Triangles

CHAPTER 6

NCERT FOCUS

SOLUTIONS

EXERCISE - 6.1

1. (i) All circles are similar.

(ii) All squares are similar.

(iii) All equilateral triangles are similar.

(iv) Two polygons of the same number of sides are similar, if

(a) their corresponding angles are equal and

(b) their corresponding sides are proportional.

2. (i) (a) Any two circles are similar figures.

(b) Any two squares are similar figures.

(ii) (a) A circle and a triangle are non-similar figures.

(b) An isosceles triangle and a scalene triangle are non-similar figures.

3. On observing the given figures, we find that their corresponding sides are proportional but their corresponding angles are not equal.

$\therefore$ The given figures are not similar.

EXERCISE - 6.2

1. (i) Since $DE \parallel BC$ [Given]

$\therefore$ Using the basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Since, $AD = 1.5$ cm, $DB = 3$ cm and $AE = 1$ cm

$$\therefore \frac{1.5 \text{ cm}}{3 \text{ cm}} = \frac{1 \text{ cm}}{EC}$$

$$\Rightarrow EC \times 1.5 = 1 \times 3$$

$$\Rightarrow EC = \frac{1 \times 3}{1.5} = \frac{1 \times 3 \times 10}{15} \Rightarrow EC = 2 \text{ cm}$$

(ii) In ΔABC, $DE \parallel BC$

$\therefore$ Using the basic proportionality theorem, we have

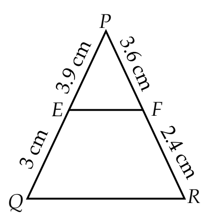
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4} \Rightarrow AD \times 5.4 = 1.8 \times 7.2$$

$$\Rightarrow AD = \frac{1.8 \times 7.2}{5.4} = \frac{18}{10} \times \frac{72}{10} \times \frac{10}{54} = \frac{24}{10} = 2.4$$

$\therefore$ $AD = 2.4$ cm

2. (i) We have, $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm



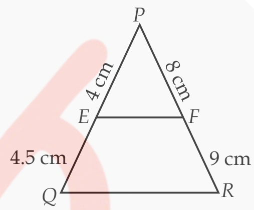
$$\therefore \frac{PE}{EQ} = \frac{3.9 \text{ cm}}{3 \text{ cm}} = \frac{1.3}{1}$$

$$\text{And } \frac{PF}{FR} = \frac{3.6 \text{ cm}}{2.4 \text{ cm}} = \frac{1.5}{1}$$

$$\because \frac{1.3}{1} \neq \frac{1.5}{1} \quad \therefore \frac{PE}{EQ} \neq \frac{PF}{FR}$$

$\Rightarrow$ EF is not parallel to QR.

(ii) We have, $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm



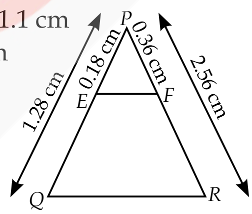
$$\therefore \frac{PE}{EQ} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$$

$$\text{And } \frac{PF}{FR} = \frac{8}{9}$$

$$\text{Since, } \frac{PE}{EQ} = \frac{PF}{FR}$$

$\Rightarrow$ EF is parallel to QR.

(iii) We have, $PE = 0.18$ cm, $PQ = 1.28$ cm, $PF = 0.36$ cm and $PR = 2.56$ cm



$\therefore$ $EQ = PQ - PE = 1.28 - 0.18 = 1.1$ cm

$FR = PR - PF = 2.56 - 0.36 = 2.2$ cm

$$\therefore \frac{PE}{EQ} = \frac{0.18}{1.1} = \frac{18}{110} = \frac{9}{55}$$

$$\text{And } \frac{PF}{FR} = \frac{0.36}{2.2} = \frac{36}{220} = \frac{9}{55}$$

$$\text{Since, } \frac{PE}{EQ} = \frac{PF}{FR} \Rightarrow EF \text{ is parallel to } QR.$$

3. In ΔABC, $LM \parallel CB$ [Given]

$\therefore$ Using the basic proportionality theorem, we have

$$\frac{AM}{MB} = \frac{AL}{LC} \Rightarrow \frac{MB}{AM} + 1 = \frac{LC}{AL} + 1$$

$$\Rightarrow \frac{MB + AM}{AM} = \frac{LC + AL}{AL} \Rightarrow \frac{AB}{AM} = \frac{AC}{AL}$$

$$\Rightarrow \frac{AM}{AB} = \frac{AL}{AC} \quad \text{...(i)}$$

Similarly, in ΔACD, $LN \parallel CD$

$\therefore$ Using the basic proportionality theorem, we have

$$\frac{AL}{AC} = \frac{AN}{AD} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{AM}{AB} = \frac{AL}{AC} = \frac{AN}{AD} \Rightarrow \frac{AM}{AB} = \frac{AN}{AD}$$

4. In ΔABC, $DE \parallel AC$ [Given]

$$\therefore \frac{BD}{DA} = \frac{BE}{EC} \quad \text{[By basic proportionality theorem] ...(i)}$$

In ΔABE, $DF \parallel AE$ [Given]

$$\therefore \frac{BD}{DA} = \frac{BF}{FE} \quad \text{[By basic proportionality theorem] ...(ii)}$$

From (i) and (ii), we get

$$\frac{BF}{FE} = \frac{BD}{DA} = \frac{BE}{EC} \Rightarrow \frac{BF}{FE} = \frac{BE}{EC}$$

5. In ΔPQO, $DE \parallel OQ$ [Given]

$\therefore$ Using the basic proportionality theorem, we have

$$\frac{PE}{EQ} = \frac{PD}{DO} \quad \text{...(i)}$$

Similarly, in ΔPOR, $DF \parallel OR$ [Given]

$\therefore$ Using the basic proportionality theorem, we have

$$\frac{PD}{DO} = \frac{PF}{FR} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{PE}{EQ} = \frac{PD}{DO} = \frac{PF}{FR} \Rightarrow \frac{PE}{EQ} = \frac{PF}{FR}$$

Now, in ΔPQR, E and F are two distinct points on PQ and PR respectively and $\frac{PE}{EQ} = \frac{PF}{FR}$ *i.e.*, E and F divides the two sides PQ and PR in the same ratio.

$\therefore$ By converse of basic proportionality theorem, $EF \parallel QR$.

6. In ΔPQR, O is a point and OP, OQ and OR are joined. We have points A, B, and C on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$.

Now, in ΔOPQ, $AB \parallel PQ$ [Given]

$\therefore$ Using the basic proportionality theorem, we have

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \quad \text{...(i)}$$

Again, in ΔOPR, $AC \parallel PR$ [Given]

$\therefore$ Using the basic proportionality theorem, we have

$$\frac{OA}{AP} = \frac{OC}{CR} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{OB}{BQ} = \frac{OA}{AP} = \frac{OC}{CR} \Rightarrow \frac{OB}{BQ} = \frac{OC}{CR}$$

Now, in ΔOQR, B is a point on OQ, C is a point on OR and $\frac{OB}{BQ} = \frac{OC}{CR}$

i.e., B and C divide the sides OQ and OR in the same ratio

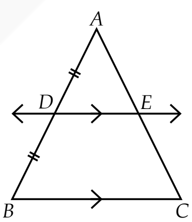
$\therefore$ $BC \parallel QR$

[By converse of basic proportionality theorem]

7. Given, ΔABC, in which D is the mid-point of AB and E is a point on AC such that $DE \parallel BC$.

$\therefore$ Using basic proportionality theorem, we get

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{...(i)}$$



But D is the mid-point of AB

$\therefore$ $AD = DB$

$$\Rightarrow \frac{AD}{DB} = 1 \quad \text{...(ii)}$$

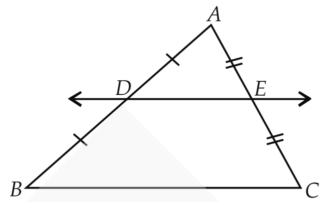
From (i) and (ii), we get

$$1 = \frac{AE}{EC} \Rightarrow EC = AE$$

$\Rightarrow$ E is the mid-point of AC. Hence, it is proved that a line through the mid-point of one side of a triangle parallel to another side bisects the third side.

8. We have ΔABC, in which D and E are the mid-points of sides AB and AC respectively.

$\therefore$ $AD = DB$ and $AE = EC$



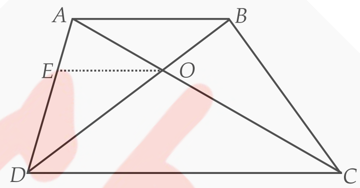
$$\Rightarrow \frac{AD}{DB} = 1 \text{ and } \frac{AE}{EC} = 1$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

$\Rightarrow$ $DE \parallel BC$

[By converse of basic proportionality theorem]

9. We have, a trapezium $ABCD$ such that $AB \parallel DC$. The diagonals AC and BD intersect each other at O.



Let us draw OE parallel to either AB or DC.

In ΔADC, $OE \parallel DC$ [By construction]

$\therefore$ Using basic proportionality theorem, we get

$$\frac{AE}{ED} = \frac{AO}{CO} \quad \text{...(i)}$$

In ΔABD, $OE \parallel AB$ [By construction]

$\therefore$ Using basic proportionality theorem, we get

$$\frac{ED}{AE} = \frac{DO}{BO} \Rightarrow \frac{AE}{ED} = \frac{BO}{DO} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{AE}{ED} = \frac{BO}{DO} = \frac{AO}{CO} \Rightarrow \frac{BO}{DO} = \frac{AO}{CO} \Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$

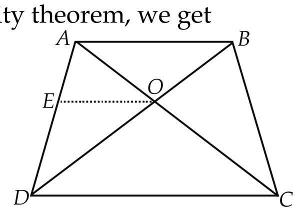
Note : Remember this as a result.

10. It is given that $\frac{AO}{BO} = \frac{CO}{DO} \Rightarrow \frac{AO}{CO} = \frac{BO}{DO}$ ...(i)

Through O, draw $OE \parallel BA$

In ΔADB, $OE \parallel AB$ [By construction]

$\therefore$ Using basic proportionality theorem, we get



$$\frac{DE}{EA} = \frac{DO}{BO}$$

$$\Rightarrow \frac{EA}{DE} = \frac{BO}{DO} \quad \text{...(ii)}$$

From (i) and (ii), we have

$$\frac{EA}{DE} = \frac{BO}{DO} = \frac{AO}{CO}$$

i.e., the points O and E on the sides AC and AD (of ΔADC) respectively are in the same ratio.

$\therefore$ Using basic proportionality theorem, we get

$OE \parallel DC$

Also, $OE \parallel AB$ [By construction]

$\Rightarrow$ $AB \parallel DC$

$\Rightarrow$ $ABCD$ is a trapezium.

EXERCISE - 6.3

1. (i) In ΔABC and ΔPQR,

$\angle A = \angle P = 60°$

$\angle B = \angle Q = 80°$

$\angle C = \angle R = 40°$

$\therefore$ The corresponding angles are equal.

$\therefore$ $\Delta ABC \sim \Delta PQR$ [By AAA similarity criterion]

(ii) In ΔABC and ΔQRP,

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}, \frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2},$$

$$\frac{CA}{PQ} = \frac{3}{6} = \frac{1}{2} \Rightarrow \frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

$\therefore$ $\Delta ABC \sim \Delta QRP$ [By SSS similarity criterion]

(iii) In ΔPML and ΔDEF,

$\because$ $\frac{PM}{DE} = \frac{2}{4} = \frac{1}{2}, \frac{ML}{EF} = \frac{2.7}{5} = \frac{27}{50}, \frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow$ $\frac{PM}{DE} \neq \frac{ML}{EF} \neq \frac{LP}{DF}$ $\Rightarrow$ Triangles are not similar.

(iv) In ΔMNL and ΔQPR,

$\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}, \frac{MN}{QP} = \frac{2.5}{5} = \frac{1}{2}$ and

$\angle NML = \angle PQR = 70°$

$\therefore$ $\Delta MNL \sim \Delta QPR$ [By SAS similarity criterion]

(v) In ΔABC and ΔFDE,

$\angle A = \angle F = 80°$

Here, $\frac{AB}{AC}$ and $\frac{FD}{DE}$ are unknown.

$\therefore$ The triangles cannot be said similar.

(vi) In ΔDEF and ΔPQR, $\angle D = \angle P = 70°$

[$\because \angle P = 180° - (80° + 30°) = 180° - 110° = 70°$]

$\angle E = \angle Q = 80°$

$\angle F = \angle R = 30°$ [$\because \angle F = 180° - (80° + 70°) = 30°$]

$\therefore$ $\Delta DEF \sim \Delta PQR$ [By AAA similarity criterion]

2. We have, $\angle BOC = 125°$ and $\angle CDO = 70°$

Since, $\angle DOC + \angle BOC = 180°$ [Linear pair]

$\Rightarrow$ $\angle DOC = 180° - 125° = 55°$ ...(i)

Using the angle sum property in ΔODC, we get

$\angle DOC + \angle ODC + \angle DCO = 180°$

$\Rightarrow$ $55° + 70° + \angle DCO = 180°$

$\Rightarrow$ $\angle DCO = 180° - 55° - 70° = 55°$

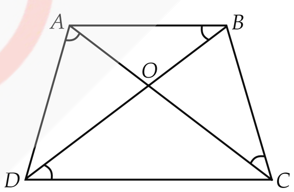
Also, $\angle OAB = \angle DCO = 55°$ ...(ii)

[Corresponding angles of similar triangles]

Thus, from (i) and (ii)

$\angle DOC = 55°, \angle DCO = 55°$ and $\angle OAB = 55°$.

3. We have a trapezium $ABCD$ in which $AB \parallel DC$. The diagonals AC and BD intersect at O.



In ΔOAB and ΔOCD,

$\because$ $AB \parallel DC$ and AC and BD are transversals.

$\therefore$ $\angle OBA = \angle ODC$ [Alternate angles]

and $\angle OAB = \angle OCD$ [Alternate angles]

$\therefore$ $\Delta OAB \sim \Delta OCD$ [By AA similarity criterion]

So, $\frac{OB}{OD} = \frac{OA}{OC}$ [Ratios of corresponding sides of the similar triangles]

4. In ΔPQR, $\angle 1 = \angle 2$ [Given]

$\therefore$ $PR = QP$ ...(i)

[Sides opposite to equal angles are equal]

Also, $\frac{QR}{QS} = \frac{QT}{PR}$ [Given] ...(ii)

From (i) and (ii), we get

$\frac{QR}{QS} = \frac{QT}{QP} \Rightarrow \frac{QS}{QR} = \frac{QP}{QT}$ [By taking reciprocals] ...(iii)

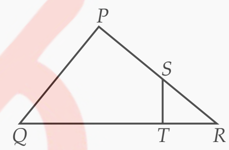
Now, in ΔPQS and ΔTQR,

$\frac{QS}{QR} = \frac{QP}{QT}$ [From (iii)]

and $\angle SQP = \angle RQT = \angle 1$

$\therefore$ $\Delta PQS \sim \Delta TQR$ [By SAS similarity criterion]

5. In ΔPQR, T is a point on QR and S is a point on PR such that $\angle RTS = \angle P$.



Now, in ΔRPQ and ΔRTS,

$\angle RPQ = \angle RTS$ [Given]

$\angle PRQ = \angle TRS$ [Common]

$\therefore$ $\Delta RPQ \sim \Delta RTS$ [By AA similarity criterion]

6. We have, $\Delta ABE \cong \Delta ACD$

$\therefore$ Their corresponding parts are equal,

i.e., $AB = AC, AE = AD$

$\Rightarrow$ $\frac{AB}{AC} = 1$ and $\frac{AE}{AD} = 1$ $\therefore$ $\frac{AB}{AC} = \frac{AE}{AD} \Rightarrow \frac{AB}{AE} = \frac{AC}{AD}$

$\Rightarrow$ $\frac{AB}{AD} = \frac{AC}{AE}$ [$\because AE = AD$] .....(i)

Now in ΔADE and ΔABC, $\frac{AB}{AD} = \frac{AC}{AE}$ [From (i)]

and $\angle DAE = \angle BAC$ [common]

$\therefore$ $\Delta ADE \sim \Delta ABC$ [By SAS similarity criterion]

7. We have a ΔABC in which altitude AD and CE intersect each other at P.

$\Rightarrow$ $\angle D = \angle E = 90°$ ...(1)

(i) In ΔAEP and ΔCDP,

$\angle AEP = \angle CDP$ [From (1)]

$\angle EPA = \angle DPC$ [Vertically opposite angles]

$\therefore$ $\Delta AEP \sim \Delta CDP$ [By AA similarity criterion]

(ii) In ΔABD and ΔCBE,

$\angle ADB = \angle CEB$ [From (1)]

Also, $\angle ABD = \angle CBE$ [Common]

$\therefore$ $\Delta ABD \sim \Delta CBE$ [By AA similarity criterion]

(iii) In ΔAEP and ΔADB,

$\angle AEP = \angle ADB$ [From (1)]

Also, $\angle EAP = \angle DAB$ [Common]

$\therefore$ $\Delta AEP \sim \Delta ADB$ [By AA similarity criterion]

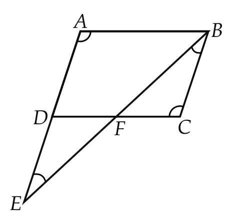
(iv) In ΔPDC and ΔBEC,

$\angle PDC = \angle BEC$ [From (1)]

Also, $\angle DCP = \angle ECB$ [Common]

$\therefore$ $\Delta PDC \sim \Delta BEC$ [By AA similarity criterion]

8. We have a parallelogram $ABCD$ in which AD is produced to E and BE is joined such that BE intersects CD at F.



Now, in ΔABE and ΔCFB,

$\angle BAE = \angle FCB$ [Opposite angles of a parallelogram]

$\angle AEB = \angle CBF$

[Alternate angles, as $AE \parallel BC$ and BE is a transversal.]

$\therefore \quad \Delta ABE \sim \Delta CFB$ [By AA similarity criterion]

9. We have ΔABC, right angled at B and ΔAMP, right angled at M.

$\therefore \quad \angle B = \angle M = 90°$ ...(1)

(i) In ΔABC and ΔAMP,

$\angle ABC = \angle AMP$ [From (1)]

and $\angle BAC = \angle MAP$ [Common]

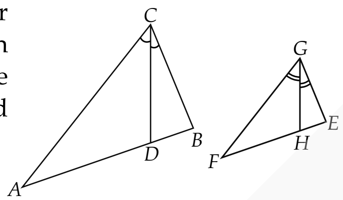
$\therefore \quad \Delta ABC \sim \Delta AMP$ [By AA similarity criterion]

(ii) $\because \quad \Delta ABC \sim \Delta AMP$ [As proved above]

$\therefore$ Their corresponding sides are proportional.

$\Rightarrow \quad \frac{CA}{PA} = \frac{BC}{MP}$

10. We have, two similar ΔABC and ΔFEG such that CD and GH are the bisectors of $\angle ACB$ and $\angle FGE$ respectively.



(i) In ΔACD and ΔFGH,

$\angle A = \angle F$

[$\because \Delta ABC \sim \Delta FEG$] ...(1)

Since $\Delta ABC \sim \Delta FEG$

$\therefore \quad \angle C = \angle G \Rightarrow \frac{1}{2}\angle C = \frac{1}{2}\angle G$

$\Rightarrow \quad \angle ACD = \angle FGH$ ...(2)

From (1) and (2), we have

$\Delta ACD \sim \Delta FGH$ [By AA similarity criterion]

$\therefore$ Their corresponding sides are proportional.

$\Rightarrow \quad \frac{CD}{GH} = \frac{AC}{FG}$

(ii) In ΔDCB and ΔHGE,

$\angle B = \angle E$ [$\because \Delta ABC \sim \Delta FEG$] ...(1)

Again, $\Delta ABC \sim \Delta FEG \Rightarrow \angle ACB = \angle FGE$

$\therefore \quad \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

$\Rightarrow \quad \angle DCB = \angle HGE$ ...(2)

From (1) and (2), we have

$\Delta DCB \sim \Delta HGE$ [By AA similarity criterion]

(iii) In ΔDCA and ΔHGF,

$\because \quad \Delta ABC \sim \Delta FEG \Rightarrow \angle CAB = \angle GFE$

$\Rightarrow \angle CAD = \angle GFH \Rightarrow \angle DAC = \angle HFG$ ...(1)

Also, $\Delta ABC \sim \Delta FEG \Rightarrow \angle ACB = \angle FGE$

$\therefore \quad \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

$\Rightarrow \quad \angle DCA = \angle HGF$ ...(2)

From (1) and (2), we have

$\Delta DCA \sim \Delta HGF$ [By AA similarity criterion]

11. We have an isosceles ΔABC in which $AB = AC$.

In ΔABD and ΔECF,

$\angle ACB = \angle ABC$

$\Rightarrow \quad \angle ECF = \angle ABD$ ...(i)

Again, $AD \perp BC$ and $EF \perp AC$

$\Rightarrow \quad \angle ADB = \angle EFC = 90°$ ...(ii)

From (i) and (ii), we have

$\Delta ABD \sim \Delta ECF$ [By AA similarity criterion]

12. We have ΔABC and ΔPQR in which AD and PM are medians corresponding to sides BC and QR respectively such that $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{(1/2)BC}{(1/2)QR} = \frac{AD}{PM} \Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$\therefore \quad \Delta ABD \sim \Delta PQM$ [By SSS similarity criterion]

$\therefore$ Their corresponding angles are equal.

$\Rightarrow \quad \angle ABD = \angle PQM \Rightarrow \angle ABC = \angle PQR$

Now, in ΔABC and ΔPQR, $\frac{AB}{PQ} = \frac{BC}{QR}$ [Given]

Also, $\angle ABC = \angle PQR$

$\therefore \quad \Delta ABC \sim \Delta PQR$ [By SAS similarity criterion]

13. We have a ΔABC and a point D on its side BC such that $\angle ADC = \angle BAC$.

In ΔBAC and ΔADC,

$\angle BAC = \angle ADC$ [Given]

and $\angle BCA = \angle ACD$ [Common]

$\therefore \quad \Delta BAC \sim \Delta ADC$ [By AA similarity criterion]

$\therefore$ Their corresponding sides are proportional.

$\Rightarrow \quad \frac{CA}{CD} = \frac{CB}{CA} \Rightarrow CA \times CA = CB \times CD$

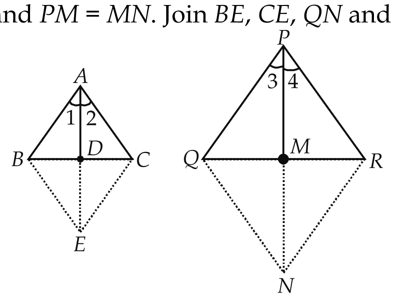
$\Rightarrow \quad CA^2 = CB \times CD$

14. **Given :** ΔABC and ΔPQR in which AD and PM are medians.

Also, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$ ...(i)

To prove : $\Delta ABC \sim \Delta PQR$

Construction : Produce AD to E and PM to N such that $AD = DE$ and $PM = MN$. Join BE, CE, QN and RN.



Proof : Quadrilaterals $ABEC$ and $PQNR$ are parallelograms, since their diagonals bisect each other at point D and M respectively.

$\Rightarrow \quad BE = AC$ and $QN = PR$

$\Rightarrow \quad \frac{BE}{QN} = \frac{AC}{PR} \Rightarrow \frac{BE}{QN} = \frac{AB}{PQ}$ [From (i)]

i.e., $\frac{AB}{PQ} = \frac{BE}{QN}$ ...(ii)

From (i), $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AE}{PN}$ ...(iii)

From (ii) and (iii), we have

$$\frac{AB}{PQ} = \frac{BE}{QN} = \frac{AE}{PN}$$

$\Rightarrow$ $\Delta ABE \sim \Delta PQN$ [By SSS similarity criterion]

$\Rightarrow$ $\angle 1 = \angle 3$ ...(iv)

Similarly, we can prove

$\Delta ACE \sim \Delta PRN \Rightarrow \angle 2 = \angle 4$ ...(v)

From (iv) and (v), $\angle 1 + \angle 2 = \angle 3 + \angle 4$

$\Rightarrow$ $\angle A = \angle P$ ...(vi)

Now, in ΔABC and ΔPQR, we have

$\frac{AB}{PQ} = \frac{AC}{PR}$ [From (i)]

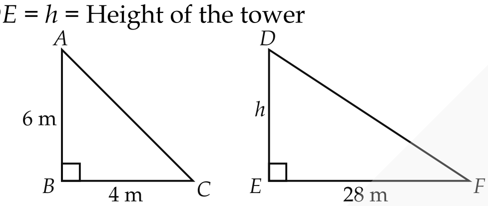
and $\angle A = \angle P$ [From (vi)]

$\therefore$ $\Delta ABC \sim \Delta PQR$ [By SAS similarity criterion]

15. Let $AB = 6$ m be the pole and $BC = 4$ m be its shadow (in right ΔABC), whereas DE and EF denote the tower and its shadow respectively.

EF = Length of the shadow of the tower = 28 m

Let $DE = h$ = Height of the tower



In ΔABC and ΔDEF, we have $\angle B = \angle E = 90°$

$\angle A = \angle D$

[$\because$ Angular elevation of the sun at the same time is equal]

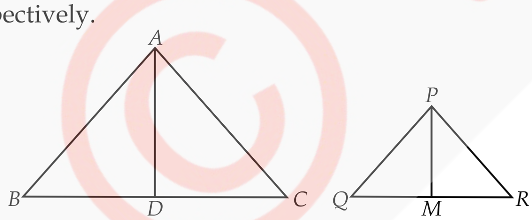
$\therefore$ $\Delta ABC \sim \Delta DEF$ [By AA similarity criterion]

$\therefore$ Their sides are proportional *i.e.*, $\frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow$ $\frac{6}{h} = \frac{4}{28} \Rightarrow h = \frac{6 \times 28}{4} = 42$ m

Thus, the required height of the tower is 42 m.

16. We have $\Delta ABC \sim \Delta PQR$ such that AD and PM are the medians corresponding to the sides BC and QR respectively.



$\because$ $\Delta ABC \sim \Delta PQR$

$\Rightarrow$ The corresponding sides of similar triangles are proportional.

$\therefore$ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ...(i)

$\because$ Corresponding angles are also equal in two similar triangles.

$\therefore$ $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$ ...(ii)

Since, AD and PM are medians.

$\therefore$ $BC = 2BD$ and $QR = 2QM$

$\therefore$ From (i), $\frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{BD}{QM}$ ...(iii)

And $\angle B = \angle Q \Rightarrow \angle ABD = \angle PQM$ ...(iv)

From (iii) and (iv), we have

$\Delta ABD \sim \Delta PQM$ [By SAS similarity criterion]

$\therefore$ Their corresponding sides are proportional.

$\Rightarrow$ $\frac{AB}{PQ} = \frac{AD}{PM}$.

EXERCISE - 6.4

1. We have, $ar(\Delta ABC) = 64$ cm^2, $ar(\Delta DEF) = 121$ cm^2 and $EF = 15.4$ cm

$\because$ $\Delta ABC \sim \Delta DEF$

$\therefore$ $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$

[$\because$ Ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides]

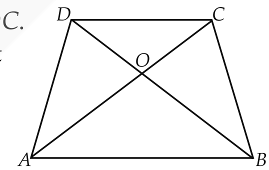
$\Rightarrow$ $\frac{64}{121} = \frac{BC^2}{(15.4)^2} \Rightarrow \frac{8}{11} = \frac{BC}{15.4}$

[Taking square root on both sides]

$\Rightarrow$ $BC = \frac{8 \times 15.4}{11} = 11.2$ cm

Thus, $BC = 11.2$ cm

2. In trapezium $ABCD$, $AB \parallel DC$. Diagonals AC and BD intersect at O.



In ΔAOB and ΔCOD,

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\angle OAB = \angle OCD$ [Alternate angles]

$\therefore$ $\Delta AOB \sim \Delta COD$ [By AA similarity criterion]

$\therefore$ $\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2}$ ...(i)

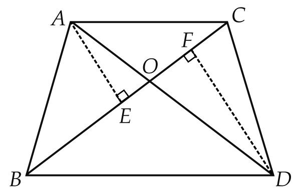
Since, $AB = 2CD$ ...(ii)

$\therefore$ From (i) and (ii), we have

$$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{(2CD)^2}{CD^2} = \frac{4CD^2}{CD^2} = \frac{4}{1}$$

i.e., $ar(\Delta AOB) : ar(\Delta COD) = 4 : 1$

3. We have, ΔABC and ΔDBC are on the same base BC. Also, BC and AD intersect at O.



Let us draw $AE \perp BC$ and $DF \perp BC$.

In ΔAOE and ΔDOF,

$\angle AEO = \angle DFO$ [Each equals 90°]

$\angle AOE = \angle DOF$ [Vertically opposite angles]

$\therefore$ $\Delta AOE \sim \Delta DOF$ [By AA similarity criterion]

$\Rightarrow$ $\frac{AE}{DF} = \frac{AO}{DO}$ ...(i)

Now, $ar(\Delta ABC) = \frac{1}{2} BC \times AE$

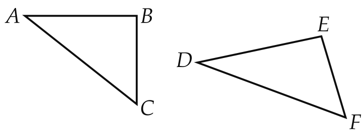
And $ar(\Delta DBC) = \frac{1}{2} BC \times DF$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2}BC \times AE}{\frac{1}{2}BC \times DF} = \frac{AE}{DF} \qquad \text{...(ii)}$$

From (i) and (ii), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

4. We have ΔABC and ΔDEF, such that $\Delta ABC \sim \Delta DEF$ and $ar(\Delta ABC) = ar(\Delta DEF)$. Since the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.



$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

But, $ar(\Delta ABC) = ar(\Delta DEF)$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = 1 \Rightarrow \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2 = 1$$

$$\Rightarrow \quad \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2 = (1)^2$$

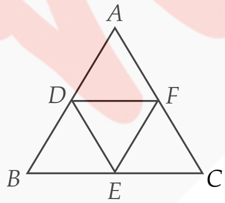
$$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1 \Rightarrow \frac{AB}{DE} = 1$$

$$\Rightarrow \quad AB = DE \Rightarrow \frac{BC}{EF} = 1 \Rightarrow BC = EF,\ \frac{AC}{DF} = 1 \Rightarrow AC = DF$$

i.e., the corresponding sides of ΔABC and ΔDEF are equal.

$\Rightarrow \quad \Delta ABC \cong \Delta DEF$ [By SSS congruency criterion]

5. We have a ΔABC in which D, E and F are mid-points of AB, BC and CA respectively. D, E and F are joined to form ΔDEF.



$$\therefore \quad DE = \frac{1}{2}CA, \quad EF = \frac{1}{2}AB$$

and $DF = \frac{1}{2}CB$

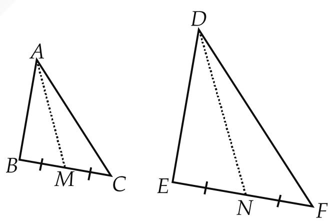
$$\Rightarrow \quad \frac{DE}{CA} = \frac{EF}{AB} = \frac{DF}{CB} = \frac{1}{2}$$

$\Rightarrow \quad \Delta DEF \sim \Delta CAB$ [By SSS similarity criterion]

$$\therefore \quad \frac{ar(\Delta DEF)}{ar(\Delta CAB)} = \left[\frac{DE}{CA}\right]^2 = \left[\frac{\frac{1}{2}CA}{CA}\right]^2 = \frac{1}{4}$$

$\Rightarrow \quad ar(\Delta DEF) : ar(\Delta CAB) = 1 : 4$

6. We have two triangles ABC and DEF such that $\Delta ABC \sim \Delta DEF, AM$ and DN are medians corresponding to sides BC and EF respectively.



$\because \quad \Delta ABC \sim \Delta DEF$

$\therefore$ The ratio of their areas is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 \qquad \text{...(i)}$$

Also, $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$ ...(ii)

$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{AB}{DE} = \frac{2BM}{2EN} = \frac{BM}{EN} \qquad \text{...(iii)}$$

In ΔABM and ΔDEN, we have

$\dfrac{AB}{DE} = \dfrac{BM}{EN}$ [From (iii)]

$\angle B = \angle E$ [Corresponding angles of similar triangles]

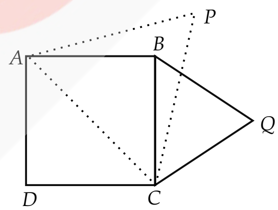
$\therefore \quad \Delta ABM \sim \Delta DEN.$ [By SAS similarity criterion]

$$\therefore \quad \frac{AB}{DE} = \frac{BM}{EN} = \frac{AM}{DN} \qquad \text{...(iv)}$$

Now, from (i) and (iv), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AM}{DN}\right)^2$$

7. We have a square $ABCD$, whose diagonal is AC. Equilateral ΔBQC is described on the side BC and another equilateral ΔAPC is described on the diagonal AC.



$\because \quad$ All equilateral triangles are similar.

$\therefore \quad \Delta APC \sim \Delta BQC$

$\therefore \quad$ The ratio of their areas is equal to the square of the ratio of their corresponding sides.

$$i.e., \quad \frac{ar(\Delta APC)}{ar(\Delta BQC)} = \left(\frac{AC}{BC}\right)^2 \qquad \text{...(i)}$$

Since, the length of a diagonal of a square = $\sqrt{2}$ × side

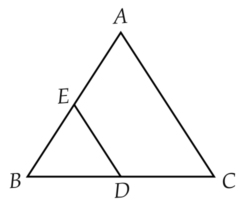
$\therefore \quad AC = \sqrt{2} \times BC$ ...(ii)

From (i) and (ii), we have

$$\frac{ar(\Delta APC)}{ar(\Delta BQC)} = \left(\frac{\sqrt{2}BC}{BC}\right)^2 = (\sqrt{2})^2 = 2$$

$$\Rightarrow \quad ar(\Delta BQC) = \frac{1}{2}ar(\Delta APC)$$

8. **(c) :** We have equilateral ΔABC and ΔBDE and D is the mid-point of BC. Since all equilateral triangles are similar.



$\therefore \quad \Delta ABC \sim \Delta BDE$

So, the ratio of their areas is equal to the square of the ratio of their corresponding sides.

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \left(\frac{AB}{BD}\right)^2 \qquad \text{...(i)}$$

$\because \quad AB = AC = BC$ [Sides of equilateral ΔABC]

and $BD = \dfrac{1}{2}BC$ [$\because D$ is the mid-point of BC]

$\Rightarrow \quad BC = 2BD = AB$ [$\because AB = BC$] ...(ii)

From (i) and (ii), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \left(\frac{2BD}{BD}\right)^2 = \frac{2^2}{1^2} = \frac{4}{1}$$

$\Rightarrow \quad ar(\Delta ABC) : ar(\Delta BDE) = 4 : 1$

9. **(d) :** We have two similar triangles, Δ_1 and Δ_2 such that the ratio of their corresponding sides is 4 : 9.

$\because$ The ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\therefore \quad \frac{ar(\Delta_1)}{ar(\Delta_2)} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

$\Rightarrow \quad ar(\Delta_1) : ar(\Delta_2) = 16 : 81$

EXERCISE - 6.5

1. (i) The sides are : 7 cm, 24 cm, 25 cm

Here, $(7)^2 = 49$; $(24)^2 = 576$; $(25)^2 = 625$

$\because \quad 49 + 576 = 625$

$\therefore$ The given triangle is a right triangle.

Hypotenuse = 25 cm.

(ii) The sides are : 3 cm, 8 cm, 6 cm

Here, $(3)^2 = 9$; $(8)^2 = 64$; $(6)^2 = 36$

$\because \quad 9 + 36 = 45 \neq 64$

$\therefore$ It is not a right triangle.

(iii) The sides are : 50 cm, 80 cm, 100 cm

Here, $(50)^2 = 2500$; $(80)^2 = 6400$; $(100)^2 = 10000$

$\because \quad 2500 + 6400 = 8900 \neq 10000$

$\therefore$ It is not a right triangle.

(iv) The sides are : 13 cm, 12 cm, 5 cm

Here $(13)^2 = 169$; $(12)^2 = 144$; $(5)^2 = 25$

$\because \quad 144 + 25 = 169$

$\therefore$ The given triangle is a right triangle.

Hypotenuse = 13 cm.

2. In ΔQMP and ΔQPR,

$\angle QMP = \angle QPR$ [Each equals 90°]

$\angle Q = \angle Q$ [Common]

$\Rightarrow \Delta QMP \sim \Delta QPR$ [By AA similarity criterion] ...(i)

Again, in ΔPMR and ΔQPR,

$\angle PMR = \angle QPR$ [Each equals 90°]

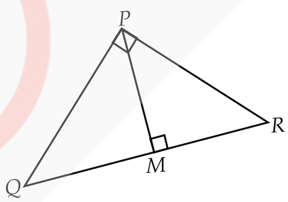
$\angle R = \angle R$ [Common]

$\Rightarrow \Delta PMR \sim \Delta QPR$ [By AA similarity criterion] ...(ii)

From (i) and (ii), we have $\Delta QMP \sim \Delta PMR$

$$\Rightarrow \frac{ar(\Delta QMP)}{ar(\Delta PMR)} = \frac{PM^2}{RM^2}$$

$$\Rightarrow \frac{\frac{1}{2}(QM)\times(PM)}{\frac{1}{2}(RM)\times(PM)} = \frac{PM^2}{RM^2}$$



$$\Rightarrow \frac{QM}{PM} = \frac{PM}{RM}$$

$\Rightarrow PM^2 = QM \times RM$

3. (i) In ΔBAC and ΔBDA,

$\angle ACB = \angle BAD$ [Each equals 90°]

$\angle B = \angle B$ [Common]

$\therefore \Delta BAC \sim \Delta BDA$ [By AA similarity criterion]

$$\Rightarrow \frac{AB}{DB} = \frac{BC}{BA} \Rightarrow \frac{AB}{BD} = \frac{BC}{AB}$$

$\Rightarrow AB \times AB = BC \times BD \Rightarrow AB^2 = BC \times BD$

(ii) In ΔABD, $\angle DAC + \angle CAB = 90°$ ...(1)

Also, in ΔABC, $\angle B + \angle CAB + \angle ACB = 180°$

$\Rightarrow \angle B + \angle CAB + 90° = 180°$

$\Rightarrow \angle B + \angle CAB = 90°$ ...(2)

From (1) and (2), $\angle DAC + \angle CAB = \angle B + \angle CAB$

$\Rightarrow \angle DAC = \angle B$ ...(3)

Now, $\angle ACB = \angle ACD$ [Each equals 90°] ...(4)

From (3) and (4), $\Delta ACB \sim \Delta DCA$

[By AA similarity criterion]

$$\therefore \quad \frac{AC}{DC} = \frac{BC}{AC}$$

$\Rightarrow AC \times AC = BC \times DC \Rightarrow AC^2 = BC \times DC$

(iii) In ΔADB and ΔCDA

$\angle D = \angle D$ [Common]

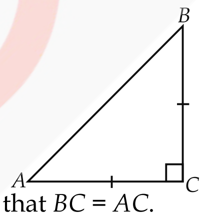
$\angle DAB = \angle DCA$ [Each equals 90°]

$\therefore \Delta ADB \sim \Delta CDA$ [By AA similarity criterion]

$$\Rightarrow \frac{AD}{CD} = \frac{BD}{AD}$$

$\Rightarrow AD \times AD = BD \times CD \Rightarrow AD^2 = BD \times CD$

4. We have, right ΔABC such that $\angle C = 90°$ and $AC = BC$.



$\therefore$ By Pythagoras theorem,we have

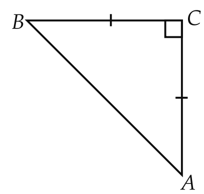
$AB^2 = AC^2 + BC^2$

$= AC^2 + AC^2$ [$\because BC = AC$ (Given)]

Thus, $AB^2 = 2AC^2$

5. We have, an isosceles ΔABC such that $BC = AC$.

Also, $AB^2 = 2AC^2$



$\Rightarrow AB^2 = AC^2 + AC^2$

But $AC = BC$

$\therefore AB^2 = AC^2 + BC^2$

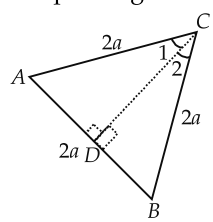
$\therefore$ By the converse of Pythagoras theorem, $\angle ACB = 90°$

i.e., ΔABC is a right triangle.

6. We have an equilateral ΔABC in which $AB = BC = CA = 2a$.

Let us draw $CD \perp AB$ *i.e.*, CD is altitude corresponding to AB.

Now, in ΔACD and ΔBCD,



$AC = BC$

$CD = CD$ [Common]

$\angle ADC = \angle BDC$ [Each equals 90°]

$\therefore \Delta ACD \cong \Delta BCD$

[By RHS congruency criterion]

$\Rightarrow AD = BD$ [By CPCT]

i.e., D is the mid-point of AB

i.e., $AD = \frac{1}{2}AB = \frac{1}{2}(2a) = a$

Now, in right ΔADC, we have $AC^2 = AD^2 + CD^2$

$\Rightarrow CD^2 = AC^2 - AD^2 = (2a)^2 - (a)^2 = 4a^2 - a^2 = 3a^2$

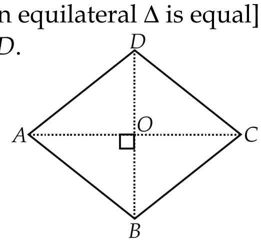
$\therefore \quad CD = \sqrt{3a^2} = a \cdot \sqrt{3}$

Similarly, each of the other altitude is $a\sqrt{3}$.

[$\because$ Each side of an equilateral Δ is equal]

7. Let us have a rhombus $ABCD$.

$\because$ Diagonals of a rhombus bisect each other at right angles.



$\therefore \quad OA = OC$ and $OB = OD$

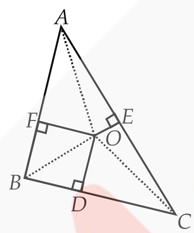
and $\angle AOB = \angle BOC$

$= \angle COD = \angle DOA$

In right ΔAOB, we have,
$AB^2 = OA^2 + OB^2$ [Using Pythagoras theorem] ...(1)
Similarly, $BC^2 = OB^2 + OC^2$ ...(2)
$CD^2 = OC^2 + OD^2$ ...(3)
and $DA^2 = OD^2 + OA^2$ ...(4)
Adding (1), (2), (3) and (4), we get
$AB^2 + BC^2 + CD^2 + DA^2$
$= [OA^2 + OB^2] + [OB^2 + OC^2] + [OC^2 + OD^2] + [OD^2 + OA^2]$
$= 2OA^2 + 2OB^2 + 2OC^2 + 2OD^2$
$= 2[OA^2 + OB^2 + OC^2 + OD^2]$
$= 2[OA^2 + OB^2 + OA^2 + OB^2]$
[$\because OA^2 = OC^2$ and $OB^2 = OD^2$]
$= 2[2OA^2 + 2OB^2] = 2\left[2\left(\frac{1}{2}AC\right)^2 + 2\left(\frac{1}{2}BD\right)^2\right]$
[$\because O$ is the mid-point of AC and BD]
$= 2\left[\frac{AC^2}{2} + \frac{BD^2}{2}\right] = \frac{2}{2}AC^2 + \frac{2}{2}BD^2 = AC^2 + BD^2$

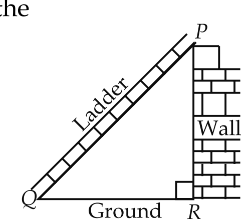
Thus, the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

8. We have a point in the interior of a ΔABC such that $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$.



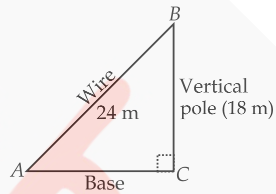
(i) Let us join OA, OB and OC.
In right ΔOAF, we have
$OA^2 = OF^2 + AF^2$ ...(1)
[Using Pythagoras theorem]
Similarly, from right triangle ODB and OEC, we have
$OB^2 = BD^2 + OD^2$ ...(2)
and $OC^2 = CE^2 + OE^2$ ...(3)
Adding (1), (2) and (3), we get $OA^2 + OB^2 + OC^2$
$= (AF^2 + OF^2) + (BD^2 + OD^2) + (CE^2 + OE^2)$
$\Rightarrow OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + (OF^2 + OD^2 + OE^2)$
$\Rightarrow OA^2 + OB^2 + OC^2 - (OD^2 + OE^2 + OF^2)$
$= AF^2 + BD^2 + CE^2$
$\Rightarrow OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$
(ii) In right triangle OBD and OCD,
$OB^2 = OD^2 + BD^2$ and $OC^2 = OD^2 + CD^2$
[Using Pythagoras theorem]
$\Rightarrow OB^2 - OC^2 = OD^2 + BD^2 - OD^2 - CD^2$
$\Rightarrow OB^2 - OC^2 = BD^2 - CD^2$ ...(4)
Similarly, we have $OC^2 - OA^2 = CE^2 - AE^2$ ...(5)
and $OA^2 - OB^2 = AF^2 - BF^2$ ...(6)
Adding (4), (5) and (6), we get
$(OB^2 - OC^2) + (OC^2 - OA^2) + (OA^2 - OB^2)$
$= (BD^2 - CD^2) + (CE^2 - AE^2) + (AF^2 - BF^2)$
$\Rightarrow 0 = BD^2 + CE^2 + AF^2 - (CD^2 + AE^2 + BF^2)$
$\Rightarrow BD^2 + CE^2 + AF^2 = CD^2 + AE^2 + BF^2$
or $AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2$

9. Let PQ be the ladder, PR be the wall and RQ be the base.
$\therefore PQ = 10$ m, $PR = 8$ m
Now, in the right ΔPQR,
$PQ^2 = PR^2 + QR^2 \Rightarrow 10^2 = 8^2 + QR^2$
[Using Pythagoras theorem]
$\Rightarrow QR^2 = 10^2 - 8^2$
$= (10 + 8)(10 - 8) = 18 \times 2 = 36$



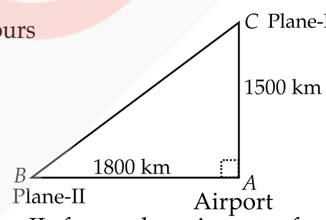
$\therefore QR = \sqrt{36} = 6$ m
Thus, the distance of the foot of the ladder from the base of the wall is 6 m.

10. Let AB is the wire and BC is the vertical pole. The point A is the stake.
$\therefore AB = 24$ m, $BC = 18$ m
Now, in the right ΔABC, using Pythagoras theorem, we have
$AB^2 = AC^2 + BC^2$
$\Rightarrow 24^2 = AC^2 + 18^2$
$\Rightarrow AC^2 = 24^2 - 18^2$
$= (24 - 18)(24 + 18)$
$= 6 \times 42 = 252 = 7 \times 36$
$\therefore AC = \sqrt{7 \times 36} = 6\sqrt{7}$ m



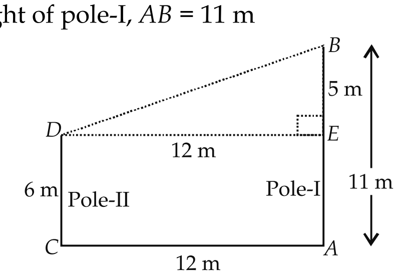
Thus, the stake is required to be taken at $6\sqrt{7}$ m from the base of the pole to make the wire taut.

11. Let the point A represent the airport.
Let the plane-I fly towards North,
$\therefore$ Distance of the plane-I from the airport after $1\frac{1}{2}$ hours
= speed $\times$ time
$= 1000 \times 1\frac{1}{2}$ km $= 1500$ km
Let the plane-II fly towards West.



$\therefore$ Distance of the plane- II from the airport after $1\frac{1}{2}$ hours $= 1200 \times 1\frac{1}{2}$ km $= 1800$ km
Now, in right ΔABC, using Pythagoras theorem, we have
$BC^2 = AB^2 + AC^2$
$\Rightarrow BC^2 = (1800)^2 + (1500)^2$
$= 3240000 + 2250000 = 5490000$
$\Rightarrow BC = \sqrt{5490000} = \sqrt{61 \times 90000}$
$= \sqrt{61} \times \sqrt{90000} = 300\sqrt{61}$ km
Thus, after $1\frac{1}{2}$ hours, the two planes will be $300\sqrt{61}$ km apart from each other.

12. Let the two poles AB and CD are such that the distance between their feet, $AC = 12$ m.
$\because$ Height of pole-I, $AB = 11$ m



Height of pole-II, $CD = 6$ m
$\therefore BE = AB - AE = 11$ m $- 6$ m $= 5$ m [$\because AE = CD$]
Let us join the tops of the poles, D and B.
Now, in right ΔBED, using the Pythagoras theorem, we have $DB^2 = DE^2 + EB^2$

$\Rightarrow \quad DB^2 = 12^2 + 5^2 = 144 + 25 = 169$

$\Rightarrow \quad DB = \sqrt{169} = 13$ m

Thus, the required distance between the tops is 13 m.

13. We have a right ΔABC such that $\angle C = 90°$.

Also, D and E are points on CA and CB respectively.

Let us join AE and BD.

In right ΔACB, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$ ...(1)

In right ΔDCE, by Pythagoras theorem

$DE^2 = CD^2 + CE^2$ ...(2)

Adding (1) and (2), we get

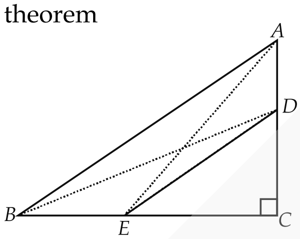
$AB^2 + DE^2 = [AC^2 + BC^2] + [CD^2 + CE^2]$

$= AC^2 + BC^2 + CD^2 + CE^2$

$= [AC^2 + CE^2] + [BC^2 + CD^2]$ ...(3)

In right ΔACE, by Pythagoras theorem

$AC^2 + CE^2 = AE^2$ ...(4)



In right ΔBCD, by Pythagoras theorem

$BC^2 + CD^2 = BD^2$ ...(5)

$\therefore$ From (3), (4) and (5), we have

$AB^2 + DE^2 = AE^2 + BD^2$

or $AE^2 + BD^2 = AB^2 + DE^2$

14. We have a ΔABC such that $AD \perp BC$.

The position of D is such that $BD = 3CD$.

In right ΔABD, by Pythagoras theorem

$AB^2 = AD^2 + BD^2$ ...(1)

Similarly, from right ΔACD, we have

$AC^2 = AD^2 + CD^2$ ...(2)

Subtracting (2) from (1), we get

$AB^2 - AC^2 = BD^2 - CD^2$ ...(3)

Now, $BC = DB + CD = 3CD + CD = 4CD \Rightarrow CD = \frac{1}{4}BC$

$\therefore BD = BC - CD = BC - \frac{1}{4}BC = \frac{3}{4}BC$

Now, substituting the values of CD and BD in (3), we get

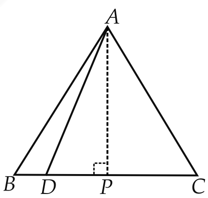
$$AB^2 - AC^2 = \left[\frac{3}{4}BC\right]^2 - \left[\frac{1}{4}BC\right]^2$$

$$AB^2 - AC^2 = BC^2\left[\left(\frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right] = BC^2\left[\left(\frac{3}{4}+\frac{1}{4}\right)\left(\frac{3}{4}-\frac{1}{4}\right)\right]$$

$$= BC^2\left[(1)\left(\frac{1}{2}\right)\right] = \frac{1}{2}BC^2$$

$\Rightarrow \quad 2AB^2 - 2AC^2 = BC^2$ or $2AB^2 = 2AC^2 + BC^2$

15. We have an equilateral ΔABC in which D is a point on BC such that $BD = \frac{1}{3}BC$.



Let us draw $AP \perp BC$

$\Rightarrow \quad BP = \frac{1}{2}BC$

In right ΔAPB, by Pythagoras theorem

$AB^2 = AP^2 + BP^2$ ...(1)

In right ΔAPD, by Pythagoras theorem

$AD^2 = AP^2 + DP^2 \quad \Rightarrow \quad AP^2 = AD^2 - DP^2$

From (1), we have $AB^2 = AP^2 + BP^2$

$\Rightarrow \quad AB^2 = AD^2 - DP^2 + \left(\frac{BC}{2}\right)^2$

$\Rightarrow \quad AD^2 = AB^2 + DP^2 - \frac{BC^2}{4}$ ...(2)

Since, $BD = \frac{1}{3}BC$ and $BP = \frac{1}{2}BC$

$\therefore \quad DP = BP - BD \Rightarrow DP = \frac{1}{2}BC - \frac{1}{3}BC = \frac{1}{6}BC$

Now, substituting $DP = \frac{1}{6}BC$ in (2), we have

$$AD^2 = AB^2 + \left[\frac{1}{6}BC\right]^2 - \frac{BC^2}{4}$$

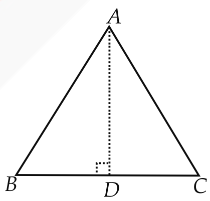
$$= AB^2 + \frac{BC^2}{36} - \frac{BC^2}{4} = AB^2 + \frac{BC^2 - 9BC^2}{36}$$

$$= AB^2 - \frac{2}{9}BC^2 = AB^2 - \frac{2}{9}AB^2$$

[$\because \Delta ABC$ is an equilateral Δ $\therefore AB = BC = CA$]

$$= \frac{9AB^2 - 2AB^2}{9} = \frac{7AB^2}{9} \Rightarrow 9AD^2 = 7AB^2$$

16. We have an equilateral ΔABC in which $AD \perp BC$.



Since, an altitude in an equilateral Δ, bisects the corresponding side.

$\therefore \quad D$ is the mid-point of BC.

$\Rightarrow \quad BD = DC = \frac{1}{2}BC$

In right ΔADB, by Pythagoras theorem $AB^2 = AD^2 + BD^2$

$$= AD^2 + \left(\frac{1}{2}BC\right)^2 = AD^2 + \frac{1}{4}BC^2$$

$\Rightarrow \quad 4AB^2 = 4AD^2 + BC^2$

$\Rightarrow \quad 4AB^2 = 4AD^2 + AB^2$ [$\because AB = BC = AC$]

$\Rightarrow \quad 4AD^2 = 4AB^2 - AB^2$ or $3AB^2 = 4AD^2$

17. (c) : We have, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm

$\therefore \quad AB^2 = (6\sqrt{3})^2 = 36 \times 3 = 108$

$AC^2 = 12^2 = 144, BC^2 = 6^2 = 36$

Since, $144 = 108 + 36$ *i.e.*, $AC^2 = AB^2 + BC^2$

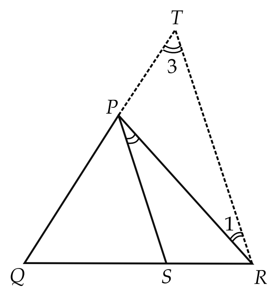
$\therefore \quad$ The given sides form a ΔABC, right angled at B.

$\Rightarrow \quad \angle B = 90°$

EXERCISE - 6.6

1. We have, ΔPQR in which PS is the bisector of $\angle QPR$.

$\therefore \quad \angle QPS = \angle RPS$



Let us draw $RT \parallel PS$ to meet QP produced at T, such that

$\angle 1 = \angle RPS$ [Alternate angles]

Also, $\angle 3 = \angle QPS$ [Corresponding angles]

But $\angle RPS = \angle QPS$ [Given]

$\therefore \quad \angle 1 = \angle 3$

$\Rightarrow \quad PT = PR$ [Sides opposite to equal angles are equal]

Now, in ΔQRT, $PS \parallel RT$ [By construction]

$\therefore$ Using the basic proportionality theorem, we have

$$\frac{QS}{SR} = \frac{PQ}{PT} \Rightarrow \frac{QS}{SR} = \frac{PQ}{PR} \quad [\because PT = PR]$$

2. We have AC as the hypotenuse of ΔABC.

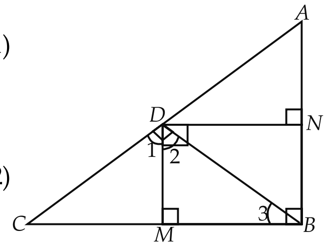
Also, $BD \perp AC$, $DM \perp BC$ and $DN \perp AB$.

$\Rightarrow$ $BMDN$ is a rectangle.

$\therefore$ $BM = ND$ [Opposite sides of a rectangle]

(i) In ΔBMD and ΔDMC,

$\angle DMB = 90° = \angle DMC$ ...(1)



$\because$ $BD \perp AC$ [Given]

$\therefore$ $\angle 1 + \angle 2 = 90°$

In ΔBDM, $\angle 3 + \angle 2 = 90°$

$\Rightarrow$ $\angle 1 = \angle 3$ ...(2)

$\therefore$ From (1) and (2),

$\Delta BMD \sim \Delta DMC$

[By AA similarity criterion]

$\therefore$ Their corresponding sides are proportional.

$$\Rightarrow \frac{BM}{DM} = \frac{MD}{MC} \Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \quad [\therefore DN = BM]$$

$\Rightarrow$ $DN \times MC = DM \times DM$

$\Rightarrow$ $DN \times MC = DM^2$ or $DM^2 = DN \times MC$

(ii) In ΔBND and ΔDNA, we have

$\angle BND = \angle DNA$ [Each equals 90°]

$\angle DBN = \angle ADN$ [Similarly, as proved in part (i)]

$\therefore$ $\Delta BND \sim \Delta DNA$ [By AA similarity criterion]

$\therefore$ Their corresponding sides are proportional.

$$\Rightarrow \frac{BN}{DN} = \frac{ND}{NA} \Rightarrow \frac{DM}{DN} = \frac{DN}{NA}$$

[$\because$ BN and DM are opposite sides of a rectangle]

$\Rightarrow$ $DM \times NA = DN \times DN$

$\Rightarrow$ $DM \times AN = DN^2$ or $DN^2 = DM \times AN$

3. We have ΔABC in which $\angle ABC > 90°$ and $AD \perp CB$ produced.

In ΔADB, $\angle D = 90°$

$\therefore$ Using Pythagoras theorem, we have

$AB^2 = AD^2 + DB^2$ ...(1)

In right ΔADC, $\angle D = 90°$

$\therefore$ Using Pythagoras theorem, we have

$AC^2 = AD^2 + DC^2 = AD^2 + [BD + BC]^2$

$= AD^2 + [BD^2 + BC^2 + 2BD \cdot BC]$

$\Rightarrow$ $AC^2 = [AD^2 + DB^2] + BC^2 + 2BC \cdot BD$

$\Rightarrow$ $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$ [From (1)]

Thus, $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$

4. We have ΔABC in which $\angle ABC < 90°$ and $AD \perp BC$

In right ΔADB, $\angle D = 90°$

$\therefore$ Using Pythagoras theorem, we have

$AB^2 = AD^2 + BD^2$ ...(1)

Also in right ΔADC, $\angle D = 90°$

$\therefore$ Using Pythagoras theorem, we have

$AC^2 = AD^2 + DC^2 = AD^2 + [BC - BD]^2$

$= AD^2 + [BC^2 + BD^2 - 2BC \cdot BD]$

$= [AD^2 + BD^2] + BC^2 - 2BC \cdot BD$

$= AB^2 + BC^2 - 2BC \cdot BD$ [From (1)]

Thus, $AC^2 = AB^2 + BC^2 - 2BC \cdot BD$

5. We have, ΔABC in which AD is median and $AM \perp BC$ such that $\angle ADC > 90°$ and $\angle ADM < 90°$.

(i) In right ΔAMC, by Pythagoras theorem

$\therefore$ $AC^2 = AM^2 + MC^2 = AM^2 + MD^2 + DC^2 + 2MD \cdot DC$

$= AD^2 + DC^2 + 2MD \cdot DC$

[$\because$ In right ΔAMD, $MD^2 + AM^2 = AD^2$]

$$= AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\frac{BC}{2} \quad [\because D \text{ is the mid-point of } BC]$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 + BC \cdot DM$$

Thus, $AC^2 = AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2$ ...(1)

(ii) In right ΔAMB, by Pythagoras theorem

$AB^2 = AM^2 + BM^2 = AM^2 + (BD - DM)^2$

$= AM^2 + BD^2 + DM^2 - 2BD \cdot DM$

$= AD^2 + BD^2 - 2BD \cdot DM$

[$\because$ In right ΔAMD, $DM^2 + AM^2 = AD^2$]

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\frac{BC}{2} \cdot DM$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \cdot DM$$

Thus, $AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2$ ...(2)

(iii) Adding (1) and (2), we get

$AB^2 + AC^2$

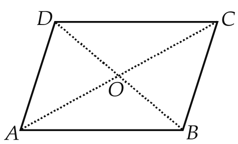
$$= AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2 + AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

$$= 2AD^2 + 2\left(\frac{BC}{2}\right)^2 = 2AD^2 + 2\frac{BC^2}{4} = 2AD^2 + \frac{BC^2}{2}$$

Thus, $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$

6. We have a parallelogram $ABCD$.

AC and BD are the diagonals of $\parallel^{gm}$ $ABCD$.



$\because$ Diagonals of a $\parallel^{gm}$ bisect each other.

$\therefore$ O is the mid-point of AC and BD.

Now, in ΔABC, BO is a median.

$$\therefore \quad AB^2 + BC^2 = 2BO^2 + \frac{1}{2}AC^2 \quad \text{...(1)}$$

[Solved in previous question]

Also, in ΔADC, DO is a median.

$$\therefore \quad AD^2 + CD^2 = 2DO^2 + \frac{1}{2}AC^2 \quad \text{...(2)}$$

Adding (1) and (2), we get

$AB^2 + BC^2 + AD^2 + CD^2$

$$= 2BO^2 + \frac{1}{2}AC^2 + 2DO^2 + \frac{1}{2}AC^2$$

$$= 2\left(\frac{BD}{2}\right)^2 + \frac{1}{2}AC^2 + 2\left(\frac{BD}{2}\right)^2 + \frac{1}{2}AC^2$$

$$\left[\because BO = DO = \frac{1}{2}BD\right]$$

$$= 2\left[\frac{BD^2}{4}+\frac{BD^2}{4}\right]+AC^2 = 2\left[\frac{2BD^2}{4}\right]+AC^2 = BD^2 + AC^2$$

Thus, $AB^2 + BC^2 + AD^2 + CD^2 = AC^2 + BD^2$

7. We have two chords AB and CD of a circle. AB and CD intersect at P.

(i) In ΔAPC and ΔDPB,

$\angle APC = \angle DPB$ [Vertically opposite angles]

$\angle CAP = \angle BDP$ [Angles in the same segment]

$\therefore$ $\Delta APC \sim \Delta DPB$ [By AA similarity criterion]

(ii) Since, $\Delta APC \sim \Delta DPB$ [As proved above]

$\therefore$ Their corresponding sides are proportional,

$\Rightarrow \dfrac{AP}{DP} = \dfrac{CP}{BP} \Rightarrow AP \cdot BP = CP \cdot DP$

8. We have two chords AB and CD, when produced meet outside the circle at P.

(i) Since, in a cyclic quadrilateral, the exterior angle is equal to the interior opposite angle.

$\therefore$ $\angle PAC = \angle PDB$

and $\angle PCA = \angle PBD$

$\therefore$ $\Delta PAC \sim \Delta PDB$ [By AA similarity criterion]

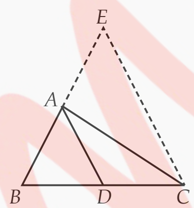
(ii) Since, $\Delta PAC \sim \Delta PDB$ [As proved above]

$\therefore$ Their corresponding sides are proportional.

$\Rightarrow \dfrac{PA}{PD} = \dfrac{PC}{PB} \Rightarrow PA \cdot PB = PC \cdot PD$

9. Let us produce BA to E such that $AE = AC$.

Join EC.



Since, $\dfrac{BD}{CD} = \dfrac{AB}{AC}$ [Given]

But $AC = AE$ [By construction]

$\therefore$ $\dfrac{BD}{CD} = \dfrac{AB}{AE}$

$\therefore$ $AD \parallel CE$

[By the converse of the basic proportionality theorem]

$\Rightarrow$ $\angle BAD = \angle AEC$ [Corresponding angles] ...(1)

Also, $\angle CAD = \angle ACE$ [Alternate angles] ...(2)

Since, $AC = AE$

$\Rightarrow$ $\angle AEC = \angle ACE$ ...(3)

[$\because$ Angles opposite to equal sides are equal]

From (1) and (3), we have

$\angle BAD = \angle ACE$ ...(4)

From (2) and (4), we have

$\angle BAD = \angle CAD$

$\Rightarrow$ AD is bisector of $\angle BAC$.

10. Let us find the length of the string that Nazima has out.

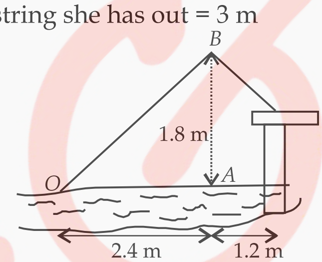
In right ΔOAB, by Pythagoras theorem $OB^2 = OA^2 + AB^2$

$\therefore$ $OB^2 = (2.4)^2 + (1.8)^2$

$\Rightarrow$ $OB^2 = 5.76 + 3.24 = 9$

$\Rightarrow$ $OB = \sqrt{9} = 3$ m

i.e., Length of string she has out = 3 m



Since, the string is pulled out at the rate of 5 cm/sec.

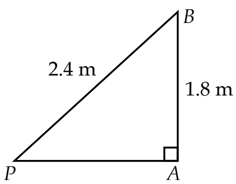
$\therefore$ Length of the string pulled out in 12 seconds

$= 5 \times 12 \text{ cm} = 60 \text{ cm} = \dfrac{60}{100} \text{ m} = 0.60 \text{ m}$

$\therefore$ Remaining string left out = (3 – 0.60) m = 2.4 m

Let PA be the horizontal distance of fly from a point A directly under the tip of rod, after pulling 0.6 m string.

In the ΔPBA, by Pythagoras theorem



$\Rightarrow$ $PB^2 = BA^2 + PA^2$

$\Rightarrow$ $PA^2 = PB^2 - BA^2$

$\therefore$ $PA^2 = (2.4)^2 - (1.8)^2$

$= 5.76 - 3.24 = 2.52$

$\Rightarrow$ $PA = \sqrt{2.52} = 1.59$ (approximately)

Thus, the horizontal distance of the fly from Nazima after 12 seconds

= (1.59 + 1.2) m (approximately)

= 2.79 m (approximately)