Post-Mid Term

SOLUTIONS

1. **(b) :** $r_1 = 24$ cm, $r_2 = 15$ cm, $h = 40$ cm

$\therefore \quad l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{40^2 + 9^2} = \sqrt{1681} = 41$ cm

2. **(d) :** Total number of cards = 52

Number of ace cards = 4

$\therefore$ Number of favourable outcomes = 52 – 4 = 48 (Non ace cards)

$\therefore \quad P(\text{not an ace card}) = \dfrac{48}{52} = \dfrac{12}{13}$

3. **(d) :** We know that mean, $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow \quad 8.1 = \dfrac{132 + 5k}{20} \qquad [\Sigma f_i x_i = 132 + 5k, \Sigma f_i = 20 \text{ (Given)}]$

$\Rightarrow \quad 132 + 5k = 162 \Rightarrow 5k = 30 \Rightarrow k = 6$

4. We have, $p(x) = x^2 - p(x + 1) - c = x^2 - px - p - c$

$\therefore \quad \alpha + \beta = p$ and $\alpha\beta = (-p - c)$

Now, $(\alpha + 1)(\beta + 1) = 0$ [Given]

$\Rightarrow \quad \alpha\beta + \alpha + \beta + 1 = 0 \Rightarrow -p - c + p + 1 = 0 \Rightarrow c = 1$

5. $2 \tan^2 45° + \cos^2 30° - \sin^2 60°$

$= 2(1)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{\sqrt{3}}{2}\right)^2 = 2$

6. Let r_1 and r_2 be the radii of two circles.

$\therefore$ According to the question,

$\dfrac{2\pi r_1}{2\pi r_2} = \dfrac{4}{9} \Rightarrow \dfrac{r_1}{r_2} = \dfrac{4}{9}$ ...(i)

Now, $\dfrac{\pi r_1^2}{\pi r_2^2} = \left(\dfrac{r_1}{r_2}\right)^2 = \left(\dfrac{4}{9}\right)^2 = \dfrac{16}{81}$ [Using (i)]

7. We have, $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

Here, $a = 3\sqrt{3}$, $b = 10$ and $c = \sqrt{3}$

$\therefore$ Discriminant $(D) = b^2 - 4ac = 10^2 - 4(3\sqrt{3})(\sqrt{3})$

$= 100 - 36 = 64$

8. Prime numbers which are less than 20 and starting from 2 are 2, 3, 5, 7, 11, 13, 17 and 19 *i.e.*, 8 in number.

$\therefore$ Number of favourable outcomes = 8

Total number of cards from 2 to 101 = 100

$\therefore$ Total number of possible outcomes = 100

$\therefore$ P(number on card is a prime number less than 20)

$= \dfrac{8}{100} = \dfrac{2}{25}$

9. In ΔABC, $\dfrac{AP}{AB} = \dfrac{3}{5}$ ...(i)

and $\dfrac{AQ}{AC} = \dfrac{6}{10} = \dfrac{3}{5}$ ...(ii)

From (i) and (ii), we get $\dfrac{AP}{AB} = \dfrac{AQ}{AC} \Rightarrow PQ \parallel BC$

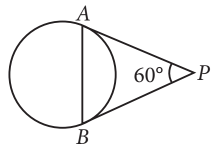
(By converse of Thales theorem)

In ΔABD, $PR \parallel BD$

$\Rightarrow \quad \dfrac{AP}{AB} = \dfrac{AR}{AD}$ (By Thales theorem)

$\Rightarrow \quad \dfrac{3}{5} = \dfrac{4.5}{AD} \Rightarrow AD = \dfrac{4.5 \times 5}{3} = 7.5$ cm

10. Tangents from an external point to a circle are equal in length.



Therefore,

$PA = PB \Rightarrow \Delta PAB$ is isosceles

$\Rightarrow \quad \angle PAB = \angle PBA$

In ΔAPB, $\angle PAB + \angle PBA + \angle APB = 180°$

$\Rightarrow \quad 2\angle PAB = 180° - 60° = 120° \Rightarrow \angle PAB = 60°$

$\Rightarrow \quad \Delta PAB$ is an equilateral triangle.

Hence, $AB = 12$ cm.

11. Let the three parts which are in A.P. = $a - d, a, a + d$.

Now, sum of three parts = 69

$\Rightarrow \quad (a - d) + (a) + (a + d) = 69$

$\Rightarrow \quad 3a = 69$

$\Rightarrow \quad a = 23$

Also, the product of two smaller parts = 483 (Given)

$\Rightarrow \quad (a - d) \times a = 483$ ...(i)

Substituting $a = 23$ in (i), we get

$(23 - d) \times 23 = 483$

$\Rightarrow \quad 23 - d = \dfrac{483}{23} = 21 \Rightarrow d = 23 - 21 = 2$

Hence, the three parts of 69 are 21, 23, 25.

12.

No. of Accidents (x_i)	Frequency (f_i)	$f_i x_i$
0	46	0
1	x	x
2	y	$2y$
3	25	75
4	10	40
5	5	25
	$\Sigma f_i = 86 + x + y = 200$	$\Sigma f_i x_i = 140 + x + 2y$

Now, mean = 1.46 (Given)

$\Rightarrow \quad 1.46 = \dfrac{140 + x + 2y}{200}$

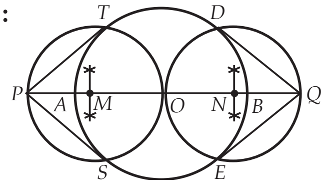
$\Rightarrow \quad 292 = 140 + x + 2y$

$\Rightarrow \quad x + 2y = 152$ ...(i)

Also, $86 + x + y = 200 \Rightarrow x + y = 114$ ...(ii)

Solving (i) and (ii), we get $x = 76$, $y = 38$.

13. Steps of Construction :



Step 1 : Draw a circle of radius 4 cm with O as its centre.

Step 2 : Draw AB as diameter of the circle.

Step 3 : Take P and Q as two points on extended diameter AB such that $OP = OQ = 6$ cm.

Step 4 : Draw perpendicular bisector of OP and OQ intersecting OP and OQ at M and N respectively.

Step 5 : With M as centre and MO as radius, draw a circle intersecting the 1st circle at T and S. With N as centre and QN as radius, draw a circle intersecting the 1st circle at D and E.

Step 6 : Join PT, PS, QD and QE.

$\therefore$ PT, PS, QD and QE are required tangents.

14. Clearly, one round of wire covers 4 mm $\left(=\frac{4}{10}\text{cm}\right)$ of the surface of the cylinder and length of the cylinder is 24 cm.

$\therefore$ Number of rounds to cover 24 cm $= \dfrac{24}{4/10} = 60$

Diameter of the cylinder = 20 cm

$\therefore$ Radius of the cylinder, $r = 10$ cm

Length of wire required in completing one round $= 2\pi r$ $= (2\pi \times 10)$ cm $= 20\,\pi$ cm.

$\therefore$ Length of wire required in covering the whole surface = Length of wire required in completing 60 rounds

$= (20\pi \times 60)$ cm $= 1200\pi$ cm.

Radius of copper wire = 2 mm $= \dfrac{2}{10}$ cm

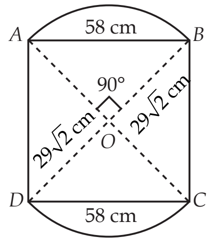
$\therefore$ Volume of wire $= \left(\pi \times \dfrac{2}{10} \times \dfrac{2}{10} \times 1200\pi\right)\text{cm}^3 = 48\pi^2\text{cm}^3$

So, weight of wire $= (48\pi^2 \times 8.88)$ gm $= 426.24\,\pi^2$ gm

15. The length of each side of a square lawn is 58 cm.

$\therefore$ Length of the diagonal of the square

$= 58\sqrt{2}$ cm



Radius of the circle = $29\sqrt{2}$ cm.

Let A be the area of one of the circular ends. Then,

A = Area of a segment of angle 90° in a circle of radius $29\sqrt{2}$ cm.

$$\Rightarrow \quad A = \left\{\frac{22}{7} \times \frac{90°}{360°} - \sin 45° \cos 45°\right\} \times (29\sqrt{2})^2 \text{ cm}^2$$

$$\left[\because \text{ Area of minor segment} = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2\right]$$

$$\Rightarrow \quad A = \left(\frac{11}{14} - \frac{1}{2}\right) \times 29 \times 29 \times 2 \text{ cm}^2 = \frac{3364}{7} \text{ cm}^2$$

$\therefore$ Area of the whole lawn = Area of the square + 2 (Area of a circular end)

$$= \left\{58 \times 58 + 2 \times \frac{3364}{7}\right\} \text{cm}^2 = \left\{3364 + 2 \times \frac{3364}{7}\right\} \text{cm}^2$$

$$= 3364\left(1 + \frac{2}{7}\right) \text{cm}^2 = 3364 \times \frac{9}{7} \text{ cm}^2 = 4325.14 \text{ cm}^2$$