Pre-Mid Term

SOLUTIONS

1. **(c) :** 95 = 5 × 19 and 152 = 2 × 2 × 2 × 19

∴ HCF (95, 152) = 19

2. **(b) :** Since, α and β are the zeroes of $f(x) = x^2 + x + 1$

∴ $\alpha + \beta = -1$ and $\alpha\beta = 1$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-1}{1} = -1$

3. **(d) :** Since, 1 is a root of the equation $ay^2 + ay + 3 = 0$

∴ $a(1)^2 + a(1) + 3 = 0$

⇒ $2a = -3 \Rightarrow a = \frac{-3}{2}$

4. Common difference, $d = a_2 - a_1 = \frac{1-6b}{2b} - \frac{1}{2b}$

$= \frac{1-6b-1}{2b} = \frac{-6b}{2b} = -3$

5. $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$

∴ Exponent of 2 in the prime factorisation of 144 = 4

6. Let two consecutive positive integers be x and $x + 1$. It is given that the product of two consecutive positive integers is 240.

∴ $x(x+1) = 240 \Rightarrow x^2 + x - 240 = 0$, which is the required quadratic equation.

7. $y = 0$ and $y = -5$ are parallel lines, hence they have no common solution.

8. Let first term = a and common difference = d.

According to the question, $5 \times a_5 = 10 \times a_{10}$

⇒ $5(a + 4d) = 10(a + 9d)$

⇒ $5a + 20d = 10a + 90d \Rightarrow a = -14d$

Now, $a_{15} = a + 14d \Rightarrow a_{15} = -14d + 14d = 0$

9. $576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$448 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7$

∴ HCF of 576 and 448 = 64

∴ Number of sections $= \frac{576}{64} + \frac{448}{64} = 9 + 7 = 16$

10. We have, $2qx^2 - (2q - p^2)x - p^2 = 0$

⇒ $2qx^2 - 2qx + p^2x - p^2 = 0$

⇒ $2qx(x - 1) + p^2(x - 1) = 0$

⇒ $(2qx + p^2)(x - 1) = 0$

⇒ $(2qx + p^2) = 0$ or $(x - 1) = 0$

⇒ $x = -\frac{p^2}{2q}$ or $x = 1$

11. We have, $x + \frac{6}{y} = 6$ ...(i)

$3x - \frac{8}{y} = 5$ ...(ii)

Multiplying (i) by 3, we get

$3x + \frac{18}{y} = 18$ ...(iii)

Subtracting (ii) from (iii), we get

$\frac{1}{y}(18 + 8) = 13 \Rightarrow \frac{26}{y} = 13 \Rightarrow y = \frac{26}{13} = 2$

Putting $y = 2$ in (i), we get

$x + \frac{6}{2} = 6 \Rightarrow x + 3 = 6 \Rightarrow x = 6 - 3 = 3$

Hence, $x = 3$ and $y = 2$.

12. Let a = first term and d = common difference

Now, $a_{12} = a + (12 - 1)d$ [$\because a_n = a + (n - 1)d$]

⇒ $-13 = a + 11d$ [Given, $a_{12} = -13$] ...(i)

Also, $S_4 = \frac{4}{2}[2a + (4-1)d]$

⇒ $24 = 2(2a + 3d)$ [Given, $S_4 = 24$]

⇒ $2a + 3d = 12$ ...(ii)

Multiplying (i) by 2 and subtracting from (ii), we get

$3d - 22d = 12 - (-26) \Rightarrow -19d = 38 \Rightarrow d = -2$

Putting the value of d in (i), we get

$a = -13 - 11d = -13 - 11 \times (-2) = -13 + 22 = 9$

∴ $S_{10} = \frac{10}{2}[2a + (10 - 1)d]$

$= 5[2 \times 9 + 9 \times (-2)] = 5(18 - 18) = 0$

13. Let $\sqrt{3}$ be a rational number.

∴ $\sqrt{3} = \frac{a}{b}$, where a and b are co-prime integers, $b \neq 0$.

Squaring both sides, we get $3 = \frac{a^2}{b^2}$.

Multiplying with b on both sides, we get $3b = \frac{a^2}{b}$

Now, LHS = $3 \times b$ = Integer

and RHS $= \frac{a^2}{b} = \frac{\text{Integer}}{\text{Integer}}$ = Rational number

Since, LHS ≠ RHS

∴ Our supposition is wrong.

⇒ $\sqrt{3}$ is an irrational number.

Let $15+17\sqrt{3}$ be a rational number

$\therefore$ $15+17\sqrt{3}=\frac{a}{b}$

$\Rightarrow$ $17\sqrt{3}=\frac{a}{b}-15$

$\Rightarrow$ $\sqrt{3}=\frac{a-15b}{17b}$, which is a contradiction because $\sqrt{3}$ is an irrational number and $\frac{a-15b}{17b}$ is a rational number.

$\therefore$ Our supposition is wrong and hence $15+17\sqrt{3}$ is irrational.

14. Let denominator of a fraction = x

$\Rightarrow$ Numerator = $x - 2$

$\therefore$ Original fraction $=\frac{x-2}{x}$ ...(i)

On adding 1 to numerator as well as denominator, the fraction becomes $\frac{x-2+1}{x+1}$ *i.e.*, $\frac{x-1}{x+1}$. ...(ii)

According to question,

$$\frac{x-2}{x}+\frac{x-1}{x+1}=\frac{19}{15} \Rightarrow \frac{(x-2)(x+1)+x(x-1)}{x(x+1)}=\frac{19}{15}$$

$$\Rightarrow \frac{x^2-2x+x-2+x^2-x}{x^2+x}=\frac{19}{15}$$

$$\Rightarrow \frac{2x^2-2x-2}{x^2+x}=\frac{19}{15}$$

$\Rightarrow$ $30x^2-30x-30=19x^2+19x$

$\Rightarrow$ $11x^2-49x-30=0$ $\Rightarrow 11x^2-55x+6x-30=0$

$\Rightarrow$ $11x(x-5)+6(x-5)=0 \Rightarrow (x-5)(11x+6)=0$

$\Rightarrow$ $x=5,-\frac{6}{11}$

$\therefore$ $x = 5$ (Rejecting fractional value)

Thus, original fraction = $\frac{5-2}{5}=\frac{3}{5}$.

15. By division algorithm, we have

$f(x) = g(x) \times q(x) + r(x)$

$\Rightarrow$ $f(x) - r(x) = g(x) \times q(x) \Rightarrow f(x) + [-r(x)] = g(x) \times q(x)$

Clearly, RHS is divisible by $g(x)$. $\therefore$ LHS is also divisible by $g(x)$. Thus, if we add $-r(x)$ to $f(x)$, then the resulting polynomial is divisible by $g(x)$. On dividing $4x^4 + 2x^3 - 2x^2 + x - 1$, by $x^2 + 2x - 3$, we get

$$\begin{array}{l} x^2+2x-3\,\big)\,4x^4+2x^3-2x^2+x-1\,\big(\,4x^2-6x+22 \\ \qquad 4x^4+8x^3-12x^2 \\ \qquad (-)\quad(-)\quad(+) \\ \hline \qquad\quad -6x^3+10x^2+x-1 \\ \qquad\quad -6x^3-12x^2+18x \\ \qquad\quad (+)\quad(+)\quad(-) \\ \hline \qquad\qquad 22x^2-17x-1 \\ \qquad\qquad 22x^2+44x-66 \\ \qquad\qquad (-)\quad(-)\quad(+) \\ \hline \qquad\qquad\quad -61x+65 \end{array}$$

$\therefore$ $r(x) = -61x + 65$

Hence, we should add $-r(x) = 61x - 65$ to $f(x)$ so that the resulting polynomial is divisible by $g(x)$.