Pre-Mid Term

SOLUTIONS

1. **(a) :** (b) Every natural number is a whole number.
(c) Every integer is a rational number.
(d) Every natural number is a rational number.

2. **(a) :** We have, $4x^4 + 0x^3 + 0x^5 + 5x + 7 = 4x^4 + 5x + 7$
Here, the highest power of x is 4.
$\therefore$ Degree of the given polynomial is 4.

3. **(a) :** We know that, line $x = a$ or $x = -a$ is always parallel to y-axis.
Also, $x = 0$ is the equation of y-axis.
$\therefore$ $x = 7$ is also parallel to $x = 0$.

4. $\left[\left((16)^{\frac{-1}{2}}\right)^{\frac{-1}{4}}\right]^2 = \left[\left((4^2)^{\frac{-1}{2}}\right)^{\frac{-1}{4}}\right]^2 = \left[4^{\frac{-1}{1}\times\frac{-1}{4}}\right]^2 = 4^{\frac{1}{4}\times 2}$
$= (4)^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2^1 = 2$

5. We have, $f(x) = 7x^2 - 3x + 7$
Now, $f(2) = 7(2)^2 - 3(2) + 7 = 28 - 6 + 7 = 29$
Also, $f(-1) = 7(-1)^2 - 3(-1) + 7 = 7 + 3 + 7 = 17$
Also, $f(0) = 7$
$\therefore$ $f(2) + f(-1) + f(0) = 29 + 17 + 7 = 53$

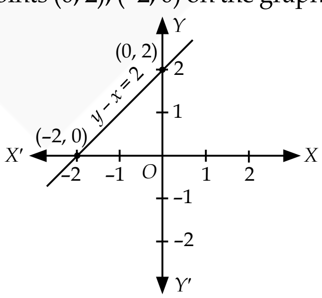
6. The given equation is $3x + 2y + 7 = 0$ ...(i)
Put $x = \alpha - 1$, $y = 2\alpha - 1$ in (i), we get
$3(\alpha - 1) + 2(2\alpha - 1) + 7 = 0 \Rightarrow 3\alpha - 3 + 4\alpha - 2 + 7 = 0$
$\Rightarrow 7\alpha - 5 + 7 = 0 \Rightarrow 7\alpha = -2 \therefore \alpha = \frac{-2}{7}$

7. We know, $\frac{1}{5} = 0.2$ and $\frac{2}{5} = 0.4$
$\therefore \frac{1}{5} < 0.303003000... < \frac{2}{5}$
0.303003000... being a non-terminating, non-recurring decimal expression, is an irrational number.

8. We have, $y - x = 2$ ...(i)
We have, the following table of solutions for (i)

x	0	–2
y	2	0

Plotting the points (0, 2), (–2, 0) on the graph paper, we get



From the graph, (0, 2) and (–2, 0) are the points of intersection of $y - x = 2$ with y-axis and x-axis respectively.

9. We have, $4^{44} + 4^{44} + 4^{44} + 4^{44} = 4^x$
$\Rightarrow 4^{44}[1 + 1 + 1 + 1] = 4^x \Rightarrow 4^{44}(4) = 4^x$
$\Rightarrow 4^{44+1} = 4^x \Rightarrow 4^{45} = 4^x$
$\therefore x = 45$ (On comparing powers)

10. (i) N, Q (ii) T, P (iii) Q, R, S (iv) L

11. We have, $\frac{5+\sqrt{3}}{7-4\sqrt{3}} = 94a + 3\sqrt{3}b$
On rationalising the L.H.S. we get
$\frac{5+\sqrt{3}}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = 94a + 3\sqrt{3}b$
$\Rightarrow \frac{35 + 20\sqrt{3} + 7\sqrt{3} + 12}{49 - 48} = 94a + 3\sqrt{3}b$
$\Rightarrow \frac{27\sqrt{3} + 47}{1} = 94a + 3\sqrt{3}b$
On comparing we get, $3b = 27$ and $47 = 94a$
$\Rightarrow b = 9$ and $a = \frac{1}{2}$
So, $a + b = \frac{1}{2} + 9 = \frac{19}{2} = 9.5$

12. We have, $(3x - 1)^3 = 6a_3x^3 + a_2x^2 + a_1x + a_0$
$\Rightarrow (3x)^3 - (1)^3 - 3 \times 3x \times 1(3x - 1)$
$= 6a_3x^3 + a_2x^2 + a_1x + a_0$ $[\because (a - b)^3 = a^3 - b^3 - 3ab(a - b)]$
$\Rightarrow 27x^3 - 1 - 9x(3x - 1) = 6a_3x^3 + a_2x^2 + a_1x + a_0$
$\Rightarrow 27x^3 - 27x^2 + 9x - 1 = 6a_3x^3 + a_2x^2 + a_1x + a_0$
Comparing the coefficient of x^3, x^2, x and x^0, we get
$6a_3 = 27 \Rightarrow a_3 = 9/2$, $a_2 = -27$, $a_1 = 9$ and $a_0 = -1$
Now, $a_3 + a_2 + a_1 + a_0 = \frac{9}{2} - 27 + 9 - 1$
$= \frac{9}{2} - 19 = \frac{9-38}{2} = \frac{-29}{2} = -14.5$

13. Given, ΔABC and ΔABD are equilateral triangles.
In ΔABC, $AB = BC = AC = a$ units
In ΔABD, $AB = BD = AD = a$ units
Altitude of triangle, $OC = \sqrt{a^2 - \left(\frac{a}{2}\right)^2}$
$\Rightarrow OC = OD = \frac{\sqrt{3}}{2}a$
$\therefore$ Coordinates of C and D are $\left(0, \frac{\sqrt{3}}{2}a\right)$ and $\left(0, \frac{-\sqrt{3}}{2}a\right)$.

14. Let total number of students in the class be y.

Let the number of boys in the class be x, then the required equation is $x = \frac{3}{4}y \Rightarrow y = \frac{4}{3}x$

When $x = 0$, $y = 0$

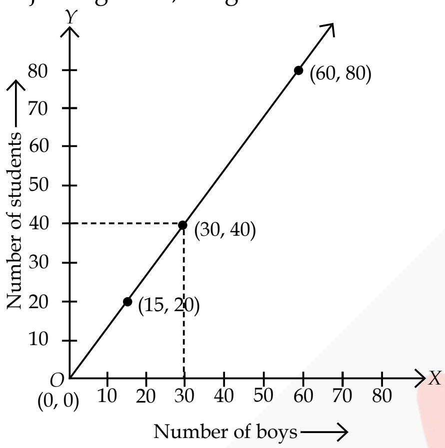
When $x = 15$, $y = 20$

When $x = 60$, $y = 80$

$\therefore$ We have the following table of solutions :

x	0	15	60
y	0	20	80

Plotting the points (0, 0), (15, 20) and (60, 80) on graph paper and joining them, we get



From the graph, it is clear that there are 30 boys in a class of 40 students.

15. Let $p(x) = 2x^3 - 7x^2 - 3x + c$

If $p(x)$ is exactly divisible by $(2x + 3)$, then by factor theorem, we have

$$p\left(-\frac{3}{2}\right) = 0 \qquad \left(\because\ 2x + 3 = 0 \Rightarrow x = -\frac{3}{2}\right)$$

$$\Rightarrow 2\left(-\frac{3}{2}\right)^3 - 7\left(-\frac{3}{2}\right)^2 - 3\left(-\frac{3}{2}\right) + c = 0$$

$$\Rightarrow 2\left(-\frac{27}{8}\right) - 7\left(\frac{9}{4}\right) + \frac{9}{2} + c = 0$$

$$\Rightarrow -\frac{27}{4} - \frac{63}{4} + \frac{9}{2} + c = 0$$

$$\Rightarrow -\frac{45}{2} + \frac{9}{2} + c = 0$$

$$\Rightarrow -18 + c = 0 \Rightarrow c = 18$$

$$\therefore p(x) = 2x^3 - 7x^2 - 3x + 18$$

$$= 2x^3 + 3x^2 - 10x^2 - 15x + 12x + 18$$

$$= x^2(2x + 3) - 5x(2x + 3) + 6(2x + 3)$$

$$= (2x + 3)(x^2 - 5x + 6)$$

$$= (2x + 3)(x^2 - 2x - 3x + 6)$$

$$= (2x + 3)[x(x - 2) - 3(x - 2)]$$

$$= (2x + 3)(x - 2)(x - 3)$$