angle 4$

$\therefore \quad \Delta PQR \cong \Delta PBR$ [By SAS]

$\Rightarrow \quad \angle PBR = \angle PQR$ [By CPCT]



But $\angle PQR = 90°$ $\quad [\because QR$ is a tangent and PQ is radius]

$\therefore \quad \angle PBR = 90°$

Thus, BR is a tangent at B.

12. $\cot\theta \cdot \tan(90° - \theta) - \sec(90° - \theta)\text{cosec}\theta + \sin^2 25°$

$+ \sin^2 65° + \sqrt{3}(\tan 5° \tan 45° \tan 85°)$

$= \cot\theta \cdot \cot\theta - \text{cosec}\,\theta \cdot \text{cosec}\,\theta + \sin^2 25° + \sin^2(90° - 25°)$

$+ \sqrt{3}[\tan 5° \cdot 1 \cdot \tan(90° - 5°)]$

$[\because \tan(90° - \theta) = \cot\theta ; \sec(90° - \theta) = \text{cosec}\,\theta]$

$= -(\text{cosec}^2\theta - \cot^2\theta) + (\sin^2 25° + \cos^2 25°) + \sqrt{3}\,(\tan 5° \cdot \cot 5°)$

$[\because \sin(90° - \theta) = \cos\theta]$

$$= -1 + 1 + \sqrt{3}.\tan 5° \frac{1}{\tan 5°} = \sqrt{3}$$

$$\left[\because \text{cosec}^2\theta - \cot^2\theta = 1 ; \sin^2\theta + \cos^2\theta = 1 ; \cot\theta = \frac{1}{\tan\theta}\right]$$

13. The given system of equations may be written as

$x + y - (a - b) = 0$

$ax - by - (a^2 + b^2) = 0$

By cross-multiplication, we have

$$\frac{x}{-(a^2+b^2)-b(a-b)} = \frac{y}{-a(a-b)+(a^2+b^2)} = \frac{1}{-b-a}$$

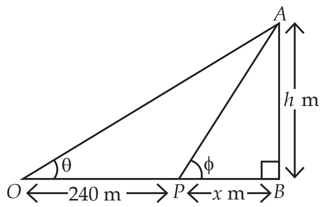
$$\Rightarrow \quad \frac{x}{-a^2-ab} = \frac{y}{ab+b^2} = \frac{1}{-b-a}$$

$$\Rightarrow \quad \frac{x}{-a(a+b)} = \frac{y}{b(a+b)} = \frac{1}{-(a+b)}$$

$$\Rightarrow \quad x = \frac{-a(a+b)}{-(a+b)} = a \text{ and } y = \frac{b(a+b)}{-(a+b)} = -b$$

Hence, $x = a$ and $y = -b$ is the solution of the given system of equations.

14. Let height of the lighthouse be h metres. Let O and P be the initial and final position of the person, making an angle of elevation θ and ϕ as shown in the figure. Let $PB = x$ metres



In right angled ΔOBA,

$$\tan\theta = \frac{AB}{OB} \Rightarrow \frac{5}{12} = \frac{h}{240+x} \quad \text{...(i)} \quad \left[\because \tan\theta = \frac{5}{12} \text{ (Given)}\right]$$

In right angled ΔPBA,

$$\tan\phi = \frac{AB}{PB} \Rightarrow \frac{3}{4} = \frac{h}{x} \quad \text{...(ii)} \quad \left[\because \tan\phi = \frac{3}{4} \text{ (Given)}\right]$$

Dividing (i) by (ii), we get

$$\frac{5}{12} \times \frac{4}{3} = \frac{h}{240+x} \times \frac{x}{h}$$

$$\Rightarrow \quad \frac{5}{9} = \frac{x}{240+x} \Rightarrow 1200 + 5x = 9x$$

$\Rightarrow \quad 4x = 1200$

$\Rightarrow \quad x = 300$

Putting $x = 300$ in (ii) we get, $h = \frac{3}{4} \times 300 = 225$

Hence height of the lighthouse is 225 metres.

15. Since $DEFG$ is a square.

$\therefore \quad \angle BDG = 90° = \angle FEC$

Also, $DG = GF = FE = DE$ ...(i)

In ΔBAC and ΔBDG,

$\angle ABC = \angle DBG$ [Common]

$\angle BAC = \angle BDG$ [Each 90°]

$\therefore \quad \Delta BAC \sim \Delta BDG$ [By AA similarity criterion]

$$\Rightarrow \quad \frac{AB}{BD} = \frac{AC}{DG} \Rightarrow \frac{AB}{AC} = \frac{BD}{DG} \quad \text{...(ii)}$$

In ΔBAC and ΔFEC,

$\angle ACB = \angle ECF$ [Common]

$\angle BAC = \angle FEC$ [Each 90°]

$\therefore \quad \Delta BAC \sim \Delta FEC$ (By AA similarity criterion)

$$\Rightarrow \quad \frac{AB}{FE} = \frac{AC}{EC} \Rightarrow \frac{AB}{AC} = \frac{FE}{EC} \quad \text{...(iii)}$$

From (ii) and (iii), we get

$$\frac{BD}{DG} = \frac{FE}{EC} \Rightarrow \frac{BD}{DE} = \frac{DE}{EC} \quad \text{[Using (i)]}$$

$\Rightarrow \quad DE^2 = BD \times EC$