Arithmetic Progressions

CHAPTER 5

NCERT FOCUS

SOLUTIONS

EXERCISE - 5.1

1. (i) Let us consider, first term, a_1 = Fare for the first 1 km = ₹ 15 since, the taxi fare after the first 1 km is ₹ 8 for each additional km.

$\therefore$ Fare for 2 km = ₹ 15 + ₹ 8 = ₹ 23

Fare for 3 km = ₹ 23 + ₹ 8 = ₹ 31 = ₹ 15 + 2 × ₹ 8

Fare for 4 km = ₹ 31 + ₹ 8 = ₹ 39 = ₹ 15 + 3 × ₹ 8

Fare for 5 km = ₹ 39 + ₹ 8 = ₹ 47 = ₹ 15 + 4 × ₹ 8

We see that fare for each km forms an A.P., with common difference 8.

(ii) Let the amount of air in the cylinder = x

$\therefore$ Air removed in 1st stroke = $x / 4$

$\Rightarrow$ Air left after 1st stroke $= x - \frac{x}{4} = \frac{3x}{4}$

Air left after 2nd stroke

$$= \frac{3x}{4} - \frac{1}{4}\left(\frac{3x}{4}\right) = \frac{3x}{4} - \frac{3x}{16} = \frac{9x}{16}$$

Air left after 3rd stroke

$$= \frac{9x}{16} - \frac{1}{4}\left(\frac{9x}{16}\right) = \frac{9x}{16} - \frac{9x}{64} = \frac{27x}{64}$$

Air left after 4th stroke

$$= \frac{27x}{64} - \frac{1}{4}\left(\frac{27x}{64}\right) = \frac{27x}{64} - \frac{27x}{256} = \frac{81x}{256}$$

Thus, the terms are $x, \frac{3x}{4}, \frac{9x}{16}, \frac{27x}{64}, \frac{81}{256}x$

Here, $\frac{3x}{4} - x = \frac{-x}{4}, \frac{9x}{16} - \frac{3x}{4} = \frac{-3x}{16}$

Since, $\left(\frac{-x}{4}\right) \neq \left(\frac{-3x}{16}\right)$.

The above terms are not in A.P.

(iii) Here,the cost of digging for first 1 metre = ₹ 150

The cost of digging for first 2 metres

= ₹ 150 + ₹ 50 = ₹ 200

The cost of digging for first 3 metres

= ₹ 200 + ₹ 50 = ₹ 250 = ₹ 150 + 2 × (₹ 50)

The cost of digging for first 4 metres

= ₹ 250 + ₹ 50 = ₹ 300 = ₹ 150 + 3 × (₹ 50)

We see that the cost of digging a well for each subsequent metre form an A.P., with common difference = 50.

(iv) $\because$ The amount at the end of 1st year

$$= 10000\left(1 + \frac{8}{100}\right)^1$$

The amount at the end of 2nd year $= 10000\left(1 + \frac{8}{100}\right)^2$

The amount at the end of 3rd year $= 10000\left(1 + \frac{8}{100}\right)^3$

The amount at the end of 4th year $= 10000\left(1 + \frac{8}{100}\right)^4$

$\therefore$ The terms are $[10000], \left[10000\left(1 + \frac{8}{100}\right)\right],$

$$\left[10000\left(1 + \frac{8}{100}\right)^2\right], \left[10000\left(1 + \frac{8}{100}\right)^3\right], ...$$

Obviously, $\left[10000\left(1 + \frac{8}{100}\right)\right] - [10000]$

$$\neq \left[10000\left(1 + \frac{8}{100}\right)^2\right] - \left[10000\left(1 + \frac{8}{100}\right)\right]$$

$\therefore$ The above terms are not in A.P.

2. (i) Here, $a = 10$ and $d = 10$

We have, first term, $a = a_1 = 10$

Second term, $a_2 = 10 + 10 = 20$

Third term, $a_3 = 20 + 10 = 30$ and

Fourth term, $a_4 = 30 + 10 = 40$

Thus, the first four terms are 10, 20, 30 and 40.

(ii) Here, $a = -2$ and $d = 0$, we have

Since, $d = 0$, so each term of given A.P. will be same as the first term of the A.P.

Thus, the first four terms of the A.P. are –2, –2, –2 and –2.

(iii) Here, $a = 4$ and $d = -3$,

We have, first term, $a = a_1 = 4$

Second term, $a_2 = 4 + (-3) = 1$

Third term, $a_3 = 1 + (-3) = -2$ and

Fourth term, $a_4 = -2 + (-3) = -5$

Thus, the first four terms are 4, 1, –2 and –5.

(iv) Here, $a = -1$ and $d = 1/2$

We have, first term, $a = a_1 = -1$,

Second term, $a_2 = -1 + \frac{1}{2} = -\frac{1}{2}$,

Third term, $a_3 = -\frac{1}{2} + \frac{1}{2} = 0$ and

Fourth term, $a_4 = 0 + \frac{1}{2} = \frac{1}{2}$

$\therefore$ Thus, the first four terms are $-1, -\frac{1}{2}, 0$ and $\frac{1}{2}$.

(v) Here, $a = -1.25$ and $d = -0.25$

We have, first term, $a = a_1 = -1.25$

Second term, $a_2 = -1.25 + (-0.25) = -1.50$,

Third term, $a_3 = -1.50 + (-0.25) = -1.75$ and

Fourth term, $a_4 = -1.75 + (-0.25) = -2.0$
Thus, the first four terms are –1.25, –1.50, –1.75 and –2.0.

3. (i) We have ; 3, 1, –1, –3, ...
$\because$ $a_1 = 3$ $\therefore$ First term = 3
Also, $a_2 = 1, a_3 = -1, a_4 = -3$
$\therefore$ $a_2 - a_1 = 1 - 3 = -2$
$a_4 - a_3 = -3 - (-1) = -3 + 1 = -2$
$\Rightarrow$ Common difference, $d = -2$

(ii) We have ; –5, –1, 3, 7, ...
$\because$ $a_1 = -5$ $\therefore$ First term = –5
Also, $a_2 = -1, a_3 = 3, a_4 = 7$
$\therefore$ $a_2 - a_1 = -1 - (-5) = -1 + 5 = 4$
and $a_4 - a_3 = 7 - 3 = 4$ $\Rightarrow$ Common difference, $d = 4$

(iii) We have ; $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$
$\because$ $a_1 = \frac{1}{3}$ $\therefore$ First term $= \frac{1}{3}$
Also, $a_2 = \frac{5}{3}, a_3 = \frac{9}{3}, a_4 = \frac{13}{3}$
$\therefore$ $a_2 - a_1 = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$ and $a_4 - a_3 = \frac{13}{3} - \frac{9}{3} = \frac{4}{3}$
$\Rightarrow$ Common difference, $d = 4/3$

(iv) We have ; 0.6, 1.7, 2.8, 3.9, ...
$\because$ $a_1 = 0.6$
$\therefore$ First term = 0.6
Also, $a_2 = 1.7, a_3 = 2.8, a_4 = 3.9$
$\therefore$ $a_2 - a_1 = 1.7 - 0.6 = 1.1$
and $a_4 - a_3 = 3.9 - 2.8 = 1.1$ $\Rightarrow$ Common difference, $d = 1.1$

4. (i) We have ; 2, 4, 8, 16, ...
Here, $a_1 = 2, a_2 = 4, a_3 = 8, a_4 = 16$
$\therefore$ $a_2 - a_1 = 4 - 2 = 2$ and $a_4 - a_3 = 16 - 8 = 8$
Since, $a_2 - a_1 \neq a_4 - a_3$
$\therefore$ The given numbers do not form an A.P.

(ii) We have ; $2, \frac{5}{2}, 3, \frac{7}{2}, ...$
Here, $a_1 = 2, a_2 = \frac{5}{2}, a_3 = 3, a_4 = \frac{7}{2}$
$\therefore$ $a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2}$, $a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$ and
$a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$
$\because$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \frac{1}{2}$
$\Rightarrow$ Common difference, $d = 1/2$
$\therefore$ The given numbers form an A.P.
Now, $a_5 = \frac{7}{2} + \frac{1}{2} = 4$, $a_6 = 4 + \frac{1}{2} = \frac{9}{2}$ and $a_7 = \frac{9}{2} + \frac{1}{2} = 5$

(iii) We have ; –1.2, –3.2, –5.2, –7.2, ...
Here, $a_1 = -1.2, a_2 = -3.2, a_3 = -5.2, a_4 = -7.2$
$\therefore$ $a_2 - a_1 = -3.2 + 1.2 = -2$,
$a_3 - a_2 = -5.2 + 3.2 = -2$ and $a_4 - a_3 = -7.2 + 5.2 = -2$
$\because$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = -2$
$\Rightarrow$ Common difference, $d = -2$
$\therefore$ The given numbers form an A.P.
Now, $a_5 = -7.2 + (-2) = -9.2$,
$a_6 = -9.2 + (-2) = -11.2$ and $a_7 = -11.2 + (-2) = -13.2$

(iv) We have ; –10, –6, –2, 2, ...
Here, $a_1 = -10, a_2 = -6, a_3 = -2, a_4 = 2$
$\therefore$ $a_2 - a_1 = -6 + 10 = 4$,
$a_3 - a_2 = -2 + 6 = 4$ and $a_4 - a_3 = 2 + 2 = 4$
$\because$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = 4$
$\Rightarrow$ Common difference, $d = 4$
$\therefore$ The given numbers form an A.P.
Now, $a_5 = 2 + 4 = 6$,
$a_6 = 6 + 4 = 10$
and $a_7 = 10 + 4 = 14$

(v) We have ; $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, ...$
Here, $a_1 = 3$, $a_2 = 3+\sqrt{2}$, $a_3 = 3+2\sqrt{2}$, $a_4 = 3+3\sqrt{2}$
$\therefore$ $a_2 - a_1 = 3+\sqrt{2} - 3 = \sqrt{2}$,
$a_3 - a_2 = 3+2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$ and
$a_4 - a_3 = 3+3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$
$\because$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \sqrt{2}$
$\Rightarrow$ Common difference, $d = \sqrt{2}$
$\therefore$ The given numbers form an A.P.
Now, $a_5 = 3+3\sqrt{2}+\sqrt{2} = 3+4\sqrt{2}$,
$a_6 = 3+4\sqrt{2}+\sqrt{2} = 3+5\sqrt{2}$ and
$a_7 = 3+5\sqrt{2}+\sqrt{2} = 3+6\sqrt{2}$

(vi) We have ; 0.2, 0.22, 0.222, 0.2222, ...
Here, $a_1 = 0.2, a_2 = 0.22, a_3 = 0.222, a_4 = 0.2222$
$\therefore$ $a_2 - a_1 = 0.22 - 0.2 = 0.02$ and
$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$
Since, $a_2 - a_1 \neq a_4 - a_3$
$\therefore$ The given numbers do not form an A.P.

(vii) We have ; 0, – 4, – 8, – 12, ...
Here, $a_1 = 0, a_2 = -4, a_3 = -8, a_4 = -12$
$\therefore$ $a_2 - a_1 = -4 - 0 = -4$,
$a_3 - a_2 = -8 + 4 = -4$
and $a_4 - a_3 = -12 + 8 = -4$
$\because$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = -4$
$\Rightarrow$ Common difference, $d = -4$
$\therefore$ The given numbers form an A.P.
Now, $a_5 = a_4 + (-4) = -12 + (-4) = -16$
$a_6 = a_5 + (-4) = -16 + (-4) = -20$
and $a_7 = a_6 + (-4) = -20 + (-4) = -24$

(viii) We have ; $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, ...$
Here, $a_1 = a_2 = a_3 = a_4 = -\frac{1}{2}$
$\therefore$ $a_2 - a_1 = 0, a_3 - a_2 = 0, a_4 - a_3 = 0$
$\Rightarrow$ Common difference, $d = 0$
$\therefore$ The given numbers form an A.P.
Now, $a_5 = -\frac{1}{2} + 0 = -\frac{1}{2}$
$a_6 = -\frac{1}{2} + 0 = -\frac{1}{2}$ and $a_7 = -\frac{1}{2} + 0 = -\frac{1}{2}$

(ix) We have ; 1, 3, 9, 27, ...
Here, $\left.\begin{matrix} a_1 = 1 \\ a_2 = 3 \end{matrix}\right\} \Rightarrow a_2 - a_1 = 3 - 1 = 2$

Also, $\left.\begin{array}{l} a_3 = 9 \\ a_4 = 27 \end{array}\right\} \Rightarrow a_4 - a_3 = 27 - 9 = 18$

Since, $a_2 - a_1 \neq a_4 - a_3$

$\therefore$ The given numbers do not form an A.P.

(x) We have ; $a, 2a, 3a, 4a, \ldots\ldots$

Here, $a_1 = a, a_2 = 2a, a_3 = 3a, a_4 = 4a$

$\therefore$ $a_2 - a_1 = 2a - a = a, a_3 - a_2 = 3a - 2a = a$

and $a_4 - a_3 = 4a - 3a = a$

$\because$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = a$

$\Rightarrow$ Common difference, $d = a$

$\therefore$ The given numbers form an A.P.

Now, $a_5 = 4a + a = 5a$,

$a_6 = 5a + a = 6a$ and $a_7 = 6a + a = 7a$

(xi) We have ; $a, a^2, a^3, a^4, \ldots$

Here, $\left.\begin{array}{l} a_1 = a \\ a_2 = a^2 \end{array}\right\} \Rightarrow a_2 - a_1 = a^2 - a = a(a-1)$

Also, $\left.\begin{array}{l} a_3 = a^3 \\ a_4 = a^4 \end{array}\right\} \Rightarrow a_4 - a_3 = a^4 - a^3 = a^3(a-1)$

Since, $a_2 - a_1 \neq a_4 - a_3$

$\therefore$ The given numbers do not form an A.P.

(xii) We have ; $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots$

$a_1 = \sqrt{2}, a_2 = \sqrt{8}, a_3 = \sqrt{18}, a_4 = \sqrt{32}$

$\therefore$ $a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$,

$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$,

and $a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

$\because$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \sqrt{2}$

$\Rightarrow$ Common difference, $d = \sqrt{2}$

$\therefore$ The given numbers form an A.P.

Now, $a_5 = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$,

$a_6 = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$ and

$a_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

(xiii) We have ; $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots\ldots$

Here, $\left.\begin{array}{l} a_1 = \sqrt{3} \\ a_2 = \sqrt{6} \end{array}\right\} \Rightarrow a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$

Also, $\left.\begin{array}{l} a_3 = \sqrt{9} \\ a_4 = \sqrt{12} \end{array}\right\} \Rightarrow a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - 3$

$= \sqrt{3}(2 - \sqrt{3})$

$\because$ $a_2 - a_1 \neq a_4 - a_3$

$\therefore$ The given numbers do not form an A.P.

(xiv) We have ; $1^2, 3^2, 5^2, 7^2, \ldots$

Here, $\left.\begin{array}{l} a_1 = 1^2 = 1 \\ a_2 = 3^2 = 9 \end{array}\right\} \Rightarrow a_2 - a_1 = 9 - 1 = 8$

Also, $\left.\begin{array}{l} a_3 = 5^2 = 25 \\ a_4 = 7^2 = 49 \end{array}\right\} \Rightarrow a_4 - a_3 = 49 - 25 = 24$

Since, $a_2 - a_1 \neq a_4 - a_3$

$\therefore$ The given numbers do not form an A.P.

(xv) We have ; $1^2, 5^2, 7^2, 73, \ldots$

Here, $a_1 = 1^2, a_2 = 5^2, a_3 = 7^2, a_4 = 73$

$\therefore$ $a_2 - a_1 = 25 - 1 = 24, a_3 - a_2 = 49 - 25 = 24$ and $a_4 - a_3 = 73 - 7^2 = 73 - 49 = 24$

$\because$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = 24$

$\Rightarrow$ Common difference, $d = 24$

$\therefore$ The given numbers form an A.P.

Now, $a_5 = 73 + 24 = 97$,

$a_6 = 97 + 24 = 121$ and $a_7 = 121 + 24 = 145$

EXERCISE - 5.2

1. (i) $a_n = a + (n-1)d$

$\Rightarrow$ $a_8 = 7 + (8-1)3 = 7 + 7 \times 3 = 7 + 21$

$\therefore$ $a_8 = 28$

(ii) $a_n = a + (n-1)d$

$\Rightarrow$ $a_{10} = -18 + (10-1)d \Rightarrow 0 = -18 + 9d$

$\Rightarrow$ $9d = 18 \Rightarrow d = 18/9 = 2$

$\therefore$ $d = 2$

(iii) $a_n = a + (n-1)d$

$\Rightarrow$ $a_{18} = a + (18-1) \times (-3) \Rightarrow -5 = a + 17 \times (-3)$

$\Rightarrow$ $-5 = a - 51 \Rightarrow a = -5 + 51 = 46$

$\therefore$ $a = 46$

(iv) $a_n = a + (n-1)d$

$\Rightarrow$ $3.6 = -18.9 + (n-1) \times 2.5$

$\Rightarrow$ $(n-1) \times 2.5 = 3.6 + 18.9$

$\Rightarrow$ $(n-1) \times 2.5 = 22.5 \Rightarrow n - 1 = \dfrac{22.5}{2.5} = 9$

$\Rightarrow$ $n = 9 + 1 = 10$

$\therefore$ $n = 10$

(v) $a_n = a + (n-1)d \Rightarrow a_{105} = 3.5 + (105-1) \times 0$

$\Rightarrow$ $a_{105} = 3.5 + 104 \times 0 \Rightarrow a_{105} = 3.5 + 0 = 3.5$

$\therefore$ $a_{105} = 3.5$

2. (i) (c) : Here, $a = 10$, $n = 30$ and $d = 7 - 10 = -3$

$\because$ $a_n = a + (n-1)d$

$\therefore$ $a_{30} = 10 + (30-1) \times (-3)$

$= 10 + 29 \times (-3) = 10 - 87 = -77$

(ii) (b) : Here, $a = -3$, $n = 11$ and

$d = -\dfrac{1}{2} - (-3) = -\dfrac{1}{2} + 3 = \dfrac{5}{2}$

$\because$ $a_n = a + (n-1)d$

$\therefore$ $a_{11} = -3 + (11-1) \times 5/2 = -3 + 25 = 22$

3. (i) Here, $a = 2, a_3 = 26$

Let common difference $= d$

$\therefore$ $a_n = a + (n-1)d$

$\Rightarrow$ $a_3 = 2 + (3-1)d \Rightarrow 26 = 2 + 2d$

$\Rightarrow$ $2d = 26 - 2 = 24 \Rightarrow d = 24/2 = 12$

$\therefore$ The missing term $= a + d = 2 + 12 = 14$

(ii) Let the first term $= a$

and common difference $= d$

Here, $a_2 = 13$ and $a_4 = 3$

$a_2 = a + d = 13, a_4 = a + 3d = 3$

$\therefore$ $a_4 - a_2 = (a + 3d) - (a + d) = 3 - 13$

$\Rightarrow$ $2d = -10 \Rightarrow d = -10/2 = -5$

Now, $a + d = 13 \Rightarrow a + (-5) = 13$

$\Rightarrow$ $a = 13 + 5 = 18$

Thus, missing terms are a and $a + 2d$

i.e., 18 and 18 + (–10) = 8

(iii) Here, $a = 5$ and $a_4 = 9\frac{1}{2} = \frac{19}{2}$

since, $a_4 = a + 3d$

$\Rightarrow \frac{19}{2} = 5 + 3d \Rightarrow 3d = \frac{19}{2} - 5 = \frac{9}{2}$

$\Rightarrow d = \frac{9}{2} \div 3 = \frac{9}{2} \times \frac{1}{3} = \frac{3}{2}$

$\therefore$ The missing terms are : $a_2 = a + d = 5 + \frac{3}{2} = 6\frac{1}{2}$

and $a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$

(iv) Here, $a = -4$, $a_6 = 6$

$\because$ $a_n = a + (n - 1)d$

$\therefore$ $a_6 = -4 + (6 - 1)d$

$\Rightarrow$ $6 = -4 + 5d \Rightarrow 5d = 10 \Rightarrow d = 2$

$\therefore$ $a_2 = a + d = -4 + 2 = -2$,

$a_3 = a + 2d = -4 + 2(2) = 0$,

$a_4 = a + 3d = -4 + 3(2) = 2$

and $a_5 = a + 4d = -4 + 4(2) = 4$

$\therefore$ The missing terms are – 2, 0, 2 and 4

(v) Here, $a_2 = 38$ and $a_6 = -22$

$\therefore$ $a_2 = a + d = 38$, $a_6 = a + 5d = -22$

$\Rightarrow$ $a_6 - a_2 = a + 5d - (a + d) = -22 - 38$

$\Rightarrow$ $4d = -60 \Rightarrow d = -60/4 = -15$

$\therefore$ $a + d = 38 \Rightarrow a + (-15) = 38$

$\Rightarrow$ $a = 38 + 15 = 53$

Now, $a_3 = a + 2d = 53 + 2(-15) = 53 - 30 = 23$,

$a_4 = a + 3d = 53 + 3(-15) = 53 - 45 = 8$

and $a_5 = a + 4d = 53 + 4(-15) = 53 - 60 = -7$

Thus, missing terms are 53, 23, 8 and –7

4. Let the n^{th} term = 78

Here, $a = 3 \Rightarrow a_1 = 3$ and $a_2 = 8$

$\therefore$ $d = a_2 - a_1 = 8 - 3 = 5$

And, $a_n = a + (n + 1)d$

$\Rightarrow$ $78 = 3 + (n - 1) \times 5 \Rightarrow 78 - 3 = (n - 1) \times 5$

$\Rightarrow$ $75 = (n - 1) \times 5 \Rightarrow (n - 1) = 15 \Rightarrow n = 16$

Thus, 78 is the 16^{th} term of the given A.P.

5. (i) Here, $a = 7$, $d = 13 - 7 = 6$

Let total number of terms be n.

$\therefore$ $a_n = 205$. Now, $a_n = a + (n - 1) \times d$

$\Rightarrow$ $7 + (n - 1) \times 6 = 205$

$\Rightarrow$ $(n - 1) \times 6 = 205 - 7 = 198$

$\therefore$ $n = 33 + 1 = 34$.

Thus, the required number of terms is 34.

(ii) Here, $a = 18$, $d = 15\frac{1}{2} - 18 = \frac{31}{2} - 18 = \frac{-5}{2}$

Let the n^{th} term = –47

$\therefore$ $a_n = a + (n - 1)d$

$\Rightarrow -47 = 18 + (n - 1) \times \left(-\frac{5}{2}\right)$

$\Rightarrow -47 - 18 = (n - 1) \times \left(\frac{-5}{2}\right) \Rightarrow -65 = (n - 1) \times \left(\frac{-5}{2}\right)$

$\Rightarrow n - 1 = -65 \times \left(\frac{-2}{5}\right) \Rightarrow n - 1 = 26$

$\Rightarrow$ $n = 26 + 1 = 27$

Thus, the required number of terms is 27.

6. For the given A.P.,

we have $a = 11$, $d = 8 - 11 = -3$

Let –150 be the n^{th} term of the given A.P.

$\therefore$ $a_n = a + (n - 1)d$

$\Rightarrow$ $-150 = 11 + (n - 1) \times (-3) \Rightarrow -150 - 11 = (n - 1) \times (-3)$

$\Rightarrow -161 = (n - 1) \times (-3) \Rightarrow n - 1 = \frac{-161}{-3} = \frac{161}{3}$

$\Rightarrow n = \frac{161}{3} + 1 = \frac{164}{3}$, which is a fraction

But, n must be a positive integer.

Thus, –150 is not a term of the given A.P.

7. Here, $a_{11} = 38$ and $a_{16} = 73$

If the first term = a and the common difference = d.

Then, $a + (11 - 1)d = 38 \Rightarrow a + 10d = 38$ ...(i)

and $a + (16 - 1)d = 73 \Rightarrow a + 15d = 73$ ...(ii)

Subtracting (i) from (ii), we get

$(a + 15d) - (a + 10d) = 73 - 38$

$\Rightarrow$ $5d = 35 \Rightarrow d = 35/5 = 7$

From (i), $a + 10(7) = 38$

$\Rightarrow$ $a + 70 = 38 \Rightarrow a = 38 - 70 = -32$

$\therefore$ $a_{31} = -32 + (31 - 1) \times 7$

$= -32 + 30 \times 7 = -32 + 210 = 178$

Thus, the 31^{st} term is 178.

8. Here, $n = 50$, $a_3 = 12$, $a_n = 106 \Rightarrow a_{50} = 106$

If the first term = a and the common difference = d

$\therefore$ $a_3 = a + 2d = 12$ ...(i)

$a_{50} = a + 49d = 106$ ...(ii)

Subtracting (i) from (ii), we get

$\Rightarrow$ $a_{50} - a_3 = a + 49d - (a + 2d) = 106 - 12$

$\Rightarrow$ $47d = 94 \Rightarrow d = 94/47 = 2$

From (i), we have $a + 2d = 12$

$\Rightarrow$ $a + 2(2) = 12 \Rightarrow a = 12 - 4 = 8$

Now, $a_{29} = a + (29 - 1)d = 8 + (28) \times 2 = 8 + 56 = 64$

Thus, the 29^{th} term is 64.

9. Here, $a_3 = 4$ and $a_9 = -8$

$\therefore$ $a_n = a + (n - 1)d$

$\Rightarrow$ $a_3 = a + 2d = 4$ ...(i)

$a_9 = a + 8d = -8$ ...(ii)

Subtracting (i) from (ii), we get

$(a + 8d) - (a + 2d) = -8 - 4$

$\Rightarrow$ $6d = -12 \Rightarrow d = -12/6 = -2$

Now, From (i), we have $a + 2d = 4$

$\Rightarrow$ $a + 2(-2) = 4 \Rightarrow a = 4 + 4 = 8$

Let the n^{th} term of the A.P. be 0.

$\therefore$ $a_n = a + (n - 1)d = 0$

$\Rightarrow$ $8 + (n - 1) \times (-2) = 0 \Rightarrow (n - 1) \times (-2) = -8$

$\Rightarrow$ $n - 1 = -8/-2 = 4 \Rightarrow n = 4 + 1 = 5$

Thus, the 5^{th} term of given A.P. is 0.

10. Let a be the first term and d the common difference of the given A.P.

Now, using $a_n = a + (n - 1)d$, we have

$a_{17} = a + 16d$, $a_{10} = a + 9d$

According to the question, $a_{10} + 7 = a_{17}$

$\Rightarrow$ $(a + 9d) + 7 = a + 16d$

$\Rightarrow$ $a + 9d - a - 16d = -7 \Rightarrow -7d = -7 \Rightarrow d = 1$

Thus, the common difference is 1.

11. Here, $a = 3$, $d = 15 - 3 = 12$

Using $a_n = a + (n - 1)d$, we get

$a_{54} = a + 53d = 3 + 53 \times 12 = 3 + 636 = 639$

Let a_n be 132 more than its 54^{th} term.

$\therefore$ $a_n = a_{54} + 132 \Rightarrow a_n = 639 + 132 = 771$

Now, $a_n = 771 \Rightarrow a + (n - 1)d = 771$

$\Rightarrow$ $3 + (n - 1) \times 12 = 771$

$\Rightarrow$ $(n - 1) \times 12 = 771 - 3 = 768$

$\Rightarrow$ $(n - 1) = 768/12 = 64 \Rightarrow n = 64 + 1 = 65$

Thus, 132 more than 54^{th} term is the 65^{th} term.

12. Let for the 1^{st} A.P., the first term = a

$\Rightarrow$ $a_{100} = a + 99d$

And for the 2^{nd} A.P., the first term = a'

$\Rightarrow$ $a'_{100} = a' + 99d$

According to the condition, we have $a_{100} - a'_{100} = 100$

$\Rightarrow$ $a + 99d - (a' + 99d) = 100$

$\Rightarrow$ $a - a' = 100$

Let, $a_{1000} - a'_{1000} = x$

$\therefore$ $a + 999d - (a' + 999d) = x$

$\Rightarrow$ $a - a' = x \Rightarrow x = 100$

$\therefore$ The difference between their 1000^{th} terms is 100.

13. The first three digit number divisible by 7 is 105.

The last such three digit number is 994.

$\therefore$ The A.P. is 105, 112, 119, ..., 994

Here, $a = 105$ and $d = 7$

Let n be the required number of terms.

$\therefore$ $a_n = a + (n - 1)d$

$\Rightarrow$ $994 = 105 + (n - 1) \times 7$

$\Rightarrow$ $(n - 1) \times 7 = 994 - 105 = 889$

$\Rightarrow$ $(n - 1) = 889/7 = 127$

$\Rightarrow$ $n = 127 + 1 = 128$

Thus, there are 128 three-digits numbers which are divisible by 7.

14. The multiple of 4 that lie between 10 and 250 are : 12, 16, ..., 248, which is an A.P.

Here, $a = 12$ and $d = 4$

Let the number of terms = n

$\therefore$ Using $a_n = a + (n - 1)d$, we get

$a_n = 12 + (n - 1) \times 4$

$\Rightarrow$ $248 = 12 + (n - 1) \times 4$

$\Rightarrow$ $(n - 1) \times 4 = 248 - 12 = 236$

$\Rightarrow$ $n - 1 = 236/4 = 59 \Rightarrow n = 59 + 1 = 60$

Thus, the required number of terms = 60.

15. For the 1^{st} A.P.

$a = 63$ and $d = 65 - 63 = 2$

$\therefore$ $a_n = a + (n - 1)d = 63 + (n - 1) \times 2$

For the 2^{nd} A.P.

$a = 3$ and $d = 10 - 3 = 7$

$\therefore$ $a_n = a + (n - 1)d = 3 + (n - 1) \times 7$

Now, according to the question

$3 + (n - 1) \times 7 = 63 + (n - 1) \times 2$

$\Rightarrow$ $(n - 1) \times 7 - (n - 1) \times 2 = 63 - 3$

$\Rightarrow$ $7n - 7 - 2n + 2 = 60$

$\Rightarrow$ $5n - 5 = 60 \Rightarrow 5n = 60 + 5 = 65 \Rightarrow n = 65/5 = 13$

Thus, the 13^{th} terms of the two given A.P.'s are equal.

16. Let the first term = a and the common difference = d

$\therefore$ Using, $a_n = a + (n - 1)d$, we have

$a_3 = a + 2d \Rightarrow a + 2d = 16$ ...(i)

And $a_7 = a + 6d$, $a_5 = a + 4d$

According to the question, $a_7 - a_5 = 12$

$\Rightarrow$ $(a + 6d) - (a + 4d) = 12$

$\Rightarrow$ $a + 6d - a - 4d = 12$

$\Rightarrow$ $2d = 12 \Rightarrow d = 6$ ...(ii)

Now, from (i) and (ii), we have $a + 2(6) = 16$

$\Rightarrow$ $a + 12 = 16 \Rightarrow a = 16 - 12 = 4$

$\therefore$ The required A.P. is 4, [4 + 6], [4 + 2(6)], [4 + 3(6)], ... or 4, 10, 16, 22, ...

17. We have, the last term, $l = 253$

Here, $d = 8 - 3 = 5$

Since the n^{th} term from the last term is given by, $l - (n - 1)d$,

$\therefore$ We have 20^{th} term from the end

$= l - (20 - 1) \times 5 = 253 - 19 \times 5 = 253 - 95 = 158$

18. Let the first term = a and the common difference = d

$\therefore$ Using $a_n = a + (n - 1)d$, we get

$a_4 + a_8 = 24 \Rightarrow (a + 3d) + (a + 7d) = 24$

$\Rightarrow$ $2a + 10d = 24 \Rightarrow a + 5d = 12$ ...(i)

And $a_6 + a_{10} = 44$

$\Rightarrow$ $(a + 5d) + (a + 9d) = 44$

$\Rightarrow$ $2a + 14d = 44 \Rightarrow a + 7d = 22$ ...(ii)

Now, subtracting (i) from (ii), we get

$(a + 7d) - (a + 5d) = 22 - 12$

$\Rightarrow$ $2d = 10 \Rightarrow d = 5$ ...(iii)

$\therefore$ From (i), $a + 5 \times 5 = 12$

$\Rightarrow$ $a = 12 - 25 = -13$

Now, the first three terms of the A.P. are given by a, $(a + d)$, $(a + 2d)$

or -13, $(-13 + 5)$, $[-13 + 2(5)]$ or $-13, -8, -3$.

19. Here, a = ₹ 5000 and d = ₹ 200

Let in the n^{th} year he gets ₹ 7000.

$\therefore$ Using $a_n = a + (n - 1)d$, we get

$7000 = 5000 + (n - 1) \times 200$

$\Rightarrow$ $(n - 1) \times 200 = 7000 - 5000 = 2000$

$\Rightarrow$ $n - 1 = 2000/200 = 10 \Rightarrow n = 10 + 1 = 11$

Thus, the income becomes ₹ 7000 in 11 years *i.e.*, in year 2006.

20. Here, a = ₹ 5 and d = ₹ 1.75

$\because$ In the n^{th} week her savings become ₹ 20.75.

$\therefore$ a_n = ₹ 20.75

$\therefore$ Using $a_n = a + (n - 1)d$, we have

$20.75 = 5 + (n - 1) \times (1.75)$

$\Rightarrow$ $(n - 1) \times 1.75 = 20.75 - 5 \Rightarrow (n - 1) \times 1.75 = 15.75$

$\Rightarrow$ $n - 1 = \dfrac{15.75}{1.75} = 9 \Rightarrow n = 9 + 1 = 10$

Thus, the required number of years = 10.

EXERCISE - 5.3

1. (i) Given A.P. is 2, 7, 12,.... to 10 terms.
Here, $a = 2$, $d = 7 - 2 = 5$, $n = 10$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{10} = \frac{10}{2}[2 \times 2 + (10-1) \times 5]$

$= 5[4 + 9 \times 5] = 5[49] = 245$

Thus, the sum of first 10 terms is 245.

(ii) Given A.P. is – 37, – 33, – 29,...., to 12 terms.
Here $a = -37$, $d = -33 - (-37) = 4$, $n = 12$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{12} = \frac{12}{2}[2(-37) + (12-1) \times 4]$

$= 6[-74 + 11 \times 4] = 6[-74 + 44] = 6 \times [-30] = -180$

Thus, the sum of first 12 terms = –180.

(iii) Given A.P. is 0.6, 1.7, 2.8,...., to 100 terms.
Here, $a = 0.6$, $d = 1.7 - 0.6 = 1.1$, $n = 100$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{100} = \frac{100}{2}[2(0.6) + (100-1) \times 1.1]$

$= 50[1.2 + 99 \times 1.1] = 50[1.2 + 108.9]$

$= 50[110.1] = 5505$

Thus, the sum of first 100 terms is 5505.

(iv) Given A.P. is $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$, to 11 terms.

Here, $a = \frac{1}{15}$, $d = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$, $n = 11$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{11} = \frac{11}{2}\left[\left(2 \times \frac{1}{15}\right) + (11-1) \times \frac{1}{60}\right]$

$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right] = \frac{11}{2} \times \frac{9}{30} = \frac{99}{60} = \frac{33}{20}$

Thus, the sum of first 11 terms = 33/20.

2. (i) The given numbers are : $7, 10\frac{1}{2}, 14, \ldots, 84$

Here, $a = 7$, $d = 10\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}$, $l = 84$

Let n be the number of terms then, $a_n = a + (n-1)d$

$\Rightarrow \quad 84 = 7 + (n-1) \times \frac{7}{2} \Rightarrow (n-1) \times \frac{7}{2} = 84 - 7 = 77$

$\Rightarrow \quad n - 1 = 77 \times \frac{2}{7} = 22 \Rightarrow n = 22 + 1 = 23$

Now, $S_n = \frac{n}{2}(a + l)$

$\therefore \quad S_{23} = \frac{23}{2}(7 + 84) = \frac{23}{2} \times 91 = \frac{2093}{2} = 1046\frac{1}{2}$

Thus, the required sum is $1046\frac{1}{2}$.

(ii) The given numbers are : 34, 32, 30,..., 10
Here, $a = 34$, $d = 32 - 34 = -2$, $l = 10$
Let the number of terms be n.
then, $a_n = a + (n-1)d$

$\Rightarrow \quad 10 = 34 + (n-1) \times (-2) \Rightarrow (n-1) \times (-2) = -24$

$\Rightarrow \quad n - 1 = \frac{-24}{-2} = 12 \Rightarrow n = 13$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{13} = \frac{13}{2}[68 + 12 \times (-2)] = \frac{13}{2}[68 - 24]$

$= \frac{13}{2}[44] = 13 \times 22 = 286$

Thus, the required sum is 286.

(iii) The given numbers are : – 5, – 8, –11, ..., – 230
Here, $a = -5$, $d = -8 - (-5) = -3$, $l = -230$
Let n be the number of terms.
then, $a_n = a + (n-1)d$

$\Rightarrow \quad -230 = -5 + (n-1) \times (-3)$

$\Rightarrow \quad (n-1) \times (-3) = -230 + 5 = -225$

$\Rightarrow \quad n - 1 = \frac{-225}{-3} = 75 \Rightarrow n = 75 + 1 = 76$

Now, $S_n = \frac{n}{2}[a + l]$

So, $S_{76} = \frac{76}{2}[(-5) + (-230)] = 38 \times (-235) = -8930.$

$\therefore$ The required sum is – 8930.

3. (i) Here, $a = 5$, $d = 3$ and $a_n = 50 = l$

$\because \quad a_n = a + (n-1)d \Rightarrow 50 = 5 + (n-1) \times 3$

$\Rightarrow \quad 50 - 5 = (n-1) \times 3 \Rightarrow (n-1) \times 3 = 45$

$\Rightarrow \quad (n-1) = \frac{45}{3} = 15 \Rightarrow n = 15 + 1 = 16$

Now, $S_n = \frac{n}{2}(a + l) \Rightarrow S_{16} = \frac{16}{2}(5 + 50) = 8(55) = 440$

Thus, $n = 16$ and $S_n = 440$

(ii) Here, $a = 7$ and $a_{13} = 35 = l$

$\therefore \quad a_{13} = a + (13-1)d \Rightarrow 35 = 7 + (13-1)d$

$\Rightarrow \quad 35 - 7 = 12d \Rightarrow 28 = 12d \Rightarrow d = \frac{28}{12} = \frac{7}{3}$

Now, $S_n = \frac{n}{2}(a + l)$

$\Rightarrow S_{12} = \frac{12}{2}(4 + 37) \quad = \frac{13}{2} \times 42 = 13 \times 21 = 273$

Thus, $S_{13} = 273$ and $d = \frac{7}{3}$

(iii) Here, $a_{12} = 37 = l$ and $d = 3$
Let the first term of the A.P. be a.
Now, $a_{12} = a + (12-1)d$

$\Rightarrow \quad 37 = a + 11d \Rightarrow 37 = a + 11 \times 3$

$\Rightarrow \quad 37 = a + 33 \Rightarrow a = 37 - 33 = 4$

Now, $S_n = \frac{n}{2}(a + l) \Rightarrow S_{12} = \frac{12}{2}(4 + 37) = 6 \times (41) = 246$

Thus, $a = 4$ and $S_{12} = 246$.

(iv) Here, $a_3 = 15$ and $S_{10} = 125$
Let the first term of the A.P. be a and d be the common difference.

$\therefore \quad a_3 = a + 2d \Rightarrow a + 2d = 15$ ...(i)

Again, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{10} = \frac{10}{2}[2a + (10-1)d]$

$\Rightarrow \quad 125 = 5[2a + 9d] \Rightarrow 2a + 9d = \frac{125}{5} = 25$

$\Rightarrow \quad 2a + 9d = 25$ ...(ii)

Multiplying (i) by 2 and subtracting (ii) from it, we get

$2a + 4d - 2a - 9d = 30 - 25$

$\Rightarrow \quad -5d = 5 \Rightarrow d = -1.$

$\therefore$ From (i), $a + 2(-1) = 15$

$\Rightarrow \quad a = 17$

Now, $a_{10} = a + (10-1)d = 17 + 9 \times (-1) = 17 - 9 = 8$

Thus, $d = -1$ and $a_{10} = 8$

(v) Here, $d = 5$ and $S_9 = 75$

Let the first term of the A.P. is a.

$\therefore \quad S_9 = \frac{9}{2}[2a + (9-1) \times 5] \Rightarrow 75 = \frac{9}{2}[2a + 40]$

$\Rightarrow \quad 75 \times \frac{2}{9} = 2a + 40 \Rightarrow \frac{50}{3} = 2a + 40$

$\Rightarrow \quad 2a = \frac{50}{3} - 40 = \frac{-70}{3} \Rightarrow a = \frac{-70}{3} \times \frac{1}{2} = \frac{-35}{3}$

Now, $a_9 = a + (9-1)d$

$= \frac{-35}{3} + (8 \times 5) = \frac{-35}{3} + 40 = \frac{-35 + 120}{3} = \frac{85}{3}$

Thus, $a = \frac{-35}{3}$ and $a_9 = \frac{85}{3}$.

(vi) Here, $a = 2$, $d = 8$ and $S_n = 90$

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad 90 = \frac{n}{2}[2 \times 2 + (n-1) \times 8]$

$\Rightarrow \quad 90 \times 2 = 4n + n(n-1) \times 8 \Rightarrow 180 = 4n + 8n^2 - 8n$

$\Rightarrow \quad 180 = 8n^2 - 4n \Rightarrow 45 = 2n^2 - n$

$\Rightarrow \quad 2n^2 - n - 45 = 0 \Rightarrow 2n^2 - 10n + 9n - 45 = 0$

$\Rightarrow \quad 2n(n-5) + 9(n-5) = 0 \Rightarrow (2n+9)(n-5) = 0$

$\therefore$ Either, $2n + 9 = 0 \Rightarrow n = -9/2$

Or $\quad n - 5 = 0 \Rightarrow n = 5$

But $n = -\frac{9}{2}$ is not possible, so $n = 5$

Now, $a_n = a + (n-1)d$

$\Rightarrow \quad a_5 = 2 + (5-1) \times 8 = 2 + 32 = 34$

Thus, $n = 5$ and $a_5 = 34$

(vii) Here, $a = 8$, $a_n = 62 = l$ and $S_n = 210$

Let the common difference $= d$

Now, $S_n = \frac{n}{2}(a + l) \Rightarrow 210 = \frac{n}{2}(8 + 62) = \frac{n}{2} \times 70 = 35n$

$\therefore \quad n = \frac{210}{35} = 6$

Again, $a_n = a + (n-1)d$

$\Rightarrow \quad 62 = 8 + (6-1) \times d \Rightarrow 62 - 8 = 5d$

$\Rightarrow \quad 54 = 5d \Rightarrow d = \frac{54}{5}$. Thus, $n = 6$ and $d = \frac{54}{5}$.

(viii) Here, $a_n = 4$, $d = 2$ and $S_n = -14$

Let the first term be 'a'.

$\because \quad a_n = 4 \quad \therefore \quad a + (n-1)2 = 4 \Rightarrow a = 4 - 2n + 2$

$\Rightarrow \quad a = 6 - 2n$ ...(i)

Also, $S_n = \frac{n}{2}(a + l) \Rightarrow -14 = \frac{n}{2}(a + 4)$

$\Rightarrow \quad n(a + 4) = -28$ ...(ii)

Substituting the value of a from (i) into (ii), we get

$n[6 - 2n + 4] = -28$

$\Rightarrow \quad n[10 - 2n] = -28 \Rightarrow 2n[5 - n] = -28$

$\Rightarrow \quad n(5 - n) = -14 \Rightarrow 5n - n^2 + 14 = 0$

$\Rightarrow \quad n^2 - 5n - 14 = 0 \Rightarrow (n-7)(n+2) = 0$

$\therefore$ Either, $n - 7 = 0 \Rightarrow n = 7$

Or $\quad n + 2 = 0 \Rightarrow n = -2$

But n cannot be negative, so $n = 7$

Now, from (i), we have $a = 6 - 2 \times 7 \Rightarrow a = -8$

Thus, $a = -8$ and $n = 7$

(ix) Here, $a = 3$, $n = 8$ and $S_n = 192$

Let d be the common difference.

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d] \quad \therefore \quad 192 = \frac{8}{2}[2(3) + (8-1)d]$

$\Rightarrow \quad 192 = 4[6 + 7d] \Rightarrow 192 = 24 + 28d$

$\Rightarrow \quad 28d = 192 - 24 = 168 \Rightarrow d = 6$

Thus, $d = 6$.

(x) Here, $l = 28$ and $S_9 = 144$

Let the first term be 'a'.

Thus $S_n = \frac{n}{2}(a + l)$

$\Rightarrow \quad S_9 = \frac{9}{2}(a + 28) \Rightarrow 144 = \frac{9}{2}(a + 28)$

$\Rightarrow \quad a + 28 = 144 \times \frac{2}{9} = 16 \times 2 = 32 \Rightarrow a = 32 - 28 = 4$

Thus, $a = 4$.

4. Here, $a = 9$, $d = 17 - 9 = 8$ and $S_n = 636$

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d] = 636$

$\therefore \quad \frac{n}{2}[(2 \times 9) + (n-1) \times 8] = 636$

$\Rightarrow \quad n[18 + (n-1) \times 8] = 1272 \Rightarrow 18n + 8n^2 - 8n = 1272$

$\Rightarrow \quad 8n^2 + 10n = 1272 \Rightarrow 4n^2 + 5n - 636 = 0$

$\Rightarrow \quad 4n^2 - 48n + 53n - 636 = 0$

$\Rightarrow \quad 4n(n-12) + 53(n-12) = 0$

$\Rightarrow \quad (n-12)(4n+53) = 0 \Rightarrow n = 12, -53/4$

As n can't be negative.

$\therefore$ Required number of terms = 12.

5. Here, $a = 5$, $l = 45 = a_n$, $S_n = 400$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad 45 = 5 + (n-1)d$

$\Rightarrow \quad (n-1)d = 45 - 5 \Rightarrow (n-1)d = 40$ ...(i)

Also $S_n = \frac{n}{2}(a + l) \Rightarrow 400 = \frac{n}{2}(5 + 45) \Rightarrow 400 \times 2 = n \times 50$

$\Rightarrow \quad n = \frac{400 \times 2}{50} = 16$

From (i), we get $(16-1)d = 40 \Rightarrow 15d = 40 \Rightarrow d = 8/3$

6. We have, first term $a = 17$, last term, $l = 350 = a_n$ and common difference $d = 9$

Let the number of terms be n.

$\because \quad a_n = a + (n-1)d$

$\therefore$ $350 = 17 + (n - 1) \times 9 \Rightarrow (n - 1) \times 9 = 350 - 17 = 333$

$\Rightarrow$ $n - 1 = 333/9 = 37 \Rightarrow n = 37 + 1 = 38$

Since, $S_n = \frac{n}{2}(a + l)$

$\therefore$ $S_{38} = \frac{38}{2}(17 + 350) = 19(367) = 6973$

Thus, $n = 38$ and $S_n = 6973$.

7. Here, $n = 22$, $a_{22} = 149 = l$, $d = 7$

Let the first term of the A.P. be a.

$\because$ $a_n = a + (n - 1)d$

$\therefore$ $a_{22} = a + (22 - 1) \times 7 \Rightarrow a + 21 \times 7 = 149$

$\Rightarrow$ $a + 147 = 149 \Rightarrow a = 149 - 147 = 2$

Now, $S_{22} = \frac{n}{2}[a + l] \Rightarrow S_{22} = \frac{22}{2}[2 + 149] = 11[151] = 1661$

Thus, $S_{22} = 1661$.

8. Here, $n = 51$, $a_2 = 14$ and $a_3 = 18$

Let the first term of the A.P. be a and the common difference is d.

We have $a_2 = a + d \Rightarrow a + d = 14$ ...(i)

$a_3 = a + 2d \Rightarrow a + 2d = 18$ ...(ii)

Subtracting (i) from (ii), we get

$a + 2d - a - d = 18 - 14 \Rightarrow d = 4$

From (i), we get

$a + 4 = 14 \Rightarrow a = 14 - 4 = 10$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow$ $S_{51} = \frac{51}{2}[(2 \times 10) + (51 - 1) \times 4]$

$= \frac{51}{2}[20 + 200] = \frac{51}{2}[220] = 51 \times 110 = 5610$

Thus, the sum of 51 terms is 5610.

9. Here, we have $S_7 = 49$ and $S_{17} = 289$

Let the first term of the A.P. be 'a' and 'd' be the common difference, then

$S_n = \frac{n}{2}[2a + (n - 1)d] \Rightarrow S_7 = \frac{7}{2}[2a + (7 - 1)d] = 49$

$\Rightarrow$ $7(2a + 6d) = 2 \times 49 = 98$

$\Rightarrow$ $2a + 6d = \frac{98}{7} = 14 \Rightarrow 2[a + 3d] = 14$

$\Rightarrow$ $a + 3d = \frac{14}{2} = 7 \Rightarrow a + 3d = 7$ ...(i)

Also, $S_{17} = \frac{17}{2}[2a + (17 - 1)d] = 289$

$\Rightarrow$ $\frac{17}{2}(2a + 16d) = 289$

$\Rightarrow$ $a + 8d = \frac{289}{17} = 17 \Rightarrow a + 8d = 17$ ...(ii)

Subtracting (i) from (ii), we have

$a + 8d - a - 3d = 17 - 7$

$\Rightarrow$ $5d = 10 \Rightarrow d = 2$

Now, from (i), we have

$a + 3(2) = 7 \Rightarrow a = 7 - 6 = 1$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}[2 \times 1 + (n - 1) \times 2]$

$= \frac{n}{2}[2 + 2n - 2] = \frac{n}{2}[2n] = n \times n = n^2$

Thus, the required sum of n terms $= n^2$.

10. (i) Here, $a_n = 3 + 4n$

Putting $n = 1, 2, 3, 4, ..., n$, we get

$a_1 = 3 + 4(1) = 7$

$a_2 = 3 + 4(2) = 11$

$a_3 = 3 + 4(3) = 15$

$a_4 = 3 + 4(4) = 19$

....

$a_n = 3 + 4n$

$\therefore$ The A.P. in which $a = 7$ and $d = 11 - 7 = 4$ is 7, 11, 15, 19, ..., $(3 + 4n)$.

Now, $S_{15} = \frac{15}{2}[(2 \times 7) + (15 - 1) \times 4]$

$= \frac{15}{2}[14 + (14 \times 4)] = \frac{15}{2}[14 + 56] = \frac{15}{2}[70]$

$= 15 \times 35 = 525$

(ii) Here, $a_n = 9 - 5n$

Putting $n = 1, 2, 3, 4, ..., n$, we get

$a_1 = 9 - 5(1) = 4$

$a_2 = 9 - 5(2) = -1$

$a_3 = 9 - 5(3) = -6$

$a_4 = 9 - 5(4) = -11$

....

$a_n = 9 - 5n$

$\therefore$ The A.P. is $4, -1, -6, -11, ..., 9 - 5n$ having first term as 4 and $d = -1 - 4 = -5$

$\therefore$ $S_{15} = \frac{15}{2}[(2 \times 4) + (15 - 1) \times (-5)]$

$= \frac{15}{2}[8 + 14 \times (-5)] = \frac{15}{2}[8 - 70] = \frac{15}{2} \times (-62)$

$= 15 \times (-31) = -465.$

11. We have $S_n = 4n - n^2$

$\therefore$ $S_1 = 4(1) - (1)^2 = 4 - 1 = 3 \Rightarrow$ First term = 3

$S_2 = 4(2) - (2)^2 = 8 - 4 = 4$

$\Rightarrow$ Sum of first two terms = 4

$\therefore$ Second term $(S_2 - S_1) = 4 - 3 = 1$

$S_3 = 4(3) - (3)^2 = 12 - 9 = 3$

$\Rightarrow$ Sum of first 3 terms = 3

$\therefore$ Third term $(S_3 - S_2) = 3 - 4 = -1$

$S_9 = 4(9) - (9)^2 = 36 - 81 = -45$

$S_{10} = 4(10) - (10)^2 = 40 - 100 = -60$

$\therefore$ Tenth term $= S_{10} - S_9 = [-60] - [-45] = -15$

Now, $S_n = 4(n) - (n)^2 = 4n - n^2$

Also, $S_{n-1} = 4(n - 1) - (n - 1)^2$

$= 4n - 4 - [n^2 - 2n + 1]$

$= 4n - 4 - n^2 + 2n - 1 = 6n - n^2 - 5$

$\therefore$ n^{th} term $= S_n - S_{n-1} = [4n - n^2] - [6n - n^2 - 5]$

$= 4n - n^2 - 6n + n^2 + 5 = 5 - 2n$

Thus, $S_1 = 3$ and $a_1 = 3$
$S_2 = 4$ and $a_2 = 1$
$S_3 = 3$ and $a_3 = -1$
$a_{10} = -15$ and $a_n = 5 - 2n$

12. $\because$ The first 40 positive integers divisible by 6 are 6, 12, 18, ..., (6 × 40)
And, these numbers are in A.P., such that $a = 6$
$d = 12 - 6 = 6$ and $a_{40} = 6 \times 40 = 240 = l$

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{40} = \frac{40}{2}[(2 \times 6) + (40-1) \times 6]$
$= 20[12 + 39 \times 6] = 20[12 + 234]$
$= 20 \times 246 = 4920$

13. The first 15 multiples of 8 are 8, (8 × 2), (8 × 3), (8 × 4), ..., (8 × 15) or 8, 16, 24, 32, ..., 120.
These number are in A.P., where $a = 8$ and $l = 120$

$\therefore \quad S_{15} = \frac{15}{2}[a + l] = \frac{15}{2}[8 + 120]$
$= \frac{15}{2} \times 128 = 15 \times 64 = 960$

Thus, the sum of first 15 multiples of 8 is 960.

14. Odd numbers between 0 and 50 are 1, 3, 5, 7, ..., 49.
These numbers are in A.P. such that $a = 1$ and $l = 49$
Here, $d = 3 - 1 = 2 \;\therefore\; a_n = a + (n-1)d$
$\Rightarrow \; 49 = 1 + (n-1)2 \Rightarrow 49 - 1 = (n-1)2$
$\Rightarrow \; (n-1) = \frac{48}{2} = 24 \quad \therefore \; n = 24 + 1 = 25$

Now, $S_{25} = \frac{25}{2}[1 + 49] = \frac{25}{2}[50] = 25 \times 25 = 625$

Thus, the sum of odd numbers between 0 and 50 is 625.

15. Here, penalty for delay on
1st day = ₹ 200
2nd day = ₹ 250
3rd day = ₹ 300
....
....

Now, 200, 250, 300, ... are in A.P. such that
$a = 200$, $d = 250 - 200 = 50$

$\therefore \quad S_{30}$ is given by $S_{30} = \frac{30}{2}[2(200) + (30-1) \times 50]$
$\left[\text{Using } S_n = \frac{n}{2}[2a + (n-1)d]\right]$
$= 15[400 + 29 \times 50] = 15[400 + 1450]$
$= 15 \times 1850 = 27750$

Thus, penalty for the delay for 30 days is ₹ 27750.

16. Sum of all the prizes = ₹ 700
Let the first prize = a
$\therefore$ 2nd prize = $(a - 20)$
3rd prize = $(a - 40)$
4th prize = $(a - 60)$
....

Thus, we have, first term = a
Common difference = –20
Sum of 7 terms, $S_7 = 700$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 700 = \frac{7}{2}[2(a) + (7-1) \times (-20)]$

$\Rightarrow \quad 700 = \frac{7}{2}[2a + 6 \times (-20)] \Rightarrow 700 \times \frac{2}{7} = 2a - 120$

$\Rightarrow \quad 200 = 2a - 120 \Rightarrow 2a = 320 \Rightarrow a = 320/2 = 160$

Thus, the values of the seven prizes are ₹ 160, ₹(160 – 20), ₹(160 – 40), ₹(160 – 60), ₹(160 – 80), ₹(160 – 100) and ₹(160 – 120) = ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60 and ₹ 40.

17. Number of classes = 12
$\because$ Each class has 3 sections.
$\therefore$ Number of plants planted by class I = 1 × 3 = 3
Number of plants planted by class II = 2 × 3 = 6
Number of plants planted by class III = 3 × 3 = 9
Number of plants planted by class IV = 4 × 3 = 12

Number of plants planted by class XII = 12 × 3 = 36
Thus, the numbers 3, 6, 9, 12, ..., 36 are in A.P.
Here, $a = 3$ and $d = 6 - 3 = 3$
$\because$ Number of classes = 12 *i.e.*, $n = 12$
$\therefore$ Sum the n terms of the above A.P., is given by

$S_{12} = \frac{12}{2}[2(3) + (12-1)3] \quad \left[\text{Using } S_n = \frac{n}{2}[2a + (n-1)d]\right]$
$= 6[6 + 11 \times 3] = 6[6 + 33] = 6 \times 39 = 234$

Thus, the total number of trees = 234.

18. Length of a semi-circle = Semi-circumference
$= \frac{1}{2}(2\pi r) = \pi r$

$\therefore \quad l_1 = \pi r_1 = 0.5\,\pi$ cm $= 1 \times 0.5\,\pi$ cm
$l_2 = \pi r_2 = 1.0\,\pi$ cm $= 2 \times 0.5\,\pi$ cm
$l_3 = \pi r_3 = 1.5\,\pi$ cm $= 3 \times 0.5\,\pi$ cm
$l_4 = \pi r_4 = 2.0\,\pi$ cm $= 4 \times 0.5\,\pi$ cm
....
$l_{13} = \pi r_{13}$ cm $= 6.5\,\pi$ cm $= 13 \times 0.5\,\pi$ cm

Now, length of the spiral = $l_1 + l_2 + l_3 + l_4 + \ldots\ldots + l_{13}$
$= 0.5\pi[1 + 2 + 3 + 4 + \ldots\ldots + 13]$ cm ...(i)
$\because$ 1, 2, 3, 4, ..., 13 are in A.P. such that
$a = 1$ and $l = 13$

$\therefore \quad S_{13} = \frac{13}{2}[1 + 13] \quad \left[\text{Using } S_n = \frac{n}{2}(a + l)\right]$
$= \frac{13}{2} \times 14 = 13 \times 7 = 91$

$\therefore$ From (i), we have
Total length of the spiral = $0.5\pi[91]$ cm
$= \frac{5}{10} \times \frac{22}{7} \times 91 \text{ cm} = 11 \times 13 \text{ cm} = 143 \text{ cm}$

19. The number of logs in
1st row = 20, 2nd row = 19 and 3rd row = 18
Obviously, the numbers 20, 19, 18, ..., are in A.P., such that $a = 20$, $d = 19 - 20 = -1$
Let the number of rows be n.

Since, $S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow\ 200 = \frac{n}{2}[2(20)+(n-1)\times(-1)] \Rightarrow 200 = \frac{n}{2}[40-(n-1)]$

$\Rightarrow\ 200 = \frac{n}{2}[40-(n-1)]$

$\Rightarrow\ 2 \times 200 = n \times 40 - n(n-1)$

$\Rightarrow\ 400 = 40n - n^2 + n \Rightarrow n^2 - 41n + 400 = 0$

$\Rightarrow\ n^2 - 16n - 25n + 400 = 0$

$\Rightarrow\ n(n-16) - 25(n-16) = 0$

$\Rightarrow\ (n-16)(n-25) = 0$

Either $n - 16 = 0 \Rightarrow n = 16$

Or $n - 25 = 0 \Rightarrow n = 25$

$a_n = 0 \Rightarrow a + (n-1)d = 0$

$\Rightarrow\ 20 + (n-1)\times(-1) = 0 \Rightarrow n - 1 = 20$

$\Rightarrow\ n = 21$ *i.e.*, 21st term becomes 0

$\therefore\ n = 25$ is not required.

$\therefore$ Number of rows = 16

Now, $a_{16} = a + (16-1)d = 20 + 15 \times (-1) = 20 - 15 = 5$

$\therefore$ Number of logs in the 16th (top) row is 5.

20. Here, number of potatoes = 10

The up-down distance of the bucket :

From the 1st potato = [5m] × 2 = 10 m

From the 2nd potato = [(5 + 3)m] × 2 = 16 m

From the 3rd potato = [(5 + 3 + 3)m] × 2 = 22 m

From the 4th potato = [(5 + 3 + 3 + 3)m] × 2 = 28 m

....

$\because$ 10, 16, 22, 28, ... are in A.P. such that

$a = 10$ and $d = 16 - 10 = 6$

$\therefore$ Using $S_n = \frac{n}{2}[2a+(n-1)d]$, we have

$$S_{10} = \frac{10}{2}[2(10)+(10-1)\times 6] = 5[20+54] = 5 \times 74 = 370$$

Thus, the sum of above distance = 370 m.

$\Rightarrow$ The competitor has to run a total distance of 370 m.

EXERCISE - 5.4

1. We have the A.P. having $a = 121$ and $d = 117 - 121 = -4$

Now, $a_n = a + (n-1)d = 121 + (n-1) \times (-4)$

$= 121 - 4n + 4 = 125 - 4n$

For the first negative term, we have $a_n < 0$

$\Rightarrow\ (125 - 4n) < 0 \Rightarrow 125 < 4n$

$\Rightarrow\ \frac{125}{4} < n \Rightarrow 31\frac{1}{4} < n$ or $n > 31\frac{1}{4}$

Thus, the first negative term is 32nd term.

2. Here, $a_3 + a_7 = 6$ and $a_3 \times a_7 = 8$

Let the first term = a and the common difference = d

$\therefore\ a_3 = a + 2d$ and $a_7 = a + 6d$

$\because\ a_3 + a_7 = 6$

$\therefore\ (a + 2d) + (a + 6d) = 6$

$\Rightarrow\ 2a + 8d = 6 \Rightarrow a + 4d = 3$ ...(i)

Again, $a_3 \times a_7 = 8$

$\therefore\ (a + 2d) \times (a + 6d) = 8$

$\Rightarrow\ [(a + 4d) - 2d] \times [(a + 4d) + 2d] = 8$

$\Rightarrow\ (3 - 2d) \times (3 + 2d) = 8$ [Using (i)]

$\Rightarrow\ 3^2 - (2d)^2 = 8 \Rightarrow 9 - 4d^2 = 8$

$\Rightarrow\ -4d^2 = 8 - 9 = -1$

$\Rightarrow\ d^2 = \frac{-1}{-4} = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2}.$

Case-I When $d = \frac{1}{2}$, from (i), we have

$a + 2 = 3 \Rightarrow a = 3 - 2 = 1$

Now, using $S_n = \frac{n}{2}[2a+(n-1)d]$, we get

The sum of first 16 terms,

$$S_{16} = \frac{16}{2}\left[2(1)+(16-1)\times\frac{1}{2}\right] = 8\left[2+\frac{15}{2}\right] = 16 + 60 = 76$$

Case-II When $d = -\frac{1}{2}$, from (i), we have

$a + 4\left(-\frac{1}{2}\right) = 3 \Rightarrow a - 2 = 3 \Rightarrow a = 5$

So, the sum of first 16 terms,

$$S_{16} = \frac{16}{2}\left[2(5)+(16-1)\times\left(-\frac{1}{2}\right)\right]$$

$$= 8\left[10+\left(\frac{-15}{2}\right)\right] = 80 - 60 = 20$$

3. Distance between bottom and top rungs $= 2\frac{1}{2}$ m $= \frac{5}{2}\times 100$ cm $= 250$ cm

Distance between two consecutive rungs = 25 cm

$\therefore$ Number of rungs, $n = 250/25 + 1 = 10 + 1 = 11$

Length of the 1st rung (bottom rung) = 45 cm

Length of the 11th rung (top rung) = 25 cm

Let the length of each successive rung decrease by x cm.

$\therefore$ Total length of the rungs = 45 cm + $(45 - x)$ cm + $(45 - 2x)$ cm + ... + 25 cm

Here, the number 45, $(45 - x)$, $(45 - 2x)$, ..., 25 are in an A.P. such that first term, $a = 45$ and last term, $l = 25$

Number of terms, $n = 11$

$\therefore$ Using, $S_n = \frac{n}{2}[a+l]$, we have $S_{11} = \frac{11}{2}[45+25]$

$\Rightarrow\ S_{11} = \frac{11}{2}\times 70 \Rightarrow S_{11} = 11\times 35 = 385$

$\therefore$ Total length of 11 rungs = 385 cm *i.e.*, Length of wood required for the rungs is 385 cm.

4. We have the following consecutive numbers on the houses of a row; 1, 2, 3, 4, 5, ..., 49.

These numbers are in A.P., such that $a = 1$, $d = 2 - 1 = 1$, $n = 49$

Let one of the houses be numbered as x

$\therefore$ Number of houses preceding it = $x - 1$

Number of houses following it = $49 - x$

Now, the sum of the house-numbers preceding x is

$$S_{x-1} = \frac{x-1}{2}[2(1)+(x-1-1)\times 1]$$
$$= \frac{x-1}{2}[2+x-2] = \frac{x(x-1)}{2} = \frac{x^2}{2} - \frac{x}{2}$$

The houses beyond x are numbered as $(x + 1)$, $(x + 2)$, $(x + 3)$, ..., 49

∴ For these house numbers (which are in an A.P.)

First term, $a = x + 1$

Last term, $l = 49$

∴ Using $S_n = \frac{n}{2}[a+l]$, we have

$$S_{49-x} = \frac{49-x}{2}[(x+1)+49]$$
$$= \frac{49-x}{2}[x+50] = \frac{49x}{2} - \frac{x^2}{2} + (49\times 25) - 25x$$
$$= \left(\frac{49x}{2} - 25x\right) - \frac{x^2}{2} + (49\times 25) = \frac{-x}{2} - \frac{x^2}{2} + (49\times 25)$$

Now, [Sum of house numbers preceding x] = [Sum of house numbers following x]

i.e., $S_{x-1} = S_{49-x}$

$$\Rightarrow \quad \frac{x^2}{2} - \frac{x}{2} = \frac{-x}{2} - \frac{x^2}{2} + (49\times 25)$$
$$\Rightarrow \quad \left(\frac{x^2}{2} + \frac{x^2}{2}\right) - \frac{x}{2} + \frac{x}{2} = (49\times 25) \Rightarrow \frac{2x^2}{2} = (49\times 25)$$
$$\Rightarrow \quad x^2 = (49 \times 25) \Rightarrow x = \pm\sqrt{49\times 25}$$
$$\Rightarrow \quad x = \pm(7 \times 5) = \pm 35$$

But x cannot be taken as negative.

∴ $x = 35$.

5. **For 1st step :** Length = 50 m, Breadth = 1/2 m, Height = 1/4 m

∴ Volume of concrete required to build the 1st step

= Volume of the cuboidal step

= Length × breadth × height

$$= 50\times\frac{1}{2}\times\frac{1}{4}\text{ m}^3 = \frac{25}{4}\times 1\text{ m}^3$$

For 2nd step : Length = 50 m, Breadth = 1/2 m, Height $= \left(\frac{1}{4}+\frac{1}{4}\right)\text{m} = 2\times\frac{1}{4}\text{m}$

∴ Volume of concrete required to build the 2nd step

$$= 50\times\frac{1}{2}\times\frac{1}{4}\times 2\text{ m}^3 = \frac{25}{4}\times 2\text{ m}^3$$

For 3rd step : Length = 50 m, Breadth = 1/2 m, Height $= \left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)\text{m} = 3\times\frac{1}{4}\text{m}$

∴ Volume of concrete required to build the 3rd step

$$= 50\times\frac{1}{2}\times\frac{1}{4}\times 3\text{ m}^3 = \frac{25}{4}\times 3\text{ m}^3$$

....

Thus, the volumes (in m^3) of concrete required to build the various steps are :

$\left(\frac{25}{4}\times 1\right), \left(\frac{25}{4}\times 2\right), \left(\frac{25}{4}\times 3\right)$,........ obviously, these numbers form an A.P. such that $a = 25/4$

$$d = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$$

Here, total number of steps, $n = 15$

Total volume of concrete required to build 15 steps is given by the sum of their individual volumes.

On using $S_n = \frac{n}{2}[2a+(n-1)d]$, we have

$$S_{15} = \frac{15}{2}\left[2\left(\frac{25}{4}\right) + (15-1)\times\frac{25}{4}\right]$$
$$= \frac{15}{2}\left[\frac{25}{2} + 14\times\frac{25}{4}\right] = \frac{15}{2}\left[\frac{25}{2}+\frac{175}{2}\right]$$
$$= 15 \times 50 = 750\text{ m}^3$$

Thus, the required volume of concrete is 750 m^3.