CHAPTER 12

# Heron's Formula

NCERT FOCUS

## SOLUTIONS

### EXERCISE - 12.1

**1.** Let each side of the equilateral triangle be $a$.

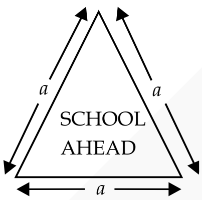
Semi-perimeter of the triangle, $s = \frac{a+a+a}{2} = \frac{3a}{2}$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{s(s-a)(s-a)(s-a)} = \sqrt{s(s-a)^3}$

$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)^3} = \sqrt{\frac{3a}{2}\times\left(\frac{a}{2}\right)^3}$

$= \sqrt{\frac{3a^4}{16}} = \frac{\sqrt{3}}{4}a^2$ sq. units.




Now, its perimeter is 180 cm.

$\therefore$ $a + a + a = 180$ cm

$\Rightarrow$ $3a = 180 \Rightarrow a = \frac{180}{3} = 60$ cm

Thus, area of the triangle $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(60)^2 = 900\sqrt{3}\text{ cm}^2$

**2.** The sides of the triangular wall are

Let $a = 122$ m, $b = 120$ m, $c = 22$ m

Semi-perimeter,

$s = \frac{a+b+c}{2} = \frac{122+120+22}{2} = \frac{264}{2} = 132$ m

The area of triangular side wall

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{132(132-122)(132-120)(132-22)}$

$= \sqrt{132\times10\times12\times110}$

$= \sqrt{12\times11\times10\times12\times11\times10} = 1320\text{ m}^2$

Rent for 1 year (*i.e.*, 12 months) per $m^2$ = ₹ 5000

$\therefore$ Rent for 3 months per $m^2$ = ₹ $5000\times\frac{3}{12}$

$\Rightarrow$ Rent for 3 months for 1320 $m^2$

$= 5000\times\frac{3}{12}\times1320 = 5000 \times 3 \times 110 =$ ₹ 16,50,000.

**3.** Sides of the wall are 15 m, 11 m and 6 m.

Let $a = 15$ m, $b = 11$ m, $c = 6$ m

Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{15+11+6}{2} = \frac{32}{2} = 16$ m

Area of the triangular wall $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-15)(16-11)(16-6)}$

$= \sqrt{16\times1\times5\times10} = \sqrt{2\times400} = 20\sqrt{2}\text{ m}^2$

Thus, the required area painted in colour is $20\sqrt{2}\text{ m}^2$.

**4.** Let the sides of the triangle be $a = 18$ cm, $b = 10$ cm and $c$.

Perimeter $= a + b + c = 42$ cm

$\Rightarrow$ $c = 42 - (18 + 10) = 14$ cm

Now, semi-perimeter, $s = \frac{42}{2} = 21$ cm

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-18)(21-10)(21-14)}$

$= \sqrt{21\times3\times11\times7}$

$= \sqrt{3\times7\times3\times11\times7} = 21\sqrt{11}\text{ cm}^2$

Thus, the required area of the triangle is $21\sqrt{11}\text{ cm}^2$.

**5.** Perimeter of the triangle = 540 cm

$\therefore$ Semi-perimeter, $s = \frac{540}{2} = 270$ cm

The sides of the triangle are in the ratio of 12 : 17 : 25.

Let $a = 12x$ cm, $b = 17x$ cm, $c = 25x$ cm

$\therefore$ $12x + 17x + 25x = 540$

$\Rightarrow$ $54x = 540 \Rightarrow x = 10$

$\therefore$ $a = 12 \times 10 = 120$ cm, $b = 17 \times 10 = 170$ cm and $c = 25 \times 10 = 250$ cm

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{270(270-120)(270-170)(270-250)}$

$= \sqrt{270\times150\times100\times20}$

$= \sqrt{3\times3\times3\times10\times3\times5\times10\times10\times10\times2\times2\times5}$

$= \sqrt{10^2\times10^2\times3^2\times3^2\times5^2\times2^2}$

$= 10 \times 10 \times 3 \times 3 \times 5 \times 2 = 9000\text{ cm}^2$

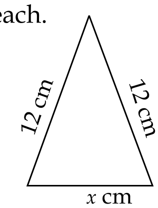
**6.** Equal sides of the triangle are 12 cm each.

Let the third side be $x$ cm.

Now, perimeter = 30 cm

$\Rightarrow$ $12 + 12 + x = 30$

$\Rightarrow$ $x = 6$




$\therefore$ Semi-perimeter, $s = \frac{30}{2} = 15$ cm

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-12)(15-12)(15-6)}$

$= \sqrt{15\times3\times3\times9} = \sqrt{5\times3\times3\times3\times3\times3}$

$= 3\times3\times\sqrt{5\times3} = 9\sqrt{15}\text{ cm}^2$

Thus, the required area of the triangle is $9\sqrt{15}\text{ cm}^2$.

## EXERCISE - 12.2

**1.** Join $B$ and $D$ such that $\Delta BCD$ is a right triangle.

In right $\Delta BCD$, $BD^2 = BC^2 + CD^2$ (By Pythagoras theorem)

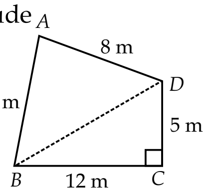
$\Rightarrow \quad BD^2 = 12^2 + 5^2$

$\Rightarrow \quad BD^2 = 144 + 25 = 169$

$\Rightarrow \quad BD = 13$ m

Now, area of $\Delta BCD = \frac{1}{2} \times \text{base} \times \text{altitude}$

$= \frac{1}{2} \times 12 \times 5 = 30 \text{ m}^2$




Now, for $\Delta ABD$, we have

$a = 9$ m, $b = 8$ m, $c = 13$ m

Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{9+8+13}{2} = \frac{30}{2} = 15$ m

Area of $\Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-9)(15-8)(15-13)}$

$= \sqrt{15 \times 6 \times 7 \times 2} = \sqrt{3 \times 5 \times 3 \times 2 \times 7 \times 2}$

$= 3 \times 2\sqrt{35} = 6 \times 5.916 \text{ m}^2 = 35.5 \text{ m}^2$ (approx.)

So, area of quadrilateral $ABCD$ = Area of $\Delta BCD$ + Area of $\Delta ABD$

$= 30 \text{ m}^2 + 35.5 \text{ m}^2 = 65.5 \text{ m}^2$ (approx.)

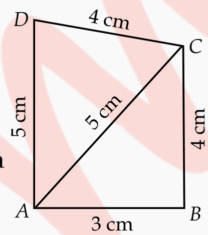
**2.** For $\Delta ABC$, $a = 3$ cm, $b = 4$ cm, $c = 5$ cm

$\therefore$ Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{3+4+5}{2} = 6$ cm

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{6(6-3)(6-4)(6-5)}$

$= \sqrt{6 \times 3 \times 2 \times 1} = 6 \text{ cm}^2$




For $\Delta ACD$, $a = 5$ cm, $b = 4$ cm, $c = 5$ cm

Semi-perimeter, $s = \frac{a+b+c}{2}$

$= \frac{5+4+5}{2} = \frac{14}{2} = 7 \text{ cm}$

Area of $\Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{7(7-5)(7-4)(7-5)}$

$= \sqrt{7 \times 2 \times 3 \times 2} = 2\sqrt{21} \text{ cm}^2$

$= 2 \times 4.6 \text{ cm}^2 = 9.2 \text{ cm}^2$ ( approx.)

So, area of quadrilateral $ABCD$

= Area of $\Delta ABC$ + Area of $\Delta ACD$

$= 6 \text{ cm}^2 + 9.2 \text{ cm}^2 = 15.2 \text{ cm}^2$ (approx.)

**3.** Area of surface I :

It is an isosceles triangle whose sides are $a = 5$ cm, $b = 5$ cm, $c = 1$ cm

Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{5+5+1}{2} = \frac{11}{2}$ cm

Area of surface I $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-1\right)}$

$= \sqrt{\frac{11}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{9}{2}} = \frac{3}{4}\sqrt{11}$

$= 0.75 \times 3.3 = 2.475 \text{ cm}^2$ (approx.)

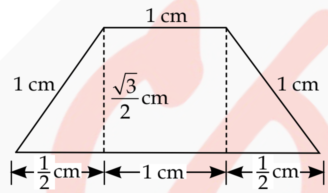
Area of surface II :

It is a rectangle with length 6.5 cm and breadth 1 cm.

Area of surface II = Length × Breadth = 6.5 × 1 = 6.5 cm$^2$

Area of surface III :

It is a trapezium whose parallel sides are 1 cm and 2 cm as shown in the adjoining figure.




Its height, $h = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$ cm

Area of surface III = $\frac{1}{2}$ [Sum of the parallel sides] × Height

$= \frac{1}{2} \times (2+1) \times \frac{\sqrt{3}}{2} = \frac{1}{2} \times 3 \times \frac{\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{4} = \frac{3 \times 1.732}{4} = 1.3 \text{ cm}^2$ (approx.)

Area of surface IV :

It is a right triangle with base as 6 cm and height as 1.5 cm.

Area of surface IV = $\frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 6 \times 1.5 = 4.5 \text{ cm}^2$

Area of surface V = Area of surface IV = 4.5 cm$^2$

Thus, the total area of the paper used

= Area of surface (I + II + III + IV + V)

= (2.475 + 6.5 + 1.3 + 4.5 + 4.5) cm$^2$

= 19.275 cm$^2$ = 19.3 cm$^2$ (approx.)

**4.** For the given triangle, we have

$a = 28$ cm, $b = 30$ cm, $c = 26$ cm

$\therefore$ Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{28+30+26}{2}$

$= \frac{84}{2} = 42$ cm

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-28)(42-30)(42-26)}$

$= \sqrt{42 \times 14 \times 12 \times 16} = \sqrt{112896} = 336 \text{ cm}^2$

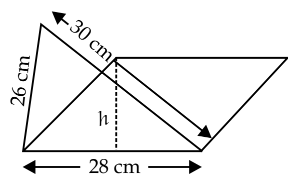
Area of the parallelogram = Area of the triangle

$\therefore$ Area of the parallelogram = 336 cm$^2$

$\Rightarrow$ Base × Height = 336

$\Rightarrow$ 28 × $h$ = 336

$\Rightarrow \quad h = \frac{336}{28}$ cm = 12 cm

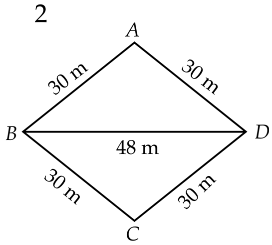



Thus, the required height of the parallelogram is 12 cm.

**5.** Each side of the rhombus = 30 m
One of the diagonal = 48 m
Let $ABCD$ be the given rhombus.
Sides of $\Delta ABD$ are $a$ = 30 m, $b$ = 30 m, $c$ = 48 m

Semi-perimeter, $s=\dfrac{a+b+c}{2}=\dfrac{30+30+48}{2}$

$=\dfrac{108}{2}=54$ m




Area of $\Delta ABD$

$=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{54(54-30)(54-30)(54-48)}$

$=\sqrt{54\times 24\times 24\times 6}=432\text{ m}^2$

Since a diagonal divides the rhombus into two congruent triangles.

$\therefore$ Area of $\Delta BCD$ = 432 $\text{m}^2$

Total area of the rhombus = 432 $\text{m}^2$ + 432 $\text{m}^2$ = 864 $\text{m}^2$

Area of grass field for 18 cows = 864 $\text{m}^2$

$\Rightarrow$ Area of grass field for 1 cow = $\dfrac{864}{18}\text{ m}^2=48\text{ m}^2$

**6.** Sides of each triangular piece are
$a$ = 20 cm, $b$ = 50 cm, $c$ = 50 cm

Semi-perimeter, $s=\dfrac{a+b+c}{2}=\dfrac{20+50+50}{2}=\dfrac{120}{2}=60$ cm

Area of each triangular piece

$=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{60(60-20)(60-50)(60-50)}$

$=\sqrt{60\times 40\times 10\times 10}=200\sqrt{6}\text{ cm}^2$

Area of 5 triangular pieces of one colour

$=5\times 200\sqrt{6}\text{ cm}^2=1000\sqrt{6}\text{ cm}^2$

Area of 5 triangular pieces of other colour $=1000\sqrt{6}\text{ cm}^2$

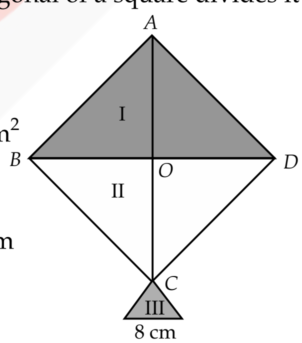
**7.** Area of triangle I :
Since, the diagonals of a square are equal and bisect each other.

$\therefore$ $AC=BD=32$ cm

$\therefore$ Height of $\Delta ABD=OA=\dfrac{1}{2}\times 32$ cm = 16 cm

Area of triangle I $=\dfrac{1}{2}\times 32\times 16=256\text{ cm}^2$

Area of triangle II : Since, diagonal of a square divides it into two congruent triangles.




So, area of triangle II
= Area of triangle I

$\therefore$ Area of triangle II = 256 $\text{cm}^2$

Area of triangle III :
The sides of triangle III are
$a$ = 8 cm, $b$ = 6 cm and $c$ = 6 cm

Semi-perimeter, $s=\dfrac{a+b+c}{2}$

$=\dfrac{8+6+6}{2}=10$ cm

Area of triangle III $=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{10(10-8)(10-6)(10-6)}=\sqrt{10\times 2\times 4\times 4}$

$=8\sqrt{5}=8\times 2.24=17.92\text{ cm}^2$ (approx.).

So, area of shade I = 256 $\text{cm}^2$,
Area of shade II = 256 $\text{cm}^2$ and
Area of shade III = 17.92 $\text{cm}^2$

**8.** Sides of the triangle are $a$ = 9 cm, $b$ = 28 cm, $c$ = 35 cm

Semi-perimeter, $s=\dfrac{a+b+c}{2}=\dfrac{9+28+35}{2}=\dfrac{72}{2}=36$ cm

Area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{36(36-9)(36-28)(36-35)}$

$=\sqrt{36\times 27\times 8\times 1}=3\times 2\times 3\times 2\sqrt{3\times 2}$

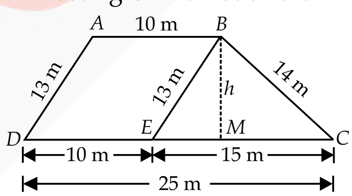
$=36\sqrt{6}=36\times 2.45=88.2\text{ cm}^2$ (approx.)

Total area of 16 triangles
= (16 × 88.2) $\text{cm}^2$ = 1411.2 $\text{cm}^2$ (approx.)

Rate of polishing = ₹ 0.5 per $\text{cm}^2$

$\therefore$ Cost of polishing the tiles
= ₹ (0.5 × 1411.2) = ₹ 705.60 (approx.)

**9.** Let the given field in the form of a trapezium be $ABCD$ such that parallel sides are $AB$ = 10 m and $DC$ = 25 m and non-parallel sides are 13 m and 14 m. Draw $BE \parallel AD$ meeting $CD$ at $E$ such that $BE$ = 13 m.




For $\Delta BCE$ :
Sides of $\Delta BCE$ are $a$ = 13 m, $b$ = 14 m, $c$ = 15 m

$\therefore$ Semi-perimeter, $s=\dfrac{a+b+c}{2}=\dfrac{13+14+15}{2}=\dfrac{42}{2}=21$ m

Area of $\Delta BCE=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{21(21-13)(21-14)(21-15)}$

$=\sqrt{21\times 8\times 7\times 6}=84\text{ m}^2$

Let the height of the $\Delta BCE$ corresponding to the side 15 m be $h$ metres.

Area of $\Delta BCE=\dfrac{1}{2}\times\text{base}\times\text{height}$

$\Rightarrow \dfrac{1}{2}\times 15\times h=84 \Rightarrow h=\dfrac{84\times 2}{15}=\dfrac{56}{5}$ m

For parallelogram $ABED$:
Area of a parallelogram = base × height

$=10\times\dfrac{56}{5}=2\times 56=112\text{ m}^2$

So, area of the field
= Area of $\Delta BCE$ + Area of parallelogram $ABED$
= 84 + 112 = 196 $\text{m}^2$