Mid Term

SOLUTIONS

1. **(d) :** π is irrational and $\frac{22}{7}$ is rational.

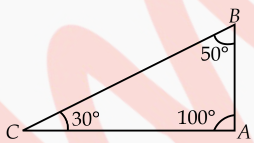
2. **(b) :** Lines parallel to same line are parallel to each other.

3. **(b) :** Diagonals of rhombus are perpendicular to each other.

4. Let $P(x) = x^{21} + 101$ ...(i)
When $P(x)$ is divided by $(x + 1)$, then for finding remainder put $x + 1 = 0$ *i.e.*, $x = -1$
Putting $x = -1$ in (i), we get
$P(-1) = (-1)^{21} + 101 = -1 + 101$
$\therefore$ $P(-1) = 100$
Hence, the remainder is 100.

5. Given, $AB = 5$ cm and $CD = 3$ cm.
or $AB > CD$
Hence, $\angle AOB > \angle COD$
[$\because$ Longer chord subtends greater angle at the centre].

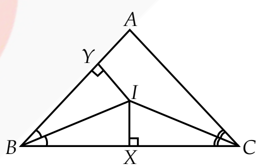
6. We know that, side opposite to greater angle is longer.



Since, $\angle A > \angle B$ $\therefore$ $BC > AC$
Since, $\angle B > \angle C$
$\therefore$ $AC > AB$
$\therefore$ $BC > AC > AB$
Thus, the descending order of sides is $BC > AC > AB$

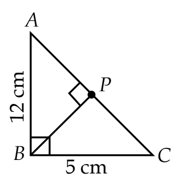
7. We have, $(x - 7, -2) = (2, x + y)$
On comparing the coordinates, we get
$x - 7 = 2 \Rightarrow x = 9$
and $x + y = -2$
$\Rightarrow$ $y = -2 - 9 = -11$
$\therefore$ $7(x - y) = 7(9 - (-11)) = 7 \times 20 = 140$

8. In triangles IBX and IBY,



$\angle IXB = \angle IYB$ (Each equal to 90°)
$\angle IBX = \angle IBY$ (Given)
$IB = IB$ (Common)
$\Delta IBX \cong \Delta IBY$
(By AAS congruency)
$IX = IY$ (By C.P.C.T.)

9. Area of $\Delta ABC = \frac{1}{2} \times BC \times AB$



$= \frac{1}{2} \times 12 \times 5$ ...(i)

Also, area of $\Delta ABC = \frac{1}{2} \times AC \times BP$ ...(ii)

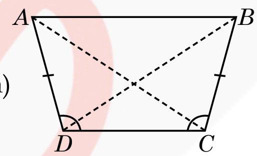
$= \frac{1}{2} \times 13 \times BP$

From (i) and (ii), we get

$\therefore$ $\frac{1}{2} \times 13 \times BP = \frac{1}{2} \times 12 \times 5$

$\Rightarrow$ $13 \times BP = 12 \times 5 \Rightarrow BP = \frac{60}{13} = 4.62$ cm

10. Join BD and AC.



In ΔDCB and ΔCDA,
$DC = DC$ (Common)
$AD = BC$ (Given)
$\angle ADC = \angle BCD$ (Given)
$\therefore$ $\Delta DCB \cong \Delta CDA$ (By SAS congruency)
$\Rightarrow$ $\angle DAC = \angle DBC$ (By C.P.C.T.)
$\therefore$ A, B, C and D are on con-cyclic.
i.e., Points A, B, C and D lie on a circle.

11. Since, $PQ \parallel BC$
$\angle QAC = \angle ACB$ (Alternate interior angles)
$\therefore$ $\angle QAC = 42°$
Also, in ΔABC, $\angle CAB + \angle B + \angle C = 180°$
[By angle sum property of triangle]
$\Rightarrow$ $\angle CAB = 180° - 64° - 42° = 74°$
$\therefore$ $\angle QAB = \angle QAC + \angle CAB = 42° + 74° = 116°$

12. In triangle OAP and OBP,
$AP = PB$ (Given)
$\angle OPA = \angle OPB$ (Each equal to 90°)
$OP = OP$ (Common)
So, $\Delta OPA \cong \Delta OPB$ (By SAS congruency)
$\therefore$ $OA = OB$ ...(i) (By C.P.C.T.)
Similarly, $\Delta OQB \cong \Delta OQC$ (By SAS congruence)
$\therefore$ $OB = OC$ ...(ii) (By C.P.C.T.)
From (i) and (ii), we get
$OA = OB = OC$

13. Let the two numbers be x and y such that $x > y$.
According to the question, we have
$4x = 10y + 2 \Rightarrow 2x = 5y + 1$, which is required linear equation in two variables.
When $x = 8$, $2 \times 8 = 5y + 1 \Rightarrow 5y = 15 \Rightarrow y = 3$
When $x = 3$, $2 \times 3 = 5y + 1 \Rightarrow 5y = 5 \Rightarrow y = 1$

14. **Given :** A $\|^{gm}$ $ABCD$ and P, Q are two points on AB and CD.
To prove : (i) $ar(\Delta AQB) = ar(\Delta APD) + ar(\Delta BPC)$
(ii) $ar(\Delta DPC) = ar(\Delta AQD) + ar(\Delta BQC)$
Proof : $ar(\Delta AQB) = \frac{1}{2}$ $ar(\|^{gm}$ $ABCD)$ ...(i)
($\because$ ΔAQB and $\|^{gm}$ $ABCD$ are on the same base AB and between the same parallel lines AB and CD)

$ar(\Delta DPC) = \frac{1}{2}\ ar(||^{gm}\ ABCD)$ ...(ii)

Now, $ar(\Delta APD) + ar(\Delta BPC) + ar(\Delta DPC) = ar(||^{gm}\ ABCD)$

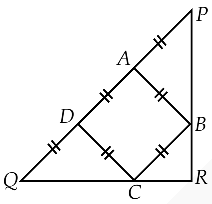
$\Rightarrow\quad ar(\Delta APD) + ar(\Delta BPC) + \frac{1}{2}\ ar(||^{gm}\ ABCD)$ [Using (ii)]
$= ar(||^{gm}\ ABCD)$

$\Rightarrow\quad ar(\Delta APD) + ar(\Delta BPC) = \frac{1}{2}\ ar\ (||^{gm}\ ABCD)$ ...(iii)

Similarly, $ar(\Delta AQD) + ar(\Delta BQC) = \frac{1}{2}\ ar(||^{gm}\ ABCD)$...(iv)

From (i) and (iii), $ar(\Delta AQB) = ar(\Delta APD) + ar(\Delta BPC)$
From (ii) and (iv), $ar(\Delta DPC) = ar(\Delta AQD) + ar(\Delta BQC)$.

15. Given : $ABCD$ is a rhombus.
And $PADQ$ is straight line such that
$PA = AD = QD$
To prove : $\angle PRQ = 90°$
Proof : ΔDQC is an isosceles triangle.
$\therefore\quad DQ = DC$



$\Rightarrow\quad \angle DQC = \angle DCQ$
(Angle opposite to equal sides are equal)

$\angle ADC = \angle DQC + \angle DCQ$ (By exterior angle property)
$= 2\angle DQC$

ΔPAB is an isosceles triangle.
$\therefore\quad AP = AB$
$\Rightarrow\quad \angle APB = \angle ABP$ (Angle opposite to equal sides are equal)

$\angle DAB = \angle APB + \angle ABP$ (By exterior angle property)
$= 2\angle APB$

Now, $\angle ADC + \angle DAB = 180°$ (Co-interior angles)
$\Rightarrow\quad 2\angle DQC + 2\angle APB = 180°$
$\Rightarrow\quad \angle DQC + \angle APB = 90°$
In ΔPQR,
$\angle QPR + \angle PQR + \angle PRQ = 180°$
$\Rightarrow\quad \angle PRQ = 180° - 90° \quad \therefore\ \angle PRQ = 90°$