CHAPTER 1

Number Systems

NCERT FOCUS

SOLUTIONS

EXERCISE - 1.1

1. Yes, zero is a rational number. Because 0 can be written in the form of p/q.

$0 = 0/1 = \frac{0}{2} = \frac{0}{3}$ etc. Denominator q can also be taken as negative integer.

2. We have, $q_1 = \frac{3+4}{2} = \frac{7}{2}; 3 < \frac{7}{2} < 4$

$$q_2 = \frac{3+\frac{7}{2}}{2} = \frac{\frac{13}{2}}{2} = \frac{13}{4} \quad \therefore\ 3 < \frac{13}{4} < \frac{7}{2} < 4$$

$$q_3 = \frac{4+\frac{7}{2}}{2} = \frac{\frac{15}{2}}{2} = \frac{15}{4} \quad \therefore\ 3 < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < 4$$

$$q_4 = \frac{\frac{7}{2}+\frac{13}{4}}{2} = \frac{\frac{14+13}{4}}{2} = \frac{\frac{27}{4}}{2} = \frac{27}{8}$$

$$\therefore \quad 3 < \frac{13}{4} < \frac{27}{8} < \frac{7}{2} < \frac{15}{4} < 4$$

$$q_5 = \frac{1}{2}\left(\frac{7}{2}+\frac{15}{4}\right) = \frac{1}{2}\left(\frac{14+15}{4}\right) = \frac{29}{8}$$

$$\therefore \quad 3 < \frac{13}{4} < \frac{27}{8} < \frac{7}{2} < \frac{29}{8} < \frac{15}{4} < 4$$

$$q_6 = \frac{1}{2}\left(\frac{13}{4}+\frac{27}{8}\right) = \frac{1}{2}\left(\frac{26+27}{8}\right) = \frac{53}{16}$$

$$\therefore \quad 3 < \frac{13}{4} < \frac{53}{16} < \frac{27}{8} < \frac{7}{2} < \frac{29}{8} < \frac{15}{4} < 4$$

Thus, the six rational numbers between 3 and 4 are

$$\frac{7}{2}, \frac{13}{4}, \frac{15}{4}, \frac{27}{8}, \frac{29}{8} \text{ and } \frac{53}{16}.$$

3. Since, we need to find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, therefore, multiply the numerator and denominator of $\frac{3}{5}$ and $\frac{4}{5}$ by 6.

$$\therefore \quad \frac{3}{5} = \frac{3\times 6}{5\times 6} = \frac{18}{30} \text{ and } \frac{4}{5} = \frac{4\times 6}{5\times 6} = \frac{24}{30}$$

$\therefore$ Five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{19}{30}$, $\frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

i.e., $\frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$.

4. (i) True, as the collection of all natural numbers and 0 is called whole numbers.

(ii) False, as negative integers are not whole numbers.

(iii) False, as rational numbers of the form p/q, where $q \neq 0$ and q does not divide p completely, are not whole numbers.

EXERCISE - 1.2

1. (i) True; because all irrational numbers can be represented on numbers line. And we know that numbers which can be represented on number line are known as real numbers.

(ii) False; because negative numbers cannot be the square root of any natural number.

(iii) False; because rational numbers are also a part of real numbers.

2. No, if we take a positive integer say 9, its square root is 3, which is a rational number.

3. Draw a line $X'OX$ and take point A on it such that OA = 1 unit. Draw $BA \perp OA$ such that BA = 1 unit. Join OB. We get, $OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2+1^2} = \sqrt{2}$ units.

Now, draw $BB_1 \perp OB$ such that BB_1 = 1 unit. Join OB_1.

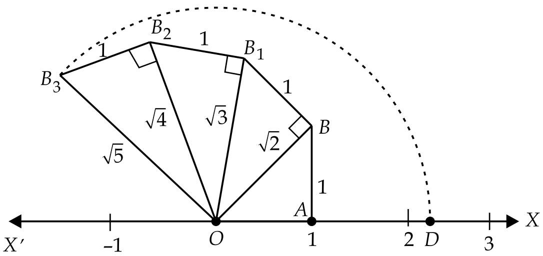
We get, $OB_1 = \sqrt{OB^2 + BB_1^2} = \sqrt{(\sqrt{2})^2+1^2} = \sqrt{3}$ units.

Next, draw $B_1B_2 \perp OB_1$ such that B_1B_2 = 1 unit. Join OB_2.

We get, $OB_2 = \sqrt{OB_1^2 + B_1B_2^2} = \sqrt{(\sqrt{3})^2+1^2} = \sqrt{4}$ units.

Again, draw $B_2B_3 \perp OB_2$ such that B_2B_3 = 1 unit. Join OB_3.

We get, $OB_3 = \sqrt{OB_2^2 + B_2B_3^2} = \sqrt{(\sqrt{4})^2+1^2} = \sqrt{5}$ units.



Take O as centre and OB_3 as radius, draw an arc which cuts OX at D. Point D represents the number $\sqrt{5}$ on number line.

4. Do it yourself.

EXERCISE - 1.3

1. (i) We have, $\frac{36}{100} = 0.36$

$\therefore$ The decimal expansion of $\frac{36}{100}$ is terminating.

(ii) On dividing 1 by 11, we have

$$\begin{array}{r} 11\overline{)1.000000}(0.090909...\\ -99\\ \hline 100\\ -99\\ \hline 100\\ -99\\ \hline 1 \end{array}$$

$\therefore \quad \frac{1}{11} = 0.090909... = 0.\overline{09}$

Thus, the given decimal expansion is non-terminating repeating.

(iii) We have, $4\frac{1}{8} = \frac{33}{8}$

Now,

$$\begin{array}{r} 8\overline{)33.000}(4.125\\ -32\\ \hline 10\\ -8\\ \hline 20\\ -16\\ \hline 40\\ -40\\ \hline 0 \end{array}$$

$\therefore \quad 4\frac{1}{8} = 4.125.$ Thus, the decimal expansion is terminating.

(iv) On dividing 3 by 13, we have

$$\begin{array}{r} 13\overline{)3.00000000}(0.23076923...\\ -26\\ \hline 40\\ -39\\ \hline 100\\ -91\\ \hline 90\\ -78\\ \hline 120\\ -117\\ \hline 30\\ -26\\ \hline 40\\ -39\\ \hline 1 \end{array}$$

$\therefore \quad 3/13 = 0.23076923... = 0.\overline{230769}$

Thus, the decimal expansion of 3/13 is non-terminating repeating.

(v) On dividing 2 by 11, we have

$$\begin{array}{r} 11\overline{)2.0000}(0.1818...\\ -11\\ \hline 90\\ -88\\ \hline 20\\ -11\\ \hline 90\\ -88\\ \hline 2 \end{array}$$

$\therefore \quad \frac{2}{11} = 0.1818... = 0.\overline{18}$

Thus, the decimal expansion of 2/11 is non-terminating repeating.

(vi) Dividing 329 by 400, we have

$$\begin{array}{r} 400\overline{)329.0000}(0.8225\\ -3200\\ \hline 900\\ -800\\ \hline 1000\\ -800\\ \hline 2000\\ -2000\\ \hline 0 \end{array}$$

$\therefore \quad \frac{329}{400} = 0.8225$

Thus, the decimal expansion of 329/400 is terminating.

2. We are given that $\frac{1}{7} = 0.\overline{142857}$

$\therefore \quad \frac{2}{7} = 2\times\frac{1}{7} = 2\times(0.\overline{142857}) = 0.\overline{285714},$

$\frac{3}{7} = 3\times\frac{1}{7} = 3\times(0.\overline{142857}) = 0.\overline{428571},$

$\frac{4}{7} = 4\times\frac{1}{7} = 4\times(0.\overline{142857}) = 0.\overline{571428},$

$\frac{5}{7} = 5\times\frac{1}{7} = 5\times(0.\overline{142857}) = 0.\overline{714285}$ and

$\frac{6}{7} = 6\times\frac{1}{7} = 6\times(0.\overline{142857}) = 0.\overline{857142}$

Thus, without actually doing the long division we can predict the decimal expansions of the given rational numbers.

3. (i) Let $x = 0.\overline{6} = 0.6666...$ ... (1)

Multiplying (1) by 10, we get

$10x = 6.6666...$ ... (2)

Subtracting (1) from (2), we get

$10x - x = 6.6666... - 0.6666...$

$\Rightarrow \quad 9x = 6 \Rightarrow x = \frac{6}{9} = \frac{2}{3}$. Thus, $0.\overline{6} = \frac{2}{3}$

(ii) Let $x = 0.4\overline{7} = 0.4777...$ ... (1)

Multiplying (1) by 10, we get

$10x = 4.777...$ ... (2)

Subtracting (1) from (2), we get

$10x - x = 4.777... - 0.4777...$

$\Rightarrow \quad 9x = 4.3 \Rightarrow x = \frac{43}{90}$. Thus, $0.4\overline{7} = \frac{43}{90}$

(iii) Let $x = 0.\overline{001} = 0.001001...$ ... (1)

Multiplying (1) by 1000, we get

$\Rightarrow \quad 1000x = 1.001001...$ ... (2)

Subtracting (1) from (2), we get

$1000x - x = (1.001...) - (0.001...)$

$\Rightarrow \quad 999x = 1 \Rightarrow x = \frac{1}{999}$. Thus, $0.\overline{001} = \frac{1}{999}$

4. Let $x = 0.99999...$ ... (1)

Multiplying (1) by 10, we get

$10x = 9.9999...$ ... (2)

Subtracting (1) from (2), we get

$10x - x = (9.9999...) - (0.9999...)$

$\Rightarrow \quad 9x = 9 \Rightarrow x = \frac{9}{9} = 1$. Thus, $0.9999... = 1$

As, 0.9999... goes on forever, there is no gap between 1 and 0.9999... . Hence, both are equal.

5. In 1/17, the number of entries in the repeating block of digits is less than the divisor *i.e.*, 17.

$\therefore$ The maximum number of digits in the repeating block is 16. To perform the long division, we have

$$\begin{array}{r l}
 & 0.0588235294117647\ldots \\
17) & 1.0000000000000000 \\
 & -85 \\
 & 150 \\
 & -136 \\
 & 140 \\
 & -136 \\
 & 40 \\
 & -34 \\
 & 60 \\
 & -51 \\
 & 90 \\
 & -85 \\
 & 50 \\
 & -34 \\
 & 160 \\
 & -153 \\
 & 70 \\
 & -68 \\
 & 20 \\
 & -17 \\
 & 30 \\
 & -17 \\
 & 130 \\
 & -119 \\
 & 110 \\
 & -102 \\
 & 80 \\
 & -68 \\
 & 120 \\
 & -119 \\
 & 1
\end{array}$$

The remainder 1 is the same digit from which we started the division.

$\therefore \frac{1}{17} = 0.\overline{0588235294117647}$

Thus, there are 16 digits in the repeating block in the decimal expansion of 1/17. Hence, our answer is verified.

6. Let us look decimal expansion of the following rational numbers:

$\frac{3}{2} = \frac{3\times 5}{2\times 5} = \frac{15}{10} = 1.5$ [Denominator = 2 = 2^1]

$\frac{1}{5} = \frac{1\times 2}{5\times 2} = \frac{2}{10} = 0.2$ [Denominator = 5 = 5^1]

$\frac{7}{8} = \frac{7\times 125}{8\times 125} = \frac{875}{1000} = 0.875$ [Denominator = 8 = 2^3]

$\frac{8}{125} = \frac{8\times 8}{125\times 8} = \frac{64}{1000} = 0.064$ [Denominator = 125 = 5^3]

$\frac{13}{20} = \frac{13\times 5}{20\times 5} = \frac{65}{100} = 0.65$ [Denominator = 20 = $2^2 \times 5^1$]

$\frac{17}{16} = \frac{17\times 625}{16\times 625} = \frac{10625}{10000} = 1.0625$ [Denominator = 16 = 2^4]

We observe that the prime factorisation of q (*i.e.*, denominator) has only powers of 2 or powers of 5 or powers of both.

7. $\sqrt{2} = 1.414213562\ldots$; $\sqrt{3} = 1.732050807\ldots$; $\sqrt{5} = 2.236067977\ldots$

8. To find irrational numbers, firstly we will divide 5 by 7 and 9 by 11.

Now,

$$\begin{array}{r l}
 & 0.714285\ldots \\
7) & 5.000000 \\
 & -49 \\
 & 10 \\
 & -7 \\
 & 30 \\
 & -28 \\
 & 20 \\
 & -14 \\
 & 60 \\
 & -56 \\
 & 40 \\
 & -35 \\
 & 5
\end{array}
\quad \text{and} \quad
\begin{array}{r l}
 & 0.8181\ldots \\
11) & 9.0000 \\
 & -88 \\
 & 20 \\
 & -11 \\
 & 90 \\
 & -88 \\
 & 20 \\
 & -11 \\
 & 9
\end{array}$$

$\therefore \frac{5}{7} = 0.\overline{714285}$ and $\frac{9}{11} = 0.\overline{81}$

Thus, three irrational numbers between $0.\overline{714285}$ and $0.\overline{81}$ are 0.750750075000750..., 0.767076700767000767..., 0.78080078008000780...

9. (i) $\because$ 23 is not a perfect square.

$\therefore$ $\sqrt{23}$ is an irrational number.

(ii) $\because$ 225 = 15 × 15 = 15^2 $\therefore$ 225 is a perfect square.

Thus, $\sqrt{225}$ is a rational number.

(iii) $\because$ 0.3796 is a terminating decimal.

$\therefore$ It is a rational number.

(iv) 7.478478... = $7.\overline{478}$. Since, $7.\overline{478}$ is a non-terminating recurring (repeating) decimal.

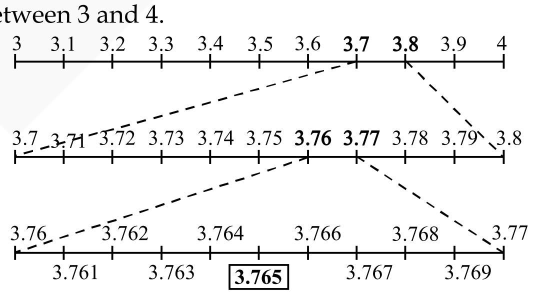
$\therefore$ It is a rational number.

(v) Since, 1.101001000100001... is a non-terminating, non-repeating decimal number.

$\therefore$ It is an irrational number.

EXERCISE - 1.4

1. 3.765 lies between 3 and 4.



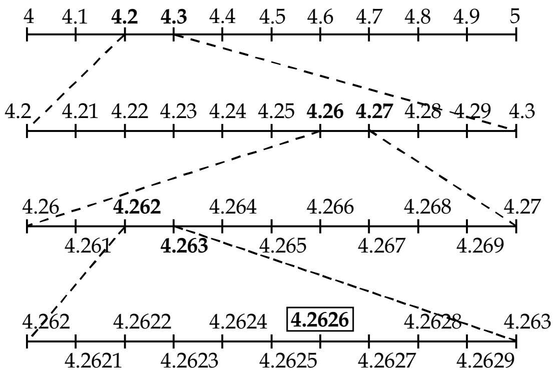
(i) 3.7 lies between 3 and 4.

(ii) 3.76 lies between 3.7 and 3.8.

(iii) 3.765 lies between 3.76 and 3.77.

2. We have, $4.\overline{26} = 4.2626\ldots$

Now, 4.2626 … lies between 4 and 5.



(i) 4.2 lies between 4 and 5.

(ii) 4.26 lies between 4.2 and 4.3.

(iii) 4.262 lies between 4.26 and 4.27.

(iv) 4.2626 lies between 4.262 and 4.263.

EXERCISE - 1.5

1. (i) We know that difference of a rational and an irrational number is always irrational.

$\therefore$ $2-\sqrt{5}$ is an irrational number.

(ii) $(3+\sqrt{23})-\sqrt{23}=3+\sqrt{23}-\sqrt{23}=3,$ which is a rational number.

(iii) Since, $\dfrac{2\sqrt{7}}{7\sqrt{7}}=\dfrac{2}{7},$ which is a rational number.

(iv) $\because$ The quotient of rational and irrational number is an irrational number.

$\therefore$ $\dfrac{1}{\sqrt{2}}$ is an irrational number.

(v) $\because$ Product of a rational and an irrational number is an irrational number.

$\therefore$ 2π is an irrational number.

2. (i) We have, $(3+\sqrt{3})(2+\sqrt{2})=2(3+\sqrt{3})+\sqrt{2}(3+\sqrt{3})$

$=6+2\sqrt{3}+3\sqrt{2}+\sqrt{6}$

(ii) We have, $(3+\sqrt{3})(3-\sqrt{3})=(3)^2-(\sqrt{3})^2$

$= 3^2 - 3 = 9 - 3 = 6$

(iii) $(\sqrt{5}+\sqrt{2})^2=(\sqrt{5})^2+(\sqrt{2})^2+2(\sqrt{5})(\sqrt{2})$

$=5+2+2\sqrt{10}=7+2\sqrt{10}$

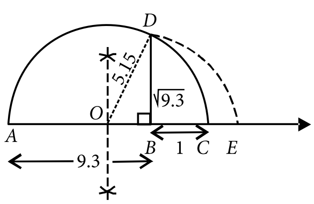
(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})=(\sqrt{5})^2-(\sqrt{2})^2 = 5 - 2 = 3$

3. When we measure the length of a line with a scale or with any other device, we only get an approximate rational value, *i.e.*, c and d both are irrational.

$\therefore$ c/d is irrational and hence π is irrational. Thus, there is no contradiction in saying that π is irrational.

4. Draw a line segment AB = 9.3 units and extend it to C such that BC = 1 unit and AC = 10.3 units.

Find mid-point of AC and mark it as O. Draw a semicircle taking O as centre and AO as radius, where $AO=\dfrac{AC}{2}=5.15$ units.



Draw $BD \perp AC$ and intersecting the semicircle at D.

In ΔOBD, $BD^2 = OD^2 - OB^2$

$\Rightarrow$ $BD^2 = (5.15)^2 - (4.15)^2 = (5.15 + 4.15)(5.15 - 4.15)$

$\Rightarrow$ $BD = \sqrt{9.3}$ units.

To represent $\sqrt{9.3}$ units on the number line, let us treat the line BC as the number line, with B as zero, C as 1, and so on.

Draw an arc with centre B and radius $BD = \sqrt{9.3}$ units, which intersects the number line BC (produced) at E.

$BD = BE = \sqrt{9.3}$ units

$\therefore$ E represents $\sqrt{9.3}$

5. (i) $\dfrac{1}{\sqrt{7}}=\dfrac{1\times\sqrt{7}}{\sqrt{7}\times\sqrt{7}}=\dfrac{\sqrt{7}}{7}$

(ii) $\dfrac{1}{\sqrt{7}-\sqrt{6}}=\dfrac{(\sqrt{7}+\sqrt{6})}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$

$=\dfrac{(\sqrt{7}+\sqrt{6})}{(\sqrt{7})^2-(\sqrt{6})^2}=\dfrac{(\sqrt{7}+\sqrt{6})}{7-6}=\sqrt{7}+\sqrt{6}$

(iii) $\dfrac{1}{\sqrt{5}+\sqrt{2}}=\dfrac{(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$

$=\dfrac{(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}=\dfrac{(\sqrt{5}-\sqrt{2})}{5-2}=\dfrac{(\sqrt{5}-\sqrt{2})}{3}$

(iv) $\dfrac{1}{\sqrt{7}-2}=\dfrac{(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)}$

$=\dfrac{(\sqrt{7}+2)}{(\sqrt{7})^2-(2)^2}=\dfrac{(\sqrt{7}+2)}{7-4}=\dfrac{\sqrt{7}+2}{3}$

EXERCISE - 1.6

1. (i) $\because$ $64 = 8 \times 8 = 8^2$

$\therefore$ $(64)^{1/2} = (8^2)^{1/2} = 8^{2 \times 1/2}$ $[\because (a^m)^n = a^{m \times n}]$

$= 8$

(ii) $\because$ $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$\therefore$ $(32)^{1/5} = (2^5)^{1/5} = 2^{5 \times 1/5}$ $[\because (a^m)^n = a^{m \times n}]$

$= 2$

(iii) $\because$ $125 = 5 \times 5 \times 5 = 5^3$

$\therefore$ $(125)^{1/3} = (5^3)^{1/3} = 5^{3 \times 1/3}$ $[\because (a^m)^n = a^{m \times n}]$

$= 5$

2. (i) $\because$ $9 = 3 \times 3 = 3^2$

$\therefore$ $(9)^{3/2} = (3^2)^{3/2} = 3^{2 \times 3/2}$ $[\because (a^m)^n = a^{m \times n}]$

$= 3^3 = 27$

(ii) $\because$ $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$\therefore$ $(32)^{2/5} = (2^5)^{2/5} = 2^{5 \times 2/5}$ $[\because (a^m)^n = a^{m \times n}]$

$= 2^2 = 4$

(iii) $\because$ $16 = 2 \times 2 \times 2 \times 2 = 2^4$

$\therefore$ $(16)^{3/4} = (2^4)^{3/4} = 2^{4 \times 3/4}$ $[\because (a^m)^n = a^{m \times n}]$

$= 2^3 = 8$

(iv) $\because$ $125 = 5 \times 5 \times 5 = 5^3$

$\therefore$ $(125)^{-1/3} = (5^3)^{-1/3} = 5^{3 \times (-1/3)}$ $[\because (a^m)^n = a^{m \times n}]$

$= 5^{-1} = 1/5$ $[\because a^{-n} = 1/a^n]$

3. (i) Since, $2^{2/3} \cdot 2^{1/5} = 2^{2/3 + 1/5}$ $[\because a^m \cdot a^n = a^{m+n}]$

$= 2^{13/15}$

(ii) $\left(\frac{1}{3^3}\right)^7 = (3^{-3})^7 = 3^{-3\times 7} = 3^{-21} = \frac{1}{3^{21}}$ $\left[\because a^{-n} = \frac{1}{a^n}\right]$

(iii) $\frac{11^{1/2}}{11^{1/4}} = 11^{\frac{1}{2}} \div 11^{\frac{1}{4}} = 11^{\frac{1}{2}-\frac{1}{4}}$ $[\because a^m \div a^n = a^{m-n}]$

$= 11^{\frac{1}{4}}$

(iv) $7^{1/2} \cdot 8^{1/2} = (7 \times 8)^{1/2}$ $[\because a^m \times b^m = (ab)^m]$

$= (56)^{1/2}$