Quadratic Equations

CHAPTER 4

NCERT FOCUS

SOLUTIONS

EXERCISE - 4.1

1. (i) We have, $(x + 1)^2 = 2(x - 3)$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 2x + 1 - 2x + 6 = 0 \Rightarrow x^2 + 7 = 0$

Since, $x^2 + 7$ is a quadratic polynomial.

$\therefore (x + 1)^2 = 2(x - 3)$ is a quadratic equation.

(ii) We have, $x^2 - 2x = (-2)(3 - x)$

$\Rightarrow x^2 - 2x = -6 + 2x \Rightarrow x^2 - 2x - 2x + 6 = 0$

$\Rightarrow x^2 - 4x + 6 = 0$

Since, $x^2 - 4x + 6$ is a quadratic polynomial.

$\therefore x^2 - 2x = (-2)(3 - x)$ is a quadratic equation.

(iii) We have, $(x - 2)(x + 1) = (x - 1)(x + 3)$

$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow x^2 - x - 2 - x^2 - 2x + 3 = 0 \Rightarrow -3x + 1 = 0$

Since, $-3x + 1$ is a linear polynomial.

$\therefore (x - 2)(x + 1) = (x - 1)(x + 3)$ is not a quadratic equation.

(iv) We have, $(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 - 5x - 3 - x^2 - 5x = 0 \Rightarrow x^2 - 10x - 3 = 0$

Since, $x^2 - 10x - 3$ is a quadratic polynomial.

$\therefore (x - 3)(2x + 1) = x(x + 5)$ is a quadratic equation.

(v) We have, $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$\Rightarrow 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\Rightarrow 2x^2 - 7x + 3 - x^2 - 4x + 5 = 0 \Rightarrow x^2 - 11x + 8 = 0$

Since, $x^2 - 11x + 8$ is a quadratic polynomial.

$\therefore (2x - 1)(x - 3) = (x + 5)(x - 1)$ is a quadratic equation.

(vi) We have, $x^2 + 3x + 1 = (x - 2)^2$

$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow x^2 + 3x + 1 - x^2 + 4x - 4 = 0 \Rightarrow 7x - 3 = 0$

Since, $7x - 3$ is a linear polynomial.

$\therefore x^2 + 3x + 1 = (x - 2)^2$ is not a quadratic equation.

(vii) We have, $(x + 2)^3 = 2x(x^2 - 1)$

$\Rightarrow x^3 + 3x^2(2) + 3x(2)^2 + (2)^3 = 2x^3 - 2x$

$\Rightarrow x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$

$\Rightarrow x^3 + 6x^2 + 12x + 8 - 2x^3 + 2x = 0$

$\Rightarrow -x^3 + 6x^2 + 14x + 8 = 0$

Since, $-x^3 + 6x^2 + 14x + 8$ is a cubic polynomial.

$\therefore (x + 2)^3 = 2x(x^2 - 1)$ is not a quadratic equation.

(viii) We have, $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 + 3x^2(-2) + 3x(-2)^2 + (-2)^3$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$

$\Rightarrow x^3 - 4x^2 - x + 1 - x^3 + 6x^2 - 12x + 8 = 0$

$\Rightarrow 2x^2 - 13x + 9 = 0$

Since, $2x^2 - 13x + 9$ is a quadratic polynomial.

$\therefore x^3 - 4x^2 - x + 1 = (x - 2)^3$ is a quadratic equation.

2. (i) Let the breadth = x metres

$\because$ Length = 2(Breadth) + 1

$\therefore$ Length = $(2x + 1)$metres

Since, length × breadth = Area

$\therefore (2x + 1) \times x = 528 \Rightarrow 2x^2 + x = 528$

$\Rightarrow 2x^2 + x - 528 = 0$

Thus, the required quadratic equation is $2x^2 + x - 528 = 0$.

(ii) Let the two consecutive positive integers be x and $(x + 1)$.

$\because$ Product of two consecutive positive integers = 306

$\therefore x(x + 1) = 306 \Rightarrow x^2 + x = 306 \Rightarrow x^2 + x - 306 = 0$

Thus, the required quadratic equation is $x^2 + x - 306 = 0$.

(iii) Let the present age of Rohan be x years.

$\therefore$ His mother's age = $(x + 26)$ years

After 3 years, Rohan's age = $(x + 3)$ years

After 3 years, his mother's age $= [(x + 26) + 3]$ years $= (x + 29)$ years

According to the condition, $(x + 3) \times (x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 29x + 3x + 87 - 360 = 0 \Rightarrow x^2 + 32x - 273 = 0$

Thus, the required quadratic equation is

$x^2 + 32x - 273 = 0$.

(iv) Let the speed of the train = u km/hr

Distance covered = 480 km

$$\text{Time taken} = \frac{\text{Distance}}{\text{Speed}} = \frac{480}{u} \text{ hours}$$

In other case, speed = $(u - 8)$ km/hour

$$\therefore \text{Time taken} = \frac{\text{Distance}}{\text{Speed}} = \frac{480}{(u-8)} \text{ hours}$$

According to the condition, $\frac{480}{u-8} - \frac{480}{u} = 3$

$\Rightarrow 480u - 480(u - 8) = 3u(u - 8)$

$\Rightarrow 480u - 480u + 3840 = 3u^2 - 24u$

$\Rightarrow 3u^2 - 24u - 3840 = 0 \Rightarrow u^2 - 8u - 1280 = 0$

Thus, the required quadratic equation is

$u^2 - 8u - 1280 = 0$.

EXERCISE - 4.2

1. (i) We have, $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0 \Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

$\Rightarrow x - 5 = 0$ or $x + 2 = 0 \Rightarrow x = 5$ or $x = -2$

Thus, the required roots are 5 and –2.

(ii) We have, $2x^2 + x - 6 = 0 \Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x + 2) - 3(x + 2) = 0 \Rightarrow (x + 2)(2x - 3) = 0$

$\Rightarrow x + 2 = 0$ or $2x - 3 = 0 \Rightarrow x = -2$ or $x = 3/2$

Thus, the required roots are –2 and 3/2.

(iii) We have, $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x(x+\sqrt{2}) + 5(x+\sqrt{2}) = 0$

$\Rightarrow \quad (x+\sqrt{2})(\sqrt{2}x+5) = 0$

$\Rightarrow \quad x+\sqrt{2} = 0$ or $\sqrt{2}x+5 = 0$

$\Rightarrow \quad x = -\sqrt{2}$ or $x = \frac{-5}{\sqrt{2}} = \frac{-5\sqrt{2}}{2}$

Thus, the required roots are $-\sqrt{2}$ and $\frac{-5\sqrt{2}}{2}$.

(iv) We have, $2x^2 - x + \frac{1}{8} = 0$

$\Rightarrow \quad 16x^2 - 8x + 1 = 0 \Rightarrow 16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow \quad 4x(4x-1) - 1(4x-1) = 0 \Rightarrow (4x-1)(4x-1) = 0$

$\Rightarrow \quad 4x - 1 = 0 \Rightarrow x = 1/4$

Thus, the required roots are 1/4 and 1/4.

(v) We have, $100x^2 - 20x + 1 = 0$

$\Rightarrow \quad 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow \quad 10x(10x-1) - 1(10x-1) = 0 \Rightarrow (10x-1)(10x-1) = 0$

$\Rightarrow \quad (10x-1) = 0 \Rightarrow x = 1/10$

Thus, the required roots are $\frac{1}{10}$ and $\frac{1}{10}$.

2. (i) Let John had x marbles and Jivanti had $(45 - x)$ marbles.

According to question,

$(x-5) \times (45 - x - 5) = 124$

$\Rightarrow \quad (x-5) \times (40 - x) = 124 \Rightarrow x^2 - 45x + 324 = 0$

$\Rightarrow \quad x^2 - 9x - 36x + 324 = 0 \Rightarrow x(x-9) - 36(x-9) = 0$

$\Rightarrow \quad (x-9)(x-36) = 0$

$\Rightarrow \quad x - 9 = 0$ or $x - 36 = 0 \Rightarrow x = 9$ or $x = 36$

$\therefore$ If John had 9 marbles, then Jivanti had $45 - 9 = 36$ marbles.

If John had 36 marbles, then Jivanti had $45 - 36 = 9$ marbles.

(ii) Let the number of toys produced in a day be x.

Then, cost of 1 toy $= \frac{750}{x}$

According to question, $\frac{750}{x} = 55 - x$

$\Rightarrow \quad 750 = 55x - x^2 \Rightarrow x^2 - 55x + 750 = 0$

$\Rightarrow \quad x^2 - 30x - 25x + 750 = 0$

$\Rightarrow \quad x(x-30) - 25(x-30) = 0 \Rightarrow (x-30)(x-25) = 0$

$\Rightarrow \quad x - 30 = 0$ or $x - 25 = 0 \Rightarrow x = 30$ or $x = 25$

Hence, number of toys produced on that day is either 30 or 25.

3. Let one of the numbers be x.

$\therefore$ Other number $= 27 - x$

According to the condition,

$x(27 - x) = 182 \Rightarrow 27x - x^2 = 182$

$\Rightarrow \quad x^2 - 27x + 182 = 0 \Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow \quad x(x-13) - 14(x-13) = 0 \Rightarrow (x-13)(x-14) = 0$

$\Rightarrow \quad x - 13 = 0$ or $x - 14 = 0 \Rightarrow x = 13$ or $x = 14$

Thus, the required numbers are 13 and 14.

4. Let the two consecutive positive integers be x and $(x + 1)$.

Since, the sum of the squares of the numbers is 365.

$\therefore \quad x^2 + (x+1)^2 = 365 \Rightarrow x^2 + x^2 + 2x + 1 = 365$

$\Rightarrow \quad 2x^2 + 2x - 364 = 0 \Rightarrow x^2 + x - 182 = 0$

$\Rightarrow \quad x^2 + 14x - 13x - 182 = 0$

$\Rightarrow \quad x(x+14) - 13(x+14) = 0 \Rightarrow (x+14)(x-13) = 0$

$\Rightarrow \quad x + 14 = 0$ or $x - 13 = 0 \Rightarrow x = -14$ or $x = 13$

Since, x has to be a positive integer, so $x = -14$ is rejected.

$\therefore \quad x = 13 \Rightarrow x + 1 = 13 + 1 = 14$

Thus, the required consecutive positive integers are 13 and 14.

5. Let the base of the given right triangle be x cm.

$\therefore$ Its altitude $= (x - 7)$ cm

$\because$ Hypotenuse $= \sqrt{(\text{Base})^2 + (\text{Altitude})^2}$

[By Pythagoras theorem]

$\therefore \quad 13 = \sqrt{x^2 + (x-7)^2}$

On squaring both sides, we get, $169 = x^2 + (x-7)^2$

$\Rightarrow \quad 169 = x^2 + x^2 - 14x + 49 \Rightarrow 2x^2 - 14x + 49 - 169 = 0$

$\Rightarrow \quad 2x^2 - 14x - 120 = 0 \Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow \quad x^2 - 12x + 5x - 60 = 0 \Rightarrow x(x-12) + 5(x-12) = 0$

$\Rightarrow \quad (x-12)(x+5) = 0$

$\Rightarrow \quad x - 12 = 0$ or $x + 5 = 0 \Rightarrow x = 12$ or $x = -5$

But the sides of a triangle can never be negative, so, $x = -5$ is rejected.

$\therefore \quad x = 12$

$\therefore$ Length of base = 12 cm

$\Rightarrow$ Length of altitude = (12 – 7) cm = 5 cm

Thus, the required base is 12 cm and altitude is 5 cm.

6. Let the number of articles produced in a day $= x$

$\therefore$ Cost of production of each article = ₹ $(2x + 3)$

Total cost = ₹ 90

$\therefore \quad x \times (2x + 3) = 90 \Rightarrow 2x^2 + 3x = 90$

$\Rightarrow \quad 2x^2 + 3x - 90 = 0 \Rightarrow 2x^2 - 12x + 15x - 90 = 0$

$\Rightarrow \quad 2x(x-6) + 15(x-6) = 0 \Rightarrow (x-6)(2x+15) = 0$

$\Rightarrow \quad x - 6 = 0$ or $2x + 15 = 0 \Rightarrow x = 6$ or $x = \frac{-15}{2}$

But the number of articles produced can never be negative,

so, $x = \frac{-15}{2}$ is rejected.

$\therefore \quad x = 6$

$\therefore$ Cost of production of each article = ₹ (2 × 6 + 3) = ₹ 15

Thus, the required number of articles produced is 6 and the cost of each article is ₹ 15.

EXERCISE - 4.3

1. (i) We have, $2x^2 - 7x + 3 = 0$

Dividing both sides by 2, we get $x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

$\Rightarrow \quad \left\{x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2\right\} - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$

$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0 \Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{24}{16} = 0$

$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 - \frac{25}{16} = 0 \Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16} = \left(\frac{5}{4}\right)^2$

$\Rightarrow \quad x-\frac{7}{4}=\pm\frac{5}{4}$

Case I : When $x-\frac{7}{4}=\frac{5}{4} \Rightarrow x=\frac{5}{4}+\frac{7}{4} \Rightarrow x=\frac{12}{4}=3$

Case II : When $x-\frac{7}{4}=\frac{-5}{4} \Rightarrow x=\frac{-5}{4}+\frac{7}{4} \Rightarrow x=\frac{2}{4}=\frac{1}{2}$

Thus, the required roots are 3 and 1/2.

(ii) We have, $2x^2 + x - 4 = 0$

Dividing both sides by 2, we get $x^2+\frac{x}{2}-2=0$

$\Rightarrow \left\{x^2+\frac{x}{2}+\left(\frac{1}{4}\right)^2\right\}-\left(\frac{1}{4}\right)^2-2=0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2-\frac{1}{16}-2=0 \Rightarrow \left(x+\frac{1}{4}\right)^2-\frac{33}{16}=0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{33}{16} \Rightarrow \left(x+\frac{1}{4}\right)^2=\left(\frac{\sqrt{33}}{4}\right)^2$

$\Rightarrow \quad x+\frac{1}{4}=\pm\frac{\sqrt{33}}{4}$

Case I : When $x+\frac{1}{4}=\frac{\sqrt{33}}{4}$

$\Rightarrow \quad x=\frac{\sqrt{33}}{4}-\frac{1}{4} \Rightarrow x=\frac{\sqrt{33}-1}{4}$

Case II : When $x+\frac{1}{4}=\frac{-\sqrt{33}}{4}$

$\Rightarrow \quad x=\frac{-\sqrt{33}}{4}-\frac{1}{4} \Rightarrow x=\frac{-\sqrt{33}-1}{4}$

Thus, the required roots are $\frac{\sqrt{33}-1}{4}$ and $\frac{-\sqrt{33}-1}{4}$.

(iii) We have, $4x^2+4\sqrt{3}x+3=0$

Dividing both sides by 4, we get $x^2+\sqrt{3}x+\frac{3}{4}=0$

$\Rightarrow \left\{x^2+\sqrt{3}x+\left(\frac{\sqrt{3}}{2}\right)^2\right\}-\left(\frac{\sqrt{3}}{2}\right)^2+\frac{3}{4}=0$

$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)^2-\frac{3}{4}+\frac{3}{4}=0 \Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)^2=0$

$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right)=0 \Rightarrow x=-\frac{\sqrt{3}}{2}$ and $x=-\frac{\sqrt{3}}{2}$

Thus, the required roots are $-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}$.

(iv) We have, $2x^2 + x + 4 = 0$

Dividing both sides by 2, we get $x^2+\frac{x}{2}+2=0$

$\Rightarrow \left\{x^2+\frac{x}{2}+\left(\frac{1}{4}\right)^2\right\}-\left(\frac{1}{4}\right)^2+2=0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2-\frac{1}{16}+2=0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2+\frac{31}{16}=0 \Rightarrow \left(x+\frac{1}{4}\right)^2=-\frac{31}{16}$

Since, the square of a number cannot be negative.

$\therefore \left(x+\frac{1}{4}\right)$ cannot give a real value.

$\therefore$ There is no real value of x satisfying the given equation.

2. (i) Comparing the given equation with $ax^2 + bx + c = 0$, we get $a = 2$, $b = -7$ and $c = 3$.

$\therefore \quad b^2 - 4ac = (-7)^2 - 4(2)(3) = 49 - 24 = 25 > 0$

Since $b^2 - 4ac > 0$, therefore the given equation has real roots, which are given by

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow x=\frac{-(-7)\pm\sqrt{25}}{2(2)}=\frac{7\pm5}{4}$$

Taking positive sign, $x=\frac{7+5}{4}=\frac{12}{4}=3$

Taking negative sign, $x=\frac{7-5}{4}=\frac{2}{4}=\frac{1}{2}$

Thus, the roots of the given equation are 3 and 1/2.

(ii) Comparing the given equation with $ax^2 + bx + c = 0$, we get $a = 2$, $b = 1$ and $c = -4$.

$\therefore \quad b^2 - 4ac = (1)^2 - 4(2)(-4) = 1 + 32 = 33 > 0$

Since $b^2 - 4ac > 0$, therefore the given equation has real roots, which are given by

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow x=\frac{-1\pm\sqrt{33}}{2(2)}=\frac{-1\pm\sqrt{33}}{4}$$

Taking positive sign, $x=\frac{-1+\sqrt{33}}{4}$

Taking negative sign, $x=\frac{-1-\sqrt{33}}{4}$

Thus, the roots of the given equation are $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$.

(iii) Comparing the given equation with $ax^2 + bx + c = 0$, we get $a = 4$, $b = 4\sqrt{3}$ and $c = 3$.

$\therefore b^2 - 4ac = (4\sqrt{3})^2 - 4(4)(3) = (16 \times 3) - 48 = 48 - 48 = 0$

Since $b^2 - 4ac = 0$, therefore the given equation has real roots, which are given by $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\Rightarrow \quad x=\frac{-4\sqrt{3}\pm\sqrt{0}}{2(4)}=\frac{-4\sqrt{3}}{8}=\frac{-\sqrt{3}}{2}$

$\Rightarrow \quad x=\frac{-\sqrt{3}}{2}$ and $x=\frac{-\sqrt{3}}{2}$

Thus, the roots of the given equation are $\frac{-\sqrt{3}}{2},\frac{-\sqrt{3}}{2}$.

(iv) Comparing the given equation with $ax^2 + bx + c = 0$, we get $a = 2$, $b = 1$ and $c = 4$.

$\therefore \quad b^2 - 4ac = (1)^2 - 4(2)(4) = 1 - 32 = -31 < 0$

Since $b^2 - 4ac < 0$, therefore the given equation does not have real roots.

3. (i) We have, $x-\frac{1}{x}=3$

$\Rightarrow \quad x^2 - 1 = 3x \Rightarrow x^2 - 3x - 1 = 0$ ...(1)

Comparing equation (1) with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -3$ and $c = -1$.

$\therefore \quad b^2 - 4ac = (-3)^2 - 4(1)(-1) = 9 + 4 = 13 > 0$

Since $b^2 - 4ac > 0$, therefore equation (1) has real roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-3) \pm \sqrt{13}}{2(1)} = \frac{3 \pm \sqrt{13}}{2}$$

Taking positive sign, $x = \frac{3 + \sqrt{13}}{2}$

Taking negative sign, $x = \frac{3 - \sqrt{13}}{2}$

Thus, the required roots of the given equation are $\frac{3 + \sqrt{13}}{2}$ and $\frac{3 - \sqrt{13}}{2}$.

(ii) We have, $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

$\Rightarrow (x-7) - (x+4) = \frac{11}{30}(x+4)(x-7)$

$\Rightarrow x - 7 - x - 4 = \frac{11}{30}(x^2 - 3x - 28)$

$\Rightarrow -11 \times 30 = 11(x^2 - 3x - 28)$

$\Rightarrow -30 = x^2 - 3x - 28 \Rightarrow x^2 - 3x - 28 + 30 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$ ...(1)

Comparing equation (1) with $ax^2 + bx + c = 0$, we get $a = 1, b = -3$ and $c = 2$.

$\therefore b^2 - 4ac = (-3)^2 - 4(1)(2) = 9 - 8 = 1 > 0$

Since $b^2 - 4ac > 0$, therefore equation (1) has real roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-3) \pm \sqrt{1}}{2(1)} = \frac{3 \pm 1}{2}$$

Taking positive sign, $x = \frac{3+1}{2} = \frac{4}{2} = 2$

Taking negative sign, $x = \frac{3-1}{2} = 1$

Thus, the required roots of the given equation are 2 and 1.

4. Let the present age of Rehman be x years.

3 years ago, Rehman's age = $(x - 3)$ years

5 years later, Rehman's age = $(x + 5)$ years

According to the condition, $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$\Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3} \Rightarrow \frac{(x+5+x-3)}{(x-3)(x+5)} = \frac{1}{3}$

$\Rightarrow 3(x + 5 + x - 3) = (x - 3)(x + 5)$

$\Rightarrow 3(2x + 2) = x^2 + 2x - 15 \Rightarrow 6x + 6 = x^2 + 2x - 15$

$\Rightarrow x^2 + 2x - 6x - 15 - 6 = 0 \Rightarrow x^2 - 4x - 21 = 0$ ...(1)

Comparing equation (1) with $ax^2 + bx + c = 0$, we get $a = 1, b = -4$ and $c = -21$.

$\therefore b^2 - 4ac = (-4)^2 - 4(1)(-21) = 16 + 84 = 100 > 0$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-4) \pm \sqrt{100}}{2(1)} = \frac{4 \pm 10}{2}$$

Taking positive sign, $x = \frac{4+10}{2} = \frac{14}{2} = 7$

Taking negative sign, $x = \frac{4-10}{2} = \frac{-6}{2} = -3$

Since, age cannot be negative, so $x = -3$ is rejected.

So, the present age of Rehman = 7 years.

5. Let Shefali's marks in Mathematics = x

$\therefore$ Marks in English = $30 - x$

[$\because$ Sum of the marks in English and Mathematics = 30]

According to the condition, $(x + 2) \times [(30 - x) - 3] = 210$

$\Rightarrow (x + 2) \times (30 - x - 3) = 210 \Rightarrow (x + 2)(-x + 27) = 210$

$\Rightarrow -x^2 + 25x + 54 = 210 \Rightarrow -x^2 + 25x + 54 - 210 = 0$

$\Rightarrow -x^2 + 25x - 156 = 0 \Rightarrow x^2 - 25x + 156 = 0$ ...(1)

Comparing equation (1) with $ax^2 + bx + c = 0$, we get $a = 1, b = -25$ and $c = 156$.

$\therefore b^2 - 4ac = (-25)^2 - 4(1)(156) = 625 - 624 = 1 > 0$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-25) \pm \sqrt{1}}{2(1)} \Rightarrow x = \frac{25 \pm 1}{2}$$

Taking positive sign, $x = \frac{25+1}{2} = \frac{26}{2} = 13$

Taking negative sign, $x = \frac{25-1}{2} = \frac{24}{2} = 12$

When $x = 13$, then $30 - x = 30 - 13 = 17$

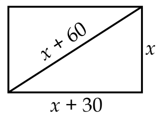
When $x = 12$, then $30 - x = 30 - 12 = 18$

Thus, marks in Mathematics = 13, marks in English = 17 or marks in Mathematics = 12, marks in English = 18.

6. Let the shorter side *i.e.*, breadth = x metres

$\therefore$ The longer side *i.e.*, length = $(x + 30)$ metres

and diagonal = $(x + 60)$ metres



In a rectangle,

$(\text{diagonal})^2 = (\text{breadth})^2 + (\text{length})^2$

$\Rightarrow (x + 60)^2 = x^2 + (x + 30)^2$

$\Rightarrow x^2 + 120x + 3600 = x^2 + x^2 + 60x + 900$

$\Rightarrow x^2 + 120x + 3600 = 2x^2 + 60x + 900$

$\Rightarrow 2x^2 - x^2 + 60x - 120x + 900 - 3600 = 0$

$\Rightarrow x^2 - 60x - 2700 = 0$ ...(1)

Comparing equation (1) with $ax^2 + bx + c = 0$, we get $a = 1, b = -60$ and $c = -2700$.

$\therefore b^2 - 4ac = (-60)^2 - 4(1)(-2700)$

$\Rightarrow b^2 - 4ac = 3600 + 10800 = 14400 > 0$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-60) \pm \sqrt{14400}}{2(1)} \Rightarrow x = \frac{60 \pm 120}{2}$$

Taking positive sign, $x = \frac{60+120}{2} = \frac{180}{2} = 90$

Taking negative sign, $x = \frac{60-120}{2} = \frac{-60}{2} = -30$

Since breadth cannot be negative.

$\therefore x \neq -30 \Rightarrow x = 90$

$\therefore x + 30 = 90 + 30 = 120$

Thus, the shorter side is 90 metres and the longer side is 120 metres.

7. Let the larger number be x.

Since, $(\text{smaller number})^2 = 8(\text{larger number})$

$\Rightarrow (\text{smaller number})^2 = 8x$

$\Rightarrow$ smaller number = $\sqrt{8x}$

According to the condition, $x^2 - (\sqrt{8x})^2 = 180$

$\Rightarrow x^2 - 8x = 180 \Rightarrow x^2 - 8x - 180 = 0$ ...(1)

Comparing equation (1) with $ax^2 + bx + c = 0$, we get $a = 1, b = -8, c = -180$

$\therefore\quad b^2 - 4ac = (-8)^2 - 4(1)(-180) = 64 + 720 = 784 > 0$

$\therefore\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \dfrac{-(-8) \pm \sqrt{784}}{2(1)} \Rightarrow x = \dfrac{8 \pm 28}{2}$

Taking positive sign, $x = \dfrac{8+28}{2} = \dfrac{36}{2} = 18$

Taking negative sign, $x = \dfrac{8-28}{2} = \dfrac{-20}{2} = -10$

But $x = -10$ is not admissible.

$\therefore$ The larger number = 18

$\Rightarrow$ Smaller number = $\sqrt{8 \times 18} = \sqrt{144} = \pm 12$

Thus, the smaller number = 12 or –12

Thus, the two numbers are 18 and 12 or 18 and –12.

8. Let the uniform speed of the train be x km/hr.

Since, time taken by the train $= \dfrac{\text{Distance}}{\text{Speed}}$

$\Rightarrow$ Time taken $= \dfrac{360}{x}$ hours

If, speed = $(x + 5)$ km/hr, then

Time taken $= \dfrac{360}{(x+5)}$ hours

$\therefore$ According to the condition,

$\dfrac{360}{x+5} - \dfrac{360}{x} = -1 \Rightarrow 360\left[\dfrac{1}{x+5} - \dfrac{1}{x}\right] = -1$

$\Rightarrow \dfrac{1}{x+5} - \dfrac{1}{x} = \dfrac{-1}{360} \Rightarrow \dfrac{x-(x+5)}{x(x+5)} = \dfrac{-1}{360}$

$\Rightarrow x - x - 5 = \dfrac{-(x+5)x}{360} \Rightarrow -5 \times 360 = -(x^2 + 5x)$

$\Rightarrow -1800 = -x^2 - 5x \Rightarrow x^2 + 5x - 1800 = 0$ ...(1)

Comparing equation (1) with $ax^2 + bx + c = 0$, we get $a = 1$, $b = 5$ and $c = -1800$.

$\therefore\quad b^2 - 4ac = (5)^2 - 4(1)(-1800) = 25 + 7200 = 7225 > 0$

$\therefore\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \dfrac{-5 \pm \sqrt{7225}}{2(1)} \Rightarrow x = \dfrac{-5 \pm 85}{2}$

Taking positive sign, $x = \dfrac{-5+85}{2} = \dfrac{80}{2} = 40$

Taking negative sign, $x = \dfrac{-5-85}{2} = \dfrac{-90}{2} = -45$

Since, the speed of a vehicle cannot be negative.

$\therefore\quad x \neq -45 \Rightarrow x = 40$

Thus, speed of the train is 40 km/hr.

9. Let the smaller tap fills the tank in x hours.

$\therefore$ The larger tap fills the tank in $(x - 10)$ hours.

Amount of water flowing through both the taps in one hour $= \dfrac{1}{x} + \dfrac{1}{x-10} = \dfrac{x-10+x}{x(x-10)} = \dfrac{2x-10}{x^2-10x}$

According to the condition, $\dfrac{8}{75} = \left(\dfrac{2x-10}{x^2-10x}\right)$

$\Rightarrow \dfrac{75(2x-10)}{8(x^2-10x)} = 1 \Rightarrow \dfrac{150x-750}{8x^2-80x} = 1$

$\Rightarrow 8x^2 - 80x = 150x - 750 \Rightarrow 8x^2 - 80x - 150x + 750 = 0$

$\Rightarrow 8x^2 - 230x + 750 = 0$ ...(1)

Comparing equation (1) with $ax^2 + bx + c = 0$, we get

$a = 8$, $b = -230$ and $c = 750$.

$\therefore\quad b^2 - 4ac = (-230)^2 - 4(8)750$

$= 52900 - 24000 = 28900 > 0$

$\therefore\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow\quad x = \dfrac{-(-230) \pm \sqrt{28900}}{2(8)} \Rightarrow x = \dfrac{230 \pm 170}{16}$

Taking positive sign, $x = \dfrac{230+170}{16} = \dfrac{400}{16} = 25$

Taking negative sign, $x = \dfrac{230-170}{16} = \dfrac{60}{16} = \dfrac{15}{4}$

For $x = \dfrac{15}{4}$, $x - 10 = \dfrac{15}{4} - 10 = \dfrac{-25}{4}$, which is not possible.

[$\because$ Time cannot be negative]

$\therefore\quad x = 25 \Rightarrow x - 10 = 25 - 10 = 15$

Thus, time taken to fill the tank by the smaller tap alone is 25 hours and by the larger tap alone is 15 hours.

10. Let the average speed of the passenger train be x km/hr.

$\therefore$ Average speed of the express train = $(x + 11)$ km/hr

Total distance covered = 132 km

Also, Time $= \dfrac{\text{Distance}}{\text{Speed}}$

Time taken by the passenger train $= \dfrac{132}{x}$ hours

Time taken by the express train $= \dfrac{132}{x+11}$ hours

According to the condition, we get $\dfrac{132}{x+11} = \left(\dfrac{132}{x}\right) - 1$

$\Rightarrow \dfrac{132}{x+11} - \dfrac{132}{x} = -1 \Rightarrow 132\left[\dfrac{1}{x+11} - \dfrac{1}{x}\right] = -1$

$\Rightarrow 132\left[\dfrac{x-x-11}{x(x+11)}\right] = -1 \Rightarrow 132\left[\dfrac{-11}{x^2+11x}\right] = -1$

$\Rightarrow -11(132) = -1(x^2 + 11x) \Rightarrow 1452 = (x^2 + 11x)$

$\Rightarrow x^2 + 11x - 1452 = 0$ ...(1)

Comparing equation (1) with $ax^2 + bx + c = 0$,

we get $a = 1$, $b = 11$ and $c = -1452$.

$\therefore\quad b^2 - 4ac = (11)^2 - 4(1)(-1452) = 121 + 5808 = 5929 > 0$

$\therefore\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \dfrac{-11 \pm \sqrt{5929}}{2(1)} = \dfrac{-11 \pm 77}{2}$

Taking positive sign, $x = \dfrac{-11+77}{2} = \dfrac{66}{2} = 33$

Taking negative sign, $x = \dfrac{-11-77}{2} = \dfrac{-88}{2} = -44$

But average speed cannot be negative.

$\therefore\quad x \neq -44 \Rightarrow x = 33$

$\therefore$ Average speed of the passenger train = 33 km/hr

And average speed of the express train

$= (x + 11) = (33 + 11) = 44$ km/hr

11. Let the side of the smaller square be x m.

$\Rightarrow$ Perimeter of the smaller square = $4x$ m

So, perimeter of the larger square = $(4x + 24)$ m

$\Rightarrow$ Side of the larger square

$= \frac{\text{Perimeter of larger square}}{4}$

$= \frac{(4x+24)}{4} = \frac{4(x+6)}{4} = (x+6)\text{m}$

Area of the smaller square = x^2 m^2

Area of the larger square = $(x + 6)^2$ m^2

According to the condition, $x^2 + (x + 6)^2 = 468$

$\Rightarrow$ $x^2 + x^2 + 12x + 36 = 468 \Rightarrow 2x^2 + 12x - 432 = 0$

$\Rightarrow$ $x^2 + 6x - 216 = 0$ ...(1)

Comparing equation (1) with $ax^2 + bx + c = 0$, we get $a = 1$, $b = 6$ and $c = -216$.

$\therefore$ $b^2 - 4ac = (6)^2 - 4(1)(-216) = 36 + 864 = 900 > 0$

$\therefore$ $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow x = \frac{-6 \pm \sqrt{900}}{2(1)} = \frac{-6 \pm 30}{2}$

Taking positive sign, $x = \frac{-6+30}{2} = \frac{24}{2} = 12$

Taking negative sign, $x = \frac{-6-30}{2} = \frac{-36}{2} = -18$

But the length of a square cannot be negative.

$\therefore$ $x \neq -18 \Rightarrow x = 12$

Length of the smaller square = 12 m

and the length of the larger square = $x + 6 = 12 + 6 = 18$ m

EXERCISE - 4.4

1. Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get $a = 2$, $b = -3$ and $c = 5$.

$\therefore$ $D = b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31 < 0$

$\therefore$ The given quadratic equation has no real roots.

(ii) Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get $a = 3$, $b = -4\sqrt{3}$ and $c = 4$.

$\therefore$ $D = b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4) = (16 \times 3) - 48$
$= 48 - 48 = 0$

$\therefore$ The given quadratic equation has two real roots which are equal. Hence, the roots are

$\frac{-b}{2a}$ and $\frac{-b}{2a}$ *i.e.*, $\frac{-(-4\sqrt{3})}{2\times 3}$ and $\frac{-(-4\sqrt{3})}{2\times 3}$ *i.e.*, $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(iii) Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get $a = 2$, $b = -6$ and $c = 3$.

$\therefore$ $D = b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12 > 0$

Since, $b^2 - 4ac$ is positive.

$\therefore$ The given quadratic equation has two real and distinct roots, which are given by $x = \frac{-b \pm \sqrt{D}}{2a}$

$\Rightarrow$ $x = \frac{-(-6) \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$

Thus, the roots are $\frac{3+\sqrt{3}}{2}$ and $\frac{3-\sqrt{3}}{2}$.

2. (i) Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get $a = 2$, $b = k$ and $c = 3$.

$\therefore$ $D = b^2 - 4ac = (k)^2 - 4(2)(3) = k^2 - 24$

$\because$ For a quadratic equation to have equal roots, $D = 0$

$\Rightarrow$ $b^2 - 4ac = 0 \Rightarrow k^2 - 24 = 0 \Rightarrow k = \pm\sqrt{24} \Rightarrow k = \pm 2\sqrt{6}$

Thus, the required values of k are $2\sqrt{6}$ and $-2\sqrt{6}$.

(ii) $kx(x - 2) + 6 = 0 \Rightarrow kx^2 - 2kx + 6 = 0$

Comparing $kx^2 - 2kx + 6 = 0$ with $ax^2 + bx + c = 0$, we get $a = k$, $b = -2k$ and $c = 6$.

$\therefore$ $D = b^2 - 4ac = (-2k)^2 - 4(k)(6) = 4k^2 - 24k$

Since, the roots are equal.

$\therefore$ $D = b^2 - 4ac = 0 \Rightarrow 4k^2 - 24k = 0$

$\Rightarrow$ $4k(k - 6) = 0 \Rightarrow 4k = 0$ or $k - 6 = 0 \Rightarrow k = 0$ or $k = 6$

But k cannot be 0, otherwise, the given equation is not quadratic. Thus, the required value of k is 6.

3. Let the breadth be x m. $\therefore$ Length = $2x$ m

Now, Area = Length × Breadth = $2x \times x = 2x^2$ m^2

According to the given condition, $2x^2 = 800$

$\Rightarrow$ $x^2 = \frac{800}{2} = 400 \Rightarrow x^2 - 400 = 0$

Here, $a = 1$, $b = 0$ and $c = -400$

$\therefore$ $D = b^2 - 4ac = 0 - 4(1)(-400) = 1600 > 0$

So, the roots are real and distinct.

$\therefore$ $x = \frac{0 \pm \sqrt{1600}}{2(1)} = \pm\frac{40}{2} = \pm 20$

Therefore, $x = 20$ or $x = -20$

But $x = -20$ is not possible. [$\because$ Breadth cannot be negative]

$\therefore$ $x = 20 \Rightarrow 2x = 2 \times 20 = 40$

Thus, it is possible to design a rectangular mango grove with length = 40 m and breadth = 20 m.

4. Let the age of one friend = x years

$\therefore$ Age of other friend = $(20 - x)$ years

Four years ago,

Age of one friend = $(x - 4)$ years

Age of other friend = $(20 - x - 4)$ years = $(16 - x)$ years

According to the condition, $(x - 4) \times (16 - x) = 48$

$\Rightarrow$ $16x - 64 - x^2 + 4x = 48 \Rightarrow -x^2 + 20x - 64 - 48 = 0$

$\Rightarrow$ $-x^2 + 20x - 112 = 0 \Rightarrow x^2 - 20x + 112 = 0$ ...(1)

Comparing equation (1) with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -20$ and $c = 112$.

$\therefore$ $D = b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48 < 0$

$\therefore$ The quadratic equation (1) has no real roots.

Thus, the given situation is not possible.

5. Let the breadth of the rectangle = x m.

Since, the perimeter of the rectangle = 80 m

$\therefore$ 2(Length + Breadth) = 80 $\Rightarrow$ 2(Length + x) = 80

$\Rightarrow$ Length + x = 80/2 = 40 $\Rightarrow$ Length = $(40 - x)$ m

$\therefore$ Area of the rectangle = $(40 - x) \times x = (40x - x^2)$ m^2

According to the given condition,

Area of the rectangle = 400 m^2

$\Rightarrow$ $40x - x^2 = 400 \Rightarrow x^2 - 40x + 400 = 0$ ...(1)

Comparing equation (1) with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -40$ and $c = 400$.

$\therefore$ $D = b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$

$\therefore$ Equation (1) has two equal and real roots. Hence, the roots are $\frac{-b}{2a}$ and $\frac{-b}{2a}$ *i.e.*, $\frac{-(-40)}{2(1)} = \frac{40}{2} = 20$

$\therefore$ Breadth = x m = 20 m, Length = $40 - x = 40 - 20 = 20$ m

Thus, it is possible to design a rectangular park of given perimeter and area.