Some Applications of Trigonometry

CHAPTER 9

NCERT FOCUS SOLUTIONS

1. Here, AB is the pole and AC is the rope tied to the point C on the ground.

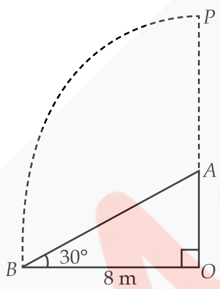
In right ΔABC,

$$\frac{AB}{AC} = \sin 30° \Rightarrow \frac{AB}{AC} = \frac{1}{2} \Rightarrow \frac{AB}{20} = \frac{1}{2}$$

$$\Rightarrow \quad AB = 20 \times \frac{1}{2} = 10 \text{ m}$$

Thus, the required height of the pole is 10 m.

2. Let the tree OP is broken at A and its top is touching the ground at B.



Now, in right ΔAOB,

$$\frac{AO}{OB} = \tan 30°$$

$$\Rightarrow \quad \frac{AO}{8} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad AO = \frac{8}{\sqrt{3}} \text{ m}$$

Also, $\frac{AB}{OB} = \sec 30°$

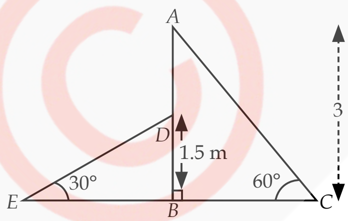
$$\Rightarrow \quad \frac{AB}{8} = \frac{2}{\sqrt{3}} \Rightarrow AB = \frac{2 \times 8}{\sqrt{3}} = \frac{16}{\sqrt{3}} \text{ m}$$

Now, height of the tree $OP = OA + AP = OA + AB$

[$\because AP = AB$]

$$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 8\sqrt{3} \text{ m}$$

3. In the figure, DE is the slide for younger children, whereas AC is the slide for elder children.



In right ΔABC,

$$\therefore \quad \frac{AB}{AC} = \sin 60°$$

$$\Rightarrow \quad \frac{3}{AC} = \frac{\sqrt{3}}{2} \Rightarrow AC = \frac{2 \times 3}{\sqrt{3}} = 2\sqrt{3} \text{ m}$$

Again, in right ΔBDE,

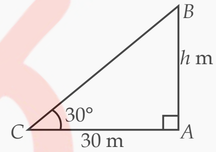
$$\frac{DE}{BD} = \operatorname{cosec} 30° = 2$$

$$\Rightarrow \quad \frac{DE}{1.5} = 2 \quad \Rightarrow \quad DE = 2 \times 1.5 = 3 \text{ m}$$

Thus, the lengths of slides are 3 m and $2\sqrt{3}$ m.

4. In right ΔABC, AB = height of the tower and point C is 30 m away from the foot of the tower.

$\therefore \quad AC = 30$ m



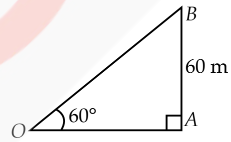
$$\text{Now, } \frac{AB}{AC} = \tan 30° \Rightarrow \frac{h}{30} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{30}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3}$$

Thus, the required height of the tower is $10\sqrt{3}$ m.

5. Let OB = Length of the string

AB = 60 m = Height of the kite.



In right ΔAOB,

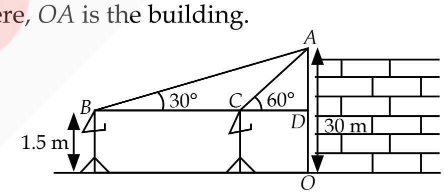
$$\frac{OB}{AB} = \operatorname{cosec} 60°$$

$$\Rightarrow \quad \frac{OB}{60} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \quad OB = \frac{2 \times 60}{\sqrt{3}} = \frac{120 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = 40\sqrt{3} \text{ m}$$

Thus, length of the string is $40\sqrt{3}$ m.

6. Here, OA is the building.



In right ΔABD,

$$\frac{AD}{BD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BD = AD\sqrt{3} = 28.5\sqrt{3} \text{ m}$$

[$\because AD = 30 \text{ m} - 1.5 \text{ m} = 28.5 \text{ m}$]

Also, in right ΔACD,

$$\frac{AD}{CD} = \tan 60° = \sqrt{3} \Rightarrow CD = \frac{AD}{\sqrt{3}} = \frac{28.5}{\sqrt{3}} \text{ m}$$

$$\text{Now, } BC = BD - CD = 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}$$

$$= 28.5\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = 28.5\left[\frac{3-1}{\sqrt{3}}\right]$$

$$= 28.5 \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{28.5 \times 2 \times \sqrt{3}}{3}$$

$$= 9.5 \times 2 \times \sqrt{3} = 19\sqrt{3} \text{ m}$$

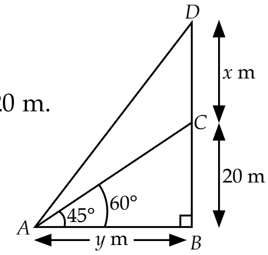
Thus, the distance walked by the boy towards the building is $19\sqrt{3}$ m.

7. Let BC be the building of height 20 m and CD be the tower of height x m.

Let the point A be at a distance of y m from the foot of the building.

Now, in right ΔABC,

$$\frac{BC}{AB} = \tan 45° = 1$$



$$\Rightarrow \frac{20}{y} = 1 \Rightarrow y = 20 \text{ i.e., } AB = 20 \text{ m.}$$

Now, in right ΔABD,

$$\frac{BD}{AB} = \tan 60°$$

$$\Rightarrow \frac{20+x}{20} = \sqrt{3} \Rightarrow 20 + x = 20\sqrt{3}$$

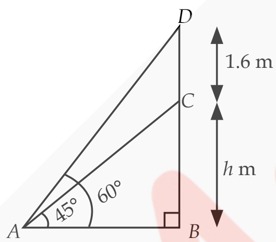
$$\Rightarrow x = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$$

Thus, the height of the tower is $20(\sqrt{3} - 1)$ m.

8. In the figure, DC represents the statue of height 1.6 m and BC represents the pedestal of height h m.

Now, in right ΔABC,

$$\frac{AB}{BC} = \cot 45^\circ = 1$$



$$\Rightarrow \frac{AB}{h} = 1 \Rightarrow AB = h \text{ m}$$

Now, in right ΔABD,

$$\frac{BD}{AB} = \tan 60^\circ$$

$$\Rightarrow \frac{h+1.6}{h} = \sqrt{3} \Rightarrow h + 1.6 = \sqrt{3}\,h$$

$$\Rightarrow h(\sqrt{3} - 1) = 1.6 \Rightarrow h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

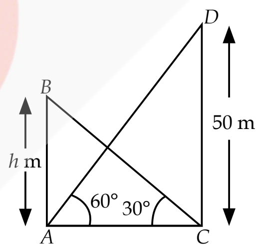
$$\Rightarrow h = \frac{1.6(\sqrt{3}+1)}{2} = 0.8(\sqrt{3}+1)$$

Thus, the height of the pedestal is $0.8(\sqrt{3}+1)$ m.

9. In the figure, let AB be the building of height h m and CD be the tower of height 50 m.

Now, in right ΔABC,

$$\frac{AC}{AB} = \cot 30^\circ = \sqrt{3}$$



$$\Rightarrow \frac{AC}{h} = \sqrt{3} \Rightarrow AC = h\sqrt{3} \quad \text{...(1)}$$

In right ΔDCA,

$$\frac{DC}{AC} = \tan 60^\circ$$

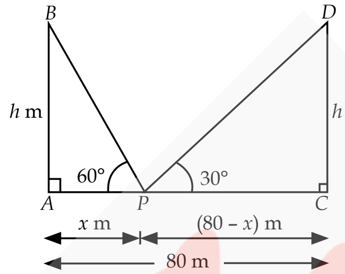
$$\Rightarrow \frac{50}{AC} = \sqrt{3} \Rightarrow AC = \frac{50}{\sqrt{3}} \quad \text{...(2)}$$

From (1) and (2), we get

$$\sqrt{3}h = \frac{50}{\sqrt{3}} \Rightarrow h = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3}$$

Thus, the height of the building is $16\frac{2}{3}$ m.

10. In the figure, let AB and CD are the poles of equal height h m and P be the point on the road at a distance of x m from the pole AB.



$\therefore \quad CP = (80 - x)$

Now, in right ΔAPB,

$$\frac{AB}{AP} = \tan 60^\circ$$

$$\Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow h = x\sqrt{3} \quad \text{...(1)}$$

Again in right ΔCPD,

$$\frac{CD}{CP} = \tan 30^\circ$$

$$\Rightarrow \frac{h}{(80-x)} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{80-x}{\sqrt{3}} \quad \text{...(2)}$$

From (1) and (2), we get

$$\sqrt{3}x = \frac{80-x}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3} \times \sqrt{3} \times x = 80 - x \Rightarrow 3x = 80 - x$$

$$\Rightarrow 3x + x = 80 \Rightarrow 4x = 80 \Rightarrow x = \frac{80}{4} = 20$$

$\therefore \quad CP = 80 - x = 80 - 20 = 60$

Now, from (1), we have $h = 20\sqrt{3}$

Thus, the required point is 20 m away from the first pole and 60 m away from the second pole and height of each pole is $20\sqrt{3}$ m.

11. In the figure, let AB be the TV tower of height h m and C be the point on the other bank of the canal at a distance of x m from B. D be another point 20 m away from point C.

$\therefore \quad BC = x$ m and $CD = 20$ m

Now, in right ΔABC,

$$\frac{AB}{BC} = \tan 60^\circ \Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow h = \sqrt{3}x \text{ m} \quad \text{...(1)}$$

In right ΔABD,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\Rightarrow \frac{h}{x+20} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{x+20}{\sqrt{3}} \text{ m} \quad \text{...(2)}$$

From (1) and (2), we get

$$\sqrt{3}x = \frac{x+20}{\sqrt{3}} \Rightarrow 3x = x + 20$$

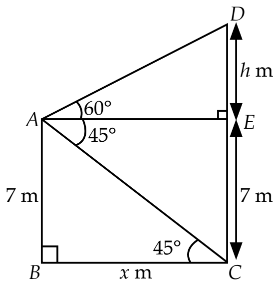
$$\Rightarrow 3x - x = 20 \Rightarrow 2x = 20 \Rightarrow x = \frac{20}{2} = 10 \text{ m}$$

Now, from (1), we get $h = 10\sqrt{3}$ m

Thus, the height of the tower is $10\sqrt{3}$ m and width of the canal is 10 m.

12. In the figure, let AB be the building of height 7 m.
Let $BC = AE = x$ m
Let CD be the height of the cable tower and $DE = h$ m.

$\therefore$ In right ΔDAE,



$$\frac{DE}{EA} = \tan 60^\circ \Rightarrow \frac{h}{x} = \sqrt{3}$$

$\Rightarrow \quad h = \sqrt{3}x$ ...(1)

Again, in right ΔABC,

$$\frac{AB}{BC} = \tan 45^\circ$$

$\Rightarrow \quad \frac{7}{x} = 1 \Rightarrow x = 7$ ...(2)

From (1) and (2), we get

$$h = 7\sqrt{3} \Rightarrow DE = 7\sqrt{3}$$

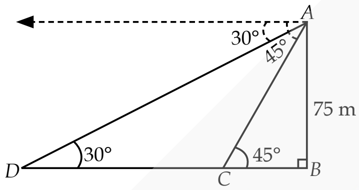
$\therefore \quad CD = CE + ED = (7 + 7\sqrt{3}) = 7(1 + \sqrt{3})$

Thus, the height of the cable tower is $7(1 + \sqrt{3})$ m.

13. In the figure, let AB be the light house.

$\therefore \quad AB = 75$ m

Let the positions of two ships be C and D such that angle of depression from A are 45° and 30° respectively.



Now, in right ΔABC,

$$\frac{AB}{BC} = \tan 45^\circ \Rightarrow \frac{75}{BC} = 1 \Rightarrow BC = 75 \text{ m}$$

Again, in right ΔABD,

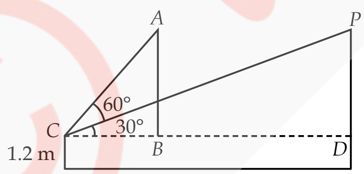
$$\frac{AB}{BD} = \tan 30^\circ \Rightarrow \frac{75}{BD} = \frac{1}{\sqrt{3}} \Rightarrow BD = 75\sqrt{3} \text{ m}$$

Now, the distance between the two ships = CD

$= BD - BC = 75\sqrt{3} - 75 = 75(\sqrt{3} - 1)$ m

Thus, the required distance between the ships is $75(\sqrt{3} - 1)$ m.

14. In the figure, let C be the position of the girl. A and P are two positions of the balloon. CD is the horizontal line from the eyes of the girl.



Here, $PD = AB = 88.2$ m – 1.2 m = 87 m

In right ΔABC,

$$\frac{AB}{BC} = \tan 60^\circ \Rightarrow \frac{87}{BC} = \sqrt{3} \Rightarrow BC = \frac{87}{\sqrt{3}} \text{ m}$$

In right ΔPDC,

$$\frac{PD}{CD} = \tan 30^\circ \Rightarrow \frac{87}{CD} = \frac{1}{\sqrt{3}} \Rightarrow CD = 87\sqrt{3} \text{ m}$$

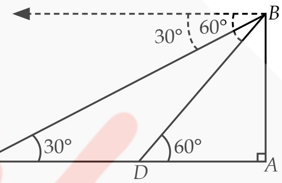
Now, $BD = CD - BC$

$$= 87\sqrt{3} - \frac{87}{\sqrt{3}} = 87\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 87 \times \left(\frac{3-1}{\sqrt{3}}\right) = \frac{2 \times 87}{\sqrt{3}}$$

$$= \frac{2 \times 87 \times \sqrt{3}}{3} = 2 \times 29 \times \sqrt{3} = 58\sqrt{3} \text{ m}$$

Thus, the required distance between the two positions of the balloon is $58\sqrt{3}$ m.

15. In the figure, let AB be the tower and C, D be the two positions of the car.



In right ΔABD,

$$\frac{AB}{AD} = \tan 60^\circ$$

$\Rightarrow \quad \frac{AB}{AD} = \sqrt{3} \Rightarrow AB = \sqrt{3}AD$ ...(1)

In right ΔABC,

$$\frac{AB}{AC} = \tan 30^\circ$$

$\Rightarrow \quad \frac{AB}{AC} = \frac{1}{\sqrt{3}} \Rightarrow AB = \frac{AC}{\sqrt{3}}$ ...(2)

From (1) and (2), we get

$$\sqrt{3}AD = \frac{AC}{\sqrt{3}}$$

$\Rightarrow \quad AC = \sqrt{3} \times \sqrt{3} \times AD = 3AD$

Now, $CD = AC - AD = 3AD - AD = 2AD$

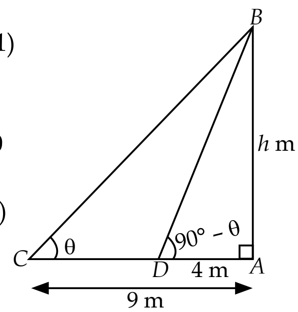
Since the distance $2AD$ is covered in 6 seconds,

$\therefore$ The distance AD will be covered in $\frac{6}{2}$ *i.e.*, 3 seconds.

Thus, the time taken by the car to reach the tower from D is 3 seconds.

16. In the figure, let AB be the tower of height h m. C and D are the two points at a distance of 9 m and 4 m respectively from AB. Let $\angle ACB = \theta \ \therefore \ \angle ADB = 90° - \theta$

In right ΔABC,



$$\frac{AB}{AC} = \tan\theta$$

$\Rightarrow \quad \frac{h}{9} = \tan\theta$ ...(1)

In right ΔABD,

$$\frac{AB}{AD} = \tan(90^\circ - \theta) = \cot\theta$$

$\Rightarrow \quad \frac{h}{4} = \cot\theta$ ...(2)

Multiplying (1) and (2), we get

$$\frac{h}{9} \times \frac{h}{4} = \tan\theta \times \cot\theta$$

$\Rightarrow \quad \frac{h^2}{36} = 1 \Rightarrow h^2 = 36 \Rightarrow h = \pm 6 \quad \therefore h = 6$

Thus, the height of the tower is 6 m.