Surface Areas and Volumes

CHAPTER 13

NCERT FOCUS

SOLUTIONS

EXERCISE - 13.1

1. (i) Here, length (l) = 1.5 m, breadth (b) = 1.25 m and height (h) $= 65 \text{ cm} = \frac{65}{100} \text{ m} = 0.65 \text{ m}$

Since, it is open from the top.

$\therefore$ Surface area of box = (Lateral surface area) + (Base area)

$= [2(l + b)h] + (lb)$

$= [2(1.50 + 1.25)0.65] + (1.50 \times 1.25)$

$= [2 \times 2.75 \times 0.65] + 1.875$

$= 3.575 + 1.875 = 5.45 \text{ m}^2$

$\therefore$ Area of the sheet required for making the box $= 5.45 \text{ m}^2$

(ii) Cost of 1 m^2 sheet = ₹ 20

$\therefore$ Cost of 5.45 m^2 sheet = ₹ (20 × 5.45) = ₹ 109

2. Length of the room (l) = 5 m

Breadth of the room (b) = 4 m

Height of the room (h) = 3 m

$\therefore$ Area for white washing

= [Lateral surface area] + [Area of the ceiling]

$= [2(l + b)h] + [l \times b] = [2(5 + 4) \times 3] + [5 \times 4]$

$= 54 + 20 = 74 \text{ m}^2$

Cost of white washing 1 m^2 area = ₹ 7.50

$\therefore$ Cost of white washing 74 m^2 area = ₹ (7.50 × 74) = ₹ 555

$\therefore$ The required cost of white washing = ₹ 555

3. Let the length, breadth and height of the hall be l, b and h respectively.

Perimeter of the floor $= 2(l + b) \Rightarrow 2(l + b) = 250$ m

$\because$ Area of four walls = Lateral surface area

$= 2(l + b) \times h = 250 \times h = 250\,h \text{ m}^2$

$\therefore$ Cost of painting the four walls

= ₹ (10 × 250 h) = ₹ 2500 h

$\Rightarrow 2500\,h = 15000 \Rightarrow h = \frac{15000}{2500} = 6$

Thus, the required height of the hall = 6 m

4. Total area that can be painted $= 9.375 \text{ m}^2$

Now, total surface area of a brick $= 2[lb + bh + hl]$

$= 2[(22.5 \times 10) + (10 \times 7.5) + (7.5 \times 22.5)]$

$= 2[225 + 75 + 168.75]$

$= 2[468.75] = 937.5 \text{ cm}^2 = \frac{937.5}{10000} \text{ m}^2$

Let the required number of bricks be n.

$\therefore$ Total surface area of n bricks $= n \times \frac{937.5}{10000} \text{ m}^2$

$\Rightarrow n \times \frac{937.5}{10000} \text{ m}^2 = \frac{9375}{1000} \text{ m}^2$ [Given]

$\Rightarrow n = \frac{9375}{1000} \times \frac{10000}{937.5} \Rightarrow n = 100$

Thus, the required number of bricks = 100

5. For the cubical box with edge (a) = 10 cm

Lateral surface area $= 4a^2$

$= 4 \times 10^2 = 4 \times 100 = 400 \text{ cm}^2$

Total surface area $= 6a^2 = 6 \times 10^2 = 6 \times 100 = 600 \text{ cm}^2$

For the cuboidal box with dimensions length (l) = 12.5 cm, breadth (b) = 10 cm and height (h) = 8 cm

Lateral surface area $= 2[l + b] \times h$

$= 2[12.5 + 10] \times 8 = 360 \text{ cm}^2$

$\therefore$ Total surface area $= 2[lb + bh + hl]$

$= 2[(12.5 \times 10) + (10 \times 8) + (8 \times 12.5)]$

$= 2[125 + 80 + 100] = 610 \text{ cm}^2$

(i) Cubical box has the greater lateral surface area by $400 - 360 = 40 \text{ cm}^2$.

(ii) Cubical box has smaller total surface area by $610 - 600 = 10 \text{ cm}^2$.

6. Here, length (l) = 30 cm, breadth (b) = 25 cm and height (h) = 25 cm

(i) Surface area of herbarium $= 2[lb + bh + hl]$

$= 2[(30 \times 25) + (25 \times 25) + (25 \times 30)]$

$= 2[750 + 625 + 750] = 2[2125] = 4250 \text{ cm}^2$

Thus, the required area of glass is 4250 cm^2.

(ii) Total length of 12 edges $= 4(l + b + h)$

$= 4(30 + 25 + 25) = 4 \times 80 = 320$ cm

Thus, required length of tape = 320 cm

7. For bigger box: length (l) = 25 cm, breadth (b) = 20 cm and height (h) = 5 cm

Total surface area of 1 bigger box $= 2(lb + bh + hl)$

$= 2[(25 \times 20) + (20 \times 5) + (5 \times 25)]$

$= 2[500 + 100 + 125] = 2[725] = 1450 \text{ cm}^2$

$\therefore$ Total surface area of 250 boxes

$= (250 \times 1450) \text{ cm}^2 = 362500 \text{ cm}^2$

For smaller box : length (l) = 15 cm, breadth (b) = 12 cm and height (h)= 5 cm

Total surface area of 1 smaller box $= 2[lb + bh + hl]$

$= 2[(15 \times 12) + (12 \times 5) + (5 \times 15)]$

$= 2[180 + 60 + 75] = 2[315] = 630 \text{ cm}^2$

$\therefore$ Total surface area of 250 boxes

$= (250 \times 630) \text{ cm}^2 = 157500 \text{ cm}^2$

Now, total surface area of both type of boxes

$= 362500 + 157500 = 520000 \text{ cm}^2$

Also, area for overlaps = 5% of [total surface area]

$= \frac{5}{100} \times 520000 = 26000 \text{ cm}^2$

$\therefore$ Total surface area of the cardboard required

= Total surface area of both type of boxes + Area of overlaps

$= 520000 + 26000 = 546000 \text{ cm}^2$

Cost of 1000 cm^2 cardboard = ₹ 4

$\therefore$ Cost of 546000 cm^2 cardboard

$= ₹\left(\frac{4\times546000}{1000}\right) = ₹\ 2184$

8. Here, height (h) = 2.5 m
Base dimension = 4 m × 3 m
∴ Length (l) = 4 m and breadth (b) = 3 m
The surface is like a cuboid
∴ Required tarpaulin surface area of the cuboid, excluding the base
= [Lateral surface area] + [Area of ceiling]
= [2(l + b)h] + (lb) = [2(4 + 3) × 2.5] + (4 × 3)
= 35 + 12 = 47 m^2

EXERCISE - 13.2

1. Let r be the radius of the cylinder.
Here, height (h) = 14 cm and curved surface area = 88 cm^2
Curved surface area of a cylinder = $2\pi rh$

$\Rightarrow 2\pi rh = 88 \Rightarrow 2\times\frac{22}{7}\times r\times14 = 88$

$\Rightarrow r = \frac{88\times7}{2\times22\times14} = 1\text{ cm}$

∴ Diameter = 2 × r = (2 × 1) cm = 2 cm

2. Here, height (h) = 1 m
Diameter of the base = 140 cm = 1.40 m

∴ Radius (r) $= \frac{1.40}{2} = 0.70\text{ m}$

Total surface area of the cylinder = $2\pi r(h + r)$

$= 2\times\frac{22}{7}\times0.70(1+0.70) = 2\times22\times0.10(1.70)$

$= 44\times\frac{17}{100} = \frac{748}{100} = 7.48\,m^2$

Hence, area of the required sheet is 7.48 m^2

3. Length of the metal pipe = 77 cm
It is in the form of a cylinder
∴ Height (h) of the cylinder = 77 cm
Inner diameter = 4 cm

$\Rightarrow$ Inner radius (r) $= \frac{4}{2} = 2$ cm

Outer diameter = 4.4 cm

$\Rightarrow$ Outer radius (R) $= \frac{4.4}{2} = 2.2$ cm

(i) Inner curved surface area = $2\pi rh$

$= 2\times\frac{22}{7}\times2\times77 = 968\ cm^2$

(ii) Outer curved surface area = $2\pi Rh$

$= 2\times\frac{22}{7}\times2.2\times77 = \frac{10648}{10} = 1064.8\ cm^2$

(iii) Total surface area = [Inner curved surface area] + [Outer curved surface area] + [Surface area of two circular bases] = $(2\pi rh) + (2\pi Rh) + [2\pi(R^2 - r^2)]$

$= 968 + 1064.8 + 2\times\frac{22}{7}\left[(2.2)^2-(2)^2\right]$

$= 2032.8 + \frac{2\times22\times0.84}{7}$

$= 2032.8 + 5.28 = 2038.08\ cm^2$

4. Diameter of roller = 84 cm

$\Rightarrow$ Radius of roller $= \frac{84}{2} = 42$ cm

Length of the roller = 120 cm
Curved surface area of the roller = $2\pi rh$

$= 2\times\frac{22}{7}\times42\times120 = 2\times22\times6\times120 = 31680\ cm^2$

∴ Area of the playground levelled in one revolution by the roller = 31680 cm^2 $= \frac{31680}{10000}\ m^2$

∴ Area levelled in 500 revolutions

$= 500\times\frac{31680}{10000} = \frac{5\times3168}{10} = 1584\ m^2$

5. Diameter of the pillar = 50 cm

∴ Radius (r) $= \frac{50}{2} = 25\text{ cm} = \frac{1}{4}\text{ m}$

and height (h) = 3.5 m
Now, curved surface area of pillar = $2\pi rh$

$= 2\times\frac{22}{7}\times\frac{1}{4}\times3.50 = \frac{44\times350}{7\times4\times100} = \frac{11}{2}\ m^2$

Cost of painting of 1 m^2 area = ₹ 12.50

∴ Cost of painting of $\frac{11}{2}\ m^2$ area $= ₹\left(\frac{11}{2}\times12.50\right)$
= ₹ 68.75.

6. Radius (r) = 0.7 m
Let height of the cylinder be h m.
Curved surface area of a cylinder = $2\pi rh$

$\Rightarrow 2\times\frac{22}{7}\times\frac{7}{10}\times h = 4.4 \Rightarrow h = \frac{44}{10}\times\frac{7}{22}\times\frac{10}{7}\times\frac{1}{2} = 1\text{ m}$

Thus, the required height is 1 m.

7. Inner diameter of the well = 3.5 m

∴ Radius of the well = $\frac{3.5}{2}$ m

and height (h) of the well = 10 m

(i) Inner curved surface area = $2\pi rh = 2\times\frac{22}{7}\times\frac{3.5}{2}\times10$

$= \frac{2\times22\times35\times10}{7\times2\times10} = 110\,m^2$

(ii) Cost of plastering per m^2 = ₹ 40
∴ Total cost of plastering the area of 110 m^2 = ₹ (110 × 40)
= ₹ 4400

8. Length of the cylindrical pipe = 28 m
i.e., h = 28 m
Diameter of the pipe = 5 cm

∴ Radius (r) = $\frac{5}{2}$ cm = $\frac{5}{200}$ m

Curved surface area of a cylinder = $2\pi rh$

$= 2\times\frac{22}{7}\times\frac{5}{200}\times28 = \frac{22\times5\times4}{100} = \frac{440}{100} = 4.4\,m^2$

Thus, the total radiating surface is 4.40 m^2.

9. The storage tank is in the form of a cylinder
∴ Diameter of the tank = 4.2 m

$\Rightarrow$ Radius (r) = $\frac{4.2}{2} = 2.1\text{m}$ and height (h) = 4.5 m

Now,

(i) Lateral (or curved) surface area = $2\pi rh$

$= 2\times\frac{22}{7}\times 2.1\times 4.5$ = 2 × 22 × 0.3 × 4.5 = 59.4 m^2

(ii) Total surface area of the tank = $2\pi r(r + h)$

$= 2\times\frac{22}{7}\times 2.1(2.1+4.5)$ = 44 × 0.3 × 6.6 = 87.12 m^2

Let actual area of the steel used be x m^2.

$\therefore$ Area of steel that wasted $=\frac{1}{12}\times x=\frac{x}{12}m^2$

$\Rightarrow$ Area of steel used $= x-\frac{x}{12}=\frac{12x-x}{12}=\frac{11x}{12}m^2$

$\Rightarrow \frac{11x}{12}=87.12 \Rightarrow x=\frac{8712}{100}\times\frac{12}{11}$

$\Rightarrow x=\frac{104544}{1100} \Rightarrow x = 95.04\ m^2$

Thus, the required area of the steel that was actually used is 95.04 m^2.

10. The lampshade is in the form of a cylinder, where radius $=\frac{20}{2}=10\,cm$ and height = 30 cm.

A margin of 2.5 cm is to be added to top and bottom

$\therefore$ Total height of the cylinder, h

= 30 cm + 2.5 cm + 2.5 cm = 35 cm

Now, curved surface area = $2\pi rh$

$=2\times\frac{22}{7}\times 10\times(35)$ = 2 × 22 × 10 × 5 = 2200 cm^2

Thus, the required area of the cloth is 2200 cm^2.

11. Here, the penholders are in the form of cylinders

Radius of penholder (r) = 3 cm

Height of penholder (h) = 10.5 cm

Since, a penholder must be open from the top

Now, Surface area of a penholder (cylinder)

= [Lateral surface area] + [Base area] = [$2\pi rh$] + πr^2

$=\left(2\times\frac{22}{7}\times 3\times 10.5\right)+\left(\frac{22}{7}\times 3\times 3\right)$

$=(44\times 3\times 1.5)+\frac{198}{7}$

$=198+\frac{198}{7}=\frac{1386+198}{7}=\frac{1584}{7}cm^2$

$\therefore$ Surface area of 35 penholders

$=35\times\frac{1584}{7}$ = 5 × 1584 = 7920 cm^2

Thus, 7920 cm^2 of cardboard was required.

EXERCISE - 13.3

1. Here, diameter of the base of a cone = 10.5 cm

$\Rightarrow$ Radius (r) = $\frac{10.5}{2}$ cm

and Slant height (l) = 10 cm

$\therefore$ Curved surface area of the cone = πrl

$=\frac{22}{7}\times\frac{10.5}{2}\times 10$ = (11 × 15 × 1) cm^2 = 165 cm^2

2. Here, diameter = 24 m and slant height (l) = 21 m

$\therefore$ Radius (r) = $\frac{24}{2}$ = 12 m

$\therefore$ Total surface area = $\pi r(r + l)$

$=\frac{22}{7}\times 12\times(12+21)=\frac{22}{7}\times 12\times 33$

$=\frac{8712}{7}$ = 1244.57 m^2 (approx.)

3. Here, curved surface area = 308 cm^2

Slant height (l) = 14 cm

(i) Let the radius of the base be 'r' cm.

$\therefore \pi rl = 308 \Rightarrow \frac{22}{7}\times r\times 14=308$

$\Rightarrow r=\frac{308\times 7}{22\times 14}=7\,cm$

Thus, radius of the cone is 7 cm.

(ii) Base area = πr^2 = $\frac{22}{7}\times 7^2=154\ cm^2$

and curved surface area = 308 cm^2 [Given]

$\therefore$ Total surface area = [Curved surface area] + [Base area]

= (308 + 154) cm^2 = 462 cm^2

4. Here, height of the tent (h) = 10 m

Radius of the base (r) = 24 m

(i) The slant height, $l=\sqrt{r^2+h^2}$

$=\sqrt{24^2+10^2}=\sqrt{576+100}=\sqrt{676}$ = 26 m

Thus, the required slant height of the tent is 26 m.

(ii) Curved surface area of the cone = πrl

$\therefore$ Area of the canvas required

$=\frac{22}{7}\times 24\times 26=\frac{13728}{7}\ m^2$

Cost of 1 m^2 canvas = ₹ 70

$\therefore$ Cost of $\frac{13728}{7}m^2$ canvas = ₹$\left(70\times\frac{13728}{7}\right)$ = ₹ 137280

5. Here, Base radius (r) = 6 m; Height (h) = 8 m

$\therefore$ Slant height (l) = $\sqrt{r^2+h^2}=\sqrt{6^2+8^2}$

$=\sqrt{36+64}=\sqrt{100}=10$ m

Now, curved surface area = πrl = 3.14 × 6 × 10 = 188.4 m^2

Thus, area of the canvas (tarpaulin) required to make the tent = 188.4 m^2

Let the length of the tarpaulin be L m.

$\therefore$ Length × breadth = 188.4

$\Rightarrow L$ × 3 = 188.4 $\Rightarrow L=\frac{188.4}{3}=62.8\,m$

Extra length of tarpaulin for margins = 20 cm

$=\frac{20}{100}m=0.2\,m$

Thus, total length of tarpaulin required

= (62.8 + 0.2) m = 63 m

6. Here, base radius $(r) = \frac{14}{2} = 7$ m and

Slant height $(l) = 25$ m

$\therefore$ Curved surface area $= \pi r l$

$= \frac{22}{7} \times 7 \times 25 = 22 \times 25 = 550 \text{ m}^2$

Cost of white washing 100 m^2 area = ₹ 210

$\therefore$ Cost of whitewashing 550 m^2 area

$= ₹\left(\frac{210}{100} \times 550\right) = ₹\ 1155$

7. Radius of the base $(r) = 7$ cm and height $(h) = 24$ cm

So, slant height $(l) = \sqrt{24^2 + 7^2} = \sqrt{625} = 25$ cm

Now, Lateral surface area $= \pi r l = \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$

So, sheet required to make 1 cap $= 550 \text{ cm}^2$

$\therefore$ Sheet required to make 10 such caps $= (10 \times 550) \text{ cm}^2 = 5500 \text{ cm}^2$

8. Diameter of the base = 40 cm

$\Rightarrow$ Radius $(r) = \frac{40}{2}$ cm $= 20$ cm $= \frac{20}{100}$ m $= 0.2$ m

Height $(h) = 1$ m

$\Rightarrow$ Slant height $(l) = \sqrt{r^2 + h^2} = \sqrt{(0.2)^2 + (1)^2} = \sqrt{1.04}$

$= 1.02$ m $\qquad [\sqrt{1.04} = 1.02 \text{ (Given)}]$

Now, curved surface area $= \pi r l$

$\therefore$ Curved surface area of 1 cone

$= 3.14 \times 0.2 \times 1.02 = \left(\frac{314}{100} \times \frac{2}{10} \times \frac{102}{100}\right) \text{m}^2$

$\Rightarrow$ Curved surface area of 50 cones

$= 50 \times \left[\frac{314}{100} \times \frac{2}{10} \times \frac{102}{100}\right] = \left(\frac{314 \times 102}{10 \times 100}\right) \text{m}^2$

Cost of painting 1 m^2 area = ₹ 12

$\therefore$ Cost of painting $\left[\frac{314 \times 102}{1000}\right] \text{m}^2$ area

$= ₹\left(\frac{12 \times 314 \times 102}{1000}\right) = ₹\ \frac{384336}{1000}$

= ₹ 384.336 = ₹ 384.34 (approx)

Thus, the required cost of painting is ₹ 384.34 (approx).

EXERCISE - 13.4

1. (i) Here, $r = 10.5$ cm

$\therefore$ Surface area of a sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times (10.5)^2 = 4 \times \frac{22}{7} \times \frac{105}{10} \times \frac{105}{10} = 1386 \text{ cm}^2$

(ii) Here, $r = 5.6$ cm

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times (5.6)^2$

$= 4 \times \frac{22}{7} \times \frac{56}{10} \times \frac{56}{10} = 394.24 \text{ cm}^2$

(iii) Here, $r = 14$ cm

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times (14)^2$

$= 4 \times \frac{22}{7} \times 14 \times 14 = 2464 \text{ cm}^2$

2. (i) Here, Diameter = 14 cm

$\Rightarrow$ Radius $(r) = \frac{14}{2} = 7$ cm

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times (7)^2$

$= 4 \times \frac{22}{7} \times 7 \times 7 = 88 \times 7 = 616 \text{ cm}^2$

(ii) Here, Diameter = 21 cm

$\Rightarrow$ Radius $(r) = \frac{21}{2}$ cm

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2$

$= 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 1386 \text{ cm}^2$

(iii) Here, Diameter = 3.5 m

$\Rightarrow$ Radius $(r) = \frac{3.5}{2}$ m $= \frac{35}{20}$ m

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times \left(\frac{35}{20}\right)^2$

$= 4 \times \frac{22}{7} \times \frac{35}{20} \times \frac{35}{20} = 38.5 \text{ m}^2$

3. Here, radius $(r) = 10$ cm

Total surface area of hemisphere $= 3\pi r^2$

$= 3 \times 3.14 \times 10 \times 10 = 942 \text{ cm}^2$

4. **Case I :** When radius $(r_1) = 7$ cm

Surface area $= 4\pi r_1^2 = 4 \times \frac{22}{7} \times (7)^2 = 616 \text{ cm}^2$

Case II : When radius $(r_2) = 14$ cm

Surface area $= 4\pi r_2^2 = 4 \times \frac{22}{7} \times 14 \times 14 = 2464 \text{ cm}^2$

$\therefore$ Required ratio $= \frac{616}{2464} = \frac{1}{4}$

Hence, the required ratio is 1 : 4.

5. Inner diameter of the hemisphere = 10.5 cm

$\therefore$ Radius $(r) = \frac{10.5}{2}$ cm $= \frac{105}{20}$ cm

Curved surface area of a hemisphere $= 2\pi r^2$

$\therefore$ Inner curved surface area of hemispherical bowl

$= 2 \times \frac{22}{7} \times \left(\frac{105}{20}\right)^2 = \frac{17325}{100} \text{cm}^2$

Cost of tin-plating 100 cm^2 area = ₹ 16

$\therefore$ Cost of tin-plating $\frac{17325}{100} \text{cm}^2$ area

$= ₹\left(\frac{16}{100} \times \frac{17325}{100}\right) = ₹\ 27.72$

6. Let the radius of the sphere be r cm.

Surface area $= 4\pi r^2$

$\Rightarrow 4\pi r^2 = 154 \Rightarrow 4 \times \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = \frac{154 \times 7}{4 \times 22} = \left(\frac{7}{2}\right)^2 \Rightarrow r = \frac{7}{2} = 3.5$

Thus, the required radius of the sphere is 3.5 cm.

7. Let the radius of the earth be r.

$\therefore$ Radius of the moon = $\frac{r}{4}$

Surface area of a sphere = $4\pi r^2$

Since, the earth as well as the moon are considered to be spheres.

$\therefore$ Surface area of the earth = $4\pi r^2$

and surface area of the moon $= 4\pi\left(\frac{r}{4}\right)^2$

$$\therefore \frac{\text{Surface area of moon}}{\text{Surface area of earth}} = \frac{4\pi\left(\frac{r}{4}\right)^2}{4\pi r^2} = \frac{\left(\frac{r}{4}\right)^2}{r^2} = \frac{r^2}{16r^2} = \frac{1}{16}$$

Thus, the required ratio = 1 : 16.

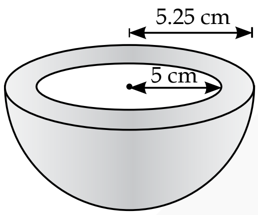
8. Inner radius (r) = 5 cm

Thickness = 0.25 cm

$\therefore$ Outer radius (R)

= (5.00 + 0.25) cm = 5.25 cm

$\therefore$ Outer curved surface area of the bowl = $2\pi R^2$



$= 2 \times \frac{22}{7} \times (5.25)^2 = 173.25\ \text{cm}^2$

9. (i) For the sphere radius = r

$\therefore$ Surface area of the sphere = $4\pi r^2$

(ii) For the right circular cylinder :

Radius of the cylinder = Radius of the sphere

$\therefore$ Radius of the cylinder = r

Height of the cylinder = Diameter of the sphere

$\therefore$ Height of the cylinder (h) = $2r$

Since, curved surface area of the cylinder = $2\pi rh$

= $2\pi r(2r) = 4\pi r^2$

(iii) $$\frac{\text{Surface area of the sphere}}{\text{Surface area of the cylinder}} = \frac{4\pi r^2}{4\pi r^2} = \frac{1}{1}$$

Thus, the required ratio is 1 : 1.

EXERCISE - 13.5

1. Measures of matchbox (cuboid) is 4 cm × 2.5 cm × 1.5 cm

$\Rightarrow$ l = 4 cm, b = 2.5 cm and h = 1.5 cm

$\therefore$ Volume of a matchbox = $l \times b \times h$

= 4 × 2.5 × 1.5 = 15 cm^3

$\therefore$ Volume of 12 such boxes = 12 × 15 cm^3 = 180 cm^3

2. Length (l) = 6 m; Breadth (b) = 5 m, Depth (h) = 4.5 m

$\therefore$ Capacity = $l \times b \times h$ = 6 × 5 × 4.5 = 135 m^3

Now, 1 m^3 = 1000 l

$\Rightarrow$ 135 m^3 = 135000 l

$\therefore$ The amount of water the tank will hold is 135000 l.

3. Length (l) = 10 m; Breadth (b) = 8 m

Let height of the cuboid be 'h'.

Volume of cuboidal vessel = 380 m^3

$\Rightarrow$ 10 × 8 × h = 380

$\Rightarrow$ $80h$ = 380

$$\Rightarrow h = \frac{380}{80} = \frac{19}{4}\ \text{m} = 4.75\ \text{m}$$

Thus, the required height of the vessel is 4.75 m

4. Length (l) = 8 m, Breadth (b) = 6 m, Depth (h) = 3 m

$\therefore$ Volume of the cuboidal pit = $l \times b \times h$

= 8 × 6 × 3 = 144 m^3

So, cost of digging the pit = ₹ 144 × 30 = ₹ 4320

5. Length of the tank (l) = 2.5 m

Depth of the tank (h) = 10 m

Let breadth of the tank be b m.

$\therefore$ Volume (capacity) of the tank = $l \times b \times h$

= 2.5 × b × 10 = $25b$ m^3

But the capacity of the tank = 50000 l = 50 m^3

[$\because$ 1000 l = 1 m^3]

$$\therefore 25b = 50 \Rightarrow b = \frac{50}{25} = 2$$

Thus, the breadth of the tank is 2 m.

6. Length of the tank (l) = 20 m

Breadth of the tank (b) = 15 m

Height of the tank (h) = 6 m

$\therefore$ Volume of the tank = $l \times b \times h$ = 20 × 15 × 6 = 1800 m^3

Since 1 m^3 = 1000 l

$\therefore$ Capacity of the tank = 1800 × 1000 l = 1800000 l

Village population = 4000

Since, 150 l of water is required per head per day.

$\therefore$ Amount of water required per day = 150 × 4000 l

= 600000 l

Let the required number of days be x.

$\therefore$ 600000 × x = 1800000

$$\Rightarrow x = \frac{1800000}{600000} = 3$$

Thus, the required number of days is 3.

7. Volume of the godown = 40 × 25 × 15 m^3

Volume of 1 crate = 1.5 × 1.25 × 0.5 m^3

$$= \frac{15}{10} \times \frac{125}{100} \times \frac{5}{10}\text{m}^3 = \frac{3}{2} \times \frac{5}{4} \times \frac{1}{2}\text{m}^3$$

Let the required number of crates be 'n'.

$$\therefore n \times \left[\frac{3}{2} \times \frac{5}{4} \times \frac{1}{2}\right] = 40 \times 25 \times 15$$

$$\Rightarrow n = \frac{40 \times 25 \times 15}{\left[\frac{3}{2} \times \frac{5}{4} \times \frac{1}{2}\right]} = \frac{40 \times 25 \times 15}{3 \times 5 \times 1} \times 2 \times 4 \times 2 = 16000$$

$\therefore$ Maximum number of wooden crates is 16000

8. Side of the given cube = 12 cm

$\therefore$ Volume of the given cube = $(\text{side})^3 = (12)^3\ \text{cm}^3$

Let the side of the new (smaller) cube be n cm.

$\Rightarrow$ Volume of smaller cube = $n^3\ \text{cm}^3$

$\Rightarrow$ Volume of 8 smaller cubes = $8n^3\ \text{cm}^3$

According to question, we have

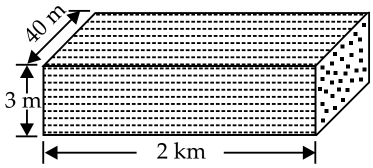
$8n^3 = (12)^3 = 12 \times 12 \times 12$

$$\Rightarrow n^3 = \frac{12 \times 12 \times 12}{8} = 6 \times 6 \times 6$$

$\Rightarrow$ $n^3 = 6^3 \Rightarrow n = 6$

Thus, the required side of the new (smaller) cube is 6 cm.
Now, surface area of the given cube
$= 6 \times (\text{side})^2 = 6 \times 12^2 = 6 \times 12 \times 12 \text{ cm}^2$
Surface area of one smaller cube $= 6 \times (\text{side})^2$
$= 6 \times 6^2 = 6 \times 6 \times 6 \text{ cm}^2$
Now,
$$\frac{\text{Surface Area of the given cube}}{\text{Surface Area of new cube}} = \frac{6 \times 12 \times 12}{6 \times 6 \times 6} = \frac{4}{1}$$
Thus, the required ratio = 4 : 1

9. The water flowing in a river can be considered in the form of a cuboid.



Length (l) = 2 km = 2000 m
Breadth (b) = 40 m, Depth (h) = 3 m
Now, volume of water flowing in 1 hr (60 minutes)
$= 2000 \times 40 \times 3 \text{ m}^3$
$\therefore$ Volume of water that will fall in 1 minute
$$= \frac{2000 \times 40 \times 3}{60} \text{m}^3 = 4000 \text{ m}^3$$

EXERCISE - 13.6

1. Let the base radius of the cylindrical vessel be r cm.
We have,
Circumference $= 2\pi r$
$\Rightarrow 2\pi r = 132$ [$\because$ Circumference = 132 cm]
$$\Rightarrow 2 \times \frac{22}{7} \times r = 132 \Rightarrow r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$
Since height of the vessel (h) = 25 cm
$$\therefore \text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 21 \times 21 \times 25$$
$= 34650 \text{ cm}^3$
Capacity of the vessel = volume of the vessel = 34650 cm^3
Since, 1000 cm^3 = 1 litre
$$\Rightarrow 34650 \text{ cm}^3 = \frac{34650}{1000} l = 34.65\ l$$
Thus, the vessel can hold 34.65 l of water.

2. Inner diameter of the cylindrical pipe = 24 cm
$$\Rightarrow \text{Inner radius of the pipe } (r) = \frac{24}{2} \text{cm} = 12 \text{cm}$$
Outer diameter of the pipe = 28 cm
$$\Rightarrow \text{Outer radius of the pipe } (R) = \frac{28}{2} \text{cm} = 14 \text{cm}$$
Length of the pipe (h) = 35 cm
$\therefore$ Amount of wood (volume) in the pipe
= Outer volume – Inner volume $= \pi R^2 h - \pi r^2 h$
$= \pi h (R + r)(R - r)$
$$= \frac{22}{7} \times 35 \times (14 + 12) \times (14 - 12) = 22 \times 5 \times 26 \times 2$$
$= 5720 \text{ cm}^3$
Mass of 1 cm^3 of wood = 0.6 g [Given]
$\Rightarrow$ Mass of 5720 cm^3 of wood = 5720 × 0.6 g = 3432 g
$$= \frac{3432}{1000} \text{kg} = 3.432 \text{kg} \quad [\because 1000 \text{ g} = 1 \text{ kg}]$$
Thus, the required mass of the pipe is 3.432 kg.

3. For rectangular pack
Length (l) = 5 cm, Breadth (b) = 4 cm
Height (h) = 15 cm
Volume $= l \times b \times h = 5 \times 4 \times 15 = 300 \text{ cm}^3$
$\therefore$ Capacity of the rectangular pack = 300 cm^3
For cylindrical pack, base diameter = 7 cm
$$\therefore \text{Radius of the base } (r) = \frac{7}{2} \text{cm}$$
Height (h) = 10 cm
$$\therefore \text{Volume} = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10 = 11 \times 7 \times 5 = 385 \text{ cm}^3$$
$\therefore$ Volume of the cylindrical pack = 385 cm^3
So, the cylindrical pack has greater capacity by 85 cm^3.

4. Height of the cylinder (h) = 5 cm
Let the base radius of the cylinder be r cm.
(i) Lateral surface of the cylinder = 94.2 cm^2
$$\Rightarrow 2\pi rh = 94.2 \Rightarrow 2 \times 3.14 \times r \times 5 = \frac{942}{10}$$
$$\Rightarrow \frac{10 \times 314}{100} \times r = \frac{942}{10} \Rightarrow r = \frac{471}{157} = 3 \text{cm}$$
Thus, the radius of the cylinder = 3 cm
(ii) Volume of a cylinder $= \pi r^2 h$
$\Rightarrow$ Volume of the given cylinder $= 3.14 \times (3)^2 \times 5$
$= 141.3 \text{ cm}^3$
Thus, the required volume is 141.3 cm^3.

5. (i) Total cost of painting = ₹ 2200
Cost of painting 1 m^2 = ₹ 20
$$\therefore \text{Area} = \frac{\text{Total cost}}{\text{Cost of 1m}^2} = \frac{2200}{20} = 110 \text{m}^2$$
$\Rightarrow$ Inner curved surface of the vessel = 110 m^2
(ii) Let r and h be the base radius and height of the cylindrical vessel.
Curved surface area of a cylinder $= 2\pi rh$
$$\therefore 2\pi rh = 110 \Rightarrow 2 \times \frac{22}{7} \times r \times 10 = 110$$
[Given, height = 10 m]
$$\Rightarrow r = \frac{110 \times 7}{2 \times 22 \times 10} = \frac{7}{4} \Rightarrow r = 1.75 \text{ m}$$
$\therefore$ The required radius of the base = 1.75 m
(iii) Since, volume of a cylinder $= \pi r^2 h$
$\Rightarrow$ Volume (capacity) of the vessel
$$= \frac{22}{7} \times \left(\frac{7}{4}\right)^2 \times 10 = \frac{385}{4} = 96.25 \text{m}^3$$
Since, 1 m^3 = 1000000 cm^3 = 1000 l = 1 kl
$\therefore$ 96.25 m^3 = 96.25 kl
Thus, the required volume = 96.25 kl

6. Capacity of the cylindrical vessel
= 15.4 l = 15.4 × 1000 cm^3 [$\because$ 1 l = 1000 cm^3]

$= \frac{15.4 \times 1000}{1000000} m^3 = \frac{15.4}{1000} m^3$ [$\because$ 1000000 cm^3 = 1m^3]

Now, volume of the vessel $= \frac{15.4}{1000} m^3$

and height of the vessel = 1 m

Let radius of the base of the vessel be r m.

Now, Volume = $\pi r^2 h \Rightarrow \pi r^2 h = \frac{15.4}{1000}$

$\Rightarrow \frac{22}{7} \times r^2 \times 1 = \frac{154}{10000} \Rightarrow r^2 = \frac{154}{10000} \times \frac{7}{22} = \frac{49}{10000}$

$\Rightarrow r^2 = \left(\frac{7}{100}\right)^2 \Rightarrow r = \frac{7}{100} m$

Now, total surface area of the cylindrical vessel

$= 2\pi r(h + r) = 2 \times \frac{22}{7} \times \frac{7}{100}\left(1 + \frac{7}{100}\right)$

$= \frac{44}{100} \times \left(1 + \frac{7}{100}\right) = 0.4708$

Thus, 0.4708 m^2 sheet is required.

7. Since, 10 mm = 1 cm

$\therefore$ $1 mm = \frac{1}{10} cm$

For graphite cylinder,

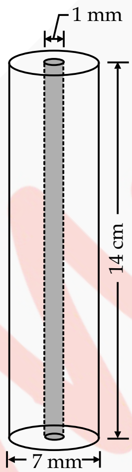
Diameter = $1 mm = \frac{1}{10} cm$

$\Rightarrow$ Radius (r) = $\frac{1}{10} \times \frac{1}{2} = \frac{1}{20} cm$

Length (h) = 14 cm

$\therefore$ Volume = $\pi r^2 h$

$= \frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 14 = 0.11$



Thus, the required volume of the graphite = 0.11 cm^3

Now, diameter of the pencil = 7 mm $= \frac{7}{10} cm$

$\therefore$ Radius of the pencil (R) $= \frac{7}{20} cm$

Height of the pencil (h) = 14 cm

Volume of the pencil = $\pi R^2 h$

$= \frac{22}{7} \times \left(\frac{7}{20}\right)^2 \times 14 = \frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 14$

$= \frac{11 \times 7 \times 7}{100} = 5.39 cm^3$

Volume of the wood = Volume of the pencil – Volume of the graphite

= 5.39 cm^3 – 0.11 cm^3 = 5.28 cm^3

Thus, the required volume of the wood is 5.28 cm^3

8. Diameter of the base of cylindrical bowl = 7 cm

$\Rightarrow$ Radius of the base (r) = $\frac{7}{2} cm$ and height (h) = 4 cm

$\therefore$ Volume of one bowl = $\pi r^2 h$

$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 4 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 4$

= 11 × 7 × 2 = 154 cm^3

i.e., Volume of soup in a bowl = 154 cm^3

$\Rightarrow$ Volume of soup in 250 bowls = 250 × 154 cm^3

= 38500 cm^3 $= \frac{38500}{1000} l = 38.5 l$ $\left[\because 1 cm^3 = \frac{1}{1000} l\right]$

Thus, the hospital needs to prepare 38.5 litres of soup daily for 250 patients.

EXERCISE - 13.7

1. (i) Radius of the cone (r) = 6 cm

Height (h) = 7 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 7$

= 22 × 2 × 6 = 264 cm^3

(ii) Here, radius of the cone (r) = 3.5 cm $= \frac{35}{10} cm$

Height (h) = 12 cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{35}{10}\right)^2 \times 12 = \frac{1}{3} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \times 12$ = 154 cm^3

2. (i) Here, r = 7 cm and l = 25 cm

$\therefore$ $h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2} = \sqrt{625 - 49} = 24 cm$

Now, volume of the conical vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24 = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$

= 22 × 7 × 8 = 1232 cm^3

$= \frac{1232}{1000} l = 1.232 l$ [$\because$ 1000 cm^3 = 1 l]

Thus, the required capacity of the conical vessel is 1.232 l.

(ii) Here, height (h) = 12 cm and l = 13 cm

$\therefore$ $r = \sqrt{l^2 - h^2} = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = 5 cm$

Now, volume of the conical vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12$

$= \frac{22 \times 5 \times 5 \times 4}{7} = \frac{2200}{7} cm^3 = \frac{2200}{7 \times 1000} l$ [$\because$ 1000 cm^3 = 1 l]

Thus, the required capacity of the conical vessel is $\frac{11}{35} l$.

3. Here, height of the cone (h) = 15 cm

Volume of the cone = 1570 cm^3 [Given]

Let the radius of the base be r cm.

$\therefore$ $\frac{1}{3}\pi r^2 h = 1570$

$\Rightarrow$ $\frac{1}{3} \times 3.14 \times r^2 \times 15 = 1570 \Rightarrow \frac{1}{3} \times \frac{314}{100} \times r^2 \times 15 = 1570$

$\Rightarrow$ $r^2 = \frac{1570 \times 3 \times 100}{314 \times 15} = \frac{5 \times 3 \times 100}{15} = 100$

$\Rightarrow$ $r^2 = 10^2 \Rightarrow r = \sqrt{10^2}$ = 10 cm

Thus, the required radius of the base is 10 cm.

4. Volume of the cone = 48π cm^3 [Given]
Height of the cone (h) = 9 cm
Let r be its base radius.

$\therefore \frac{1}{3}\pi r^2 h = 48\pi \Rightarrow \frac{1}{3}\pi r^2 \times 9 = 48\pi$

$\Rightarrow r^2 = \frac{48\times\pi\times 3}{9\times\pi} = 16 = 4^2 \Rightarrow r = \sqrt{4^2} = 4$ cm

$\therefore$ Diameter of the base of the cone = 2 × 4 = 8 cm

5. Here, diameter of the conical pit = 3.5 m

$\therefore$ Radius (r) = $\frac{3.5}{2} = \frac{35}{20}$ m, Depth (h) = 12 m

$\therefore$ Volume (capacity) = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times\left(\frac{35}{20}\right)^2\times 12 = \frac{1}{3}\times\frac{22}{7}\times\frac{35}{20}\times\frac{35}{20}\times 12$

$= \frac{11\times 35}{10} = \frac{385}{10} = 38.5$ m^3

$\because$ 1 m^3 = 1 kl $\Rightarrow$ 38.5 m^3 = 38.5 kl

Thus, the capacity of the conical pit is 38.5 kl.

6. Volume of the cone = 9856 cm^3
Diameter of the base = 28 cm

$\Rightarrow$ Radius of the base $= \frac{28}{2} = 14$ cm

(i) Let the height of the cone be h cm.

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times(14)^2\times h$

$= \frac{1}{3}\times\frac{22}{7}\times 14\times 14\times h$

$\Rightarrow \frac{1}{3}\times 22\times 2\times 14\times h = 9856$

$\Rightarrow h = \frac{9856\times 3}{22\times 2\times 14} = 16 \times 3 = 48$

Thus, the required height is 48 cm.

(ii) Let the slant height be l cm.

$\Rightarrow l^2 = r^2 + h^2 \Rightarrow l^2 = 14^2 + 48^2 = 196 + 2304 = 2500 = (50)^2$

$\therefore l = 50$

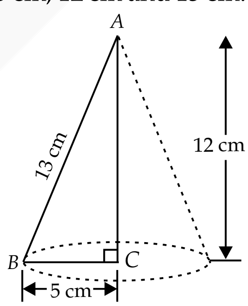
Thus, the required slant height = 50 cm.

(iii) The curved surface area of a cone is given by $\pi r l$

$\therefore$ Curved surface area $= \frac{22}{7}\times 14\times 50 = 22 \times 2 \times 50 = 2200$

Thus, the curved surface area of the cone is 2200 cm^2.

7. Sides of the right triangle are 5 cm, 12 cm and 13 cm.
When this triangle is revolved about the side of 12 cm, we get a cone as shown in the figure.



Thus, radius of the base of the cone so formed (r) = 5 cm
Height (h) = 12 cm

$\therefore$ Volume of the cone so obtained

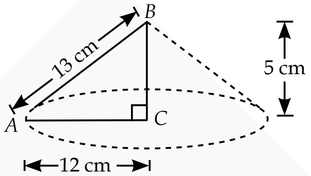
$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\pi\times(5)^2\times 12$

$= \frac{1}{3}\times\pi\times 5\times 5\times 12 = 100\,\pi$

Thus, the required volume of the cone is 100 π cm^3.

8. Since the right triangle is revolved about the side of 5 cm.

$\therefore$ Height of the cone so obtained (h) = 5 cm
Radius of the cone (r) = 12 cm



$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\pi\times(12)^2\times 5$

$= \frac{1}{3}\times\pi\times 12\times 12\times 5$

$= \pi \times 240 = 240\,\pi$ cm^3

Now, ratio of two volumes $= \frac{100\pi\,\text{cm}^3}{240\pi\,\text{cm}^3} = \frac{5}{12} = 5:12$

Thus, the required ratio is 5 : 12.

9. Here, the heap of wheat is in the form of a cone with base diameter = 10.5 m

$\therefore$ Base radius (r) $= \frac{10.5}{2}\,\text{m} = \frac{105}{20}\,\text{m}$

Height (h) = 3 m

$\therefore$ Volume of the heap $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times\left(\frac{105}{20}\right)^2\times 3 = 86.625$

Thus, the required volume = 86.625 m^3

Now the area of the canvas to cover the heap must be equal to the curved surface area of the conical heap.

$\therefore$ Area of the canvas = $\pi r l$, where $l = \sqrt{r^2 + h^2}$

$\therefore l = \sqrt{\left(\frac{105}{20}\right)^2 + 3^2} = \sqrt{\frac{11025}{400} + 9}$

$= \sqrt{36.5625} = 6.05$ m (approx.)

Now, $\pi r l = \frac{22}{7}\times\frac{105}{20}\times 6.05 = 11 \times 1.5 \times 6.05 = 99.825$ m^2

Thus, the required area of the canvas is 99.825 m^2.

EXERCISE - 13.8

1. (i) Here, radius (r) = 7 cm

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\frac{22}{7}\times(7)^3 = \frac{4}{3}\times\frac{22}{7}\times 7\times 7\times 7 = \frac{4312}{3} = 1437\frac{1}{3}$ cm^3

Thus, the required volume = $1437\frac{1}{3}$ cm^3

(ii) Here, radius (r) = 0.63 m

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\frac{22}{7}\times\left(\frac{63}{100}\right)^3 = \frac{4}{3}\times\frac{22}{7}\times\frac{63}{100}\times\frac{63}{100}\times\frac{63}{100}$

$= \frac{1047816}{1000000} = 1.047816$ m^3

Thus, the required volume is 1.05 m^3 (approx.)

2. (i) Diameter of the ball = 28 cm

$\Rightarrow$ Radius (r) $= \frac{28}{2} = 14$ cm

$\therefore$ Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times(14)^3$

$= \frac{4}{3}\times\frac{22}{7}\times14\times14\times14 = \frac{34496}{3} = 11498\frac{2}{3}\text{cm}^3$

Thus, the amount of water displaced $= 11498\frac{2}{3}\text{cm}^3$

(ii) Diameter of the ball = 0.21 m

$\Rightarrow$ Radius (r) $= \frac{0.21}{2} = \frac{21}{200}$ m

$\therefore$ Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times\left(\frac{21}{200}\right)^3$

$= \frac{4}{3}\times\frac{22}{7}\times\frac{21}{200}\times\frac{21}{200}\times\frac{21}{200}$

$= \frac{11\times21\times21}{1000000} = \frac{4851}{1000000} = 0.004851\ \text{m}^3$

3. Diameter of a metallic ball = 4.2 cm

$\Rightarrow$ Radius (r) $= \frac{4.2}{2} = 2.1$ cm

$\therefore$ Volume of the metallic ball

$= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times(2.1)^3$

$= \frac{4}{3}\times\frac{22}{7}\times\frac{21}{10}\times\frac{21}{10}\times\frac{21}{10} = \frac{4\times22\times21\times21}{10\times10\times10}\text{cm}^3$

Also, density of the metal = 8.9 g per cm^3 [Given]

$\therefore$ Mass of the ball = 8.9 × [Volume of the ball]

$= \frac{89}{10}\times\frac{4\times22\times21\times21}{10\times10\times10} = \frac{3453912}{10000} = 345.3912\,\text{g}$

= 345.39 g (approx.)

Thus, the mass of the ball is 345.39 g (approx.)

4. Let radius of the earth = r

Diameter of the moon $= \frac{1}{4}$(Diameter of the earth) [Given]

$\Rightarrow$ Radius of the moon $= \frac{1}{4}$(Radius of the earth)

$\Rightarrow$ Radius of the moon $= \frac{1}{4}(r) = \frac{r}{4}$

Volume of the earth $= \frac{4}{3}\pi r^3$

Volume of the moon $= \frac{4}{3}\pi\left(\frac{r}{4}\right)^3 = \frac{4}{3}\times\pi\times\frac{r\times r\times r}{4\times4\times4} = \frac{\pi r^3}{48}$

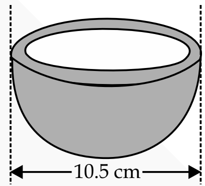
Now, $\frac{\text{Volume of the earth}}{\text{Volume of the moon}} = \frac{\frac{4}{3}\pi r^3}{\frac{\pi r^3}{48}} = \frac{4}{3}\pi r^3\times\frac{48}{\pi r^3} = \frac{64}{1}$

or Volume of the moon $= \frac{1}{64}$ × Volume of the earth

$\therefore$ The required fraction is $\frac{1}{64}$.

5. Diameter of the hemisphere = 10.5 cm

$\Rightarrow$ Radius $(r) = \frac{10.5}{2} = \frac{105}{20}$ cm



Volume of the hemispherical bowl $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\times\frac{22}{7}\times\frac{105}{20}\times\frac{105}{20}\times\frac{105}{20}$

$= \frac{11\times105\times105}{20\times20} = 303.1875\ \text{cm}^3$

$\therefore$ Capacity of the hemispherical bowl = 303.1875 cm^3

$= \frac{3031875}{10000\times1000}\,l = 0.3031875\,l$ [$\because$ 1000 cm^3 = 1 l]

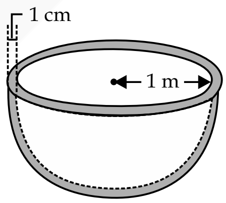
= 0.303 l (approx.)

Thus, the capacity of the bowl is 0.303 l (approx.).

6. Inner radius (r) = 1 m

$\because$ Thickness = 1 cm $= \frac{1}{100}$ m = 0.01 m

$\therefore$ Outer radius (R) = 1 + 0.01 = 1.01 m



Now, volume of outer hemispherical bowl

$= \frac{2}{3}\pi R^3 = \frac{2}{3}\times\frac{22}{7}\times(1.01)^3$

Volume of inner hemispherical bowl

$= \frac{2}{3}\pi r^3 = \frac{2}{3}\times\frac{22}{7}\times(1)^3$

$\therefore$ Volume of the iron used = [Outer volume] – [Inner volume]

$= \frac{2}{3}\times\frac{22}{7}\times(1.01)^3 - \frac{2}{3}\times\frac{22}{7}\times(1)^3$

$= \frac{2}{3}\times\frac{22}{7}\times[(1.01)^3-(1)^3]$

$= \frac{44}{21}(1.030301-1) = 0.06348\ \text{m}^3$ (approx.)

Thus, the required volume of the iron is 0.06348 m^3.

7. Let r be the radius of the sphere.

$\therefore$ Its surface area $= 4\pi r^2 \Rightarrow 4\pi r^2 = 154$

$\Rightarrow r^2 = \frac{154}{4\pi} = \frac{154\times7}{4\times22} = \frac{7\times7}{4} \Rightarrow r^2 = \left(\frac{7}{2}\right)^2 \Rightarrow r = \frac{7}{2}$ cm

Now, volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^3 = \frac{4}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times\frac{7}{2}$

$= \frac{11\times7\times7}{3} = \frac{539}{3} = 179\frac{2}{3}\text{cm}^3$

Thus, the required volume of the sphere is $179\frac{2}{3}\text{cm}^3$.

8. (i) Total cost of white washing = ₹ 4989.60

Cost of white washing of 1 m^2 area = ₹ 20

$\therefore$ Inside surface area of the dome

$= \frac{\text{Total cost}}{\text{Cost of 1 m}^2\text{ area}} = \frac{4989.6}{20} = 249.48\,\text{m}^2$

Thus, the required inside surface area of the dome is 249.48 m^2

(ii) Let r be the radius of the hemispherical dome

$\therefore$ Surface area = $2\pi r^2$

$\Rightarrow 2\pi r^2 = 249.48 \Rightarrow 2\times\frac{22}{7}\times r^2 = \frac{24948}{100}$

$\Rightarrow r^2 = \frac{24948}{100}\times\frac{7}{2\times 22} = \frac{3969}{100}$

$\Rightarrow r^2 = \left(\frac{63}{10}\right)^2 \Rightarrow r = \frac{63}{10} = 6.3\text{m}$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$\therefore$ Volume of air in the dome $= \frac{2}{3}\times\frac{22}{7}\times(6.3)^3$

$= \frac{2}{3}\times\frac{22}{7}\times\frac{63}{10}\times\frac{63}{10}\times\frac{63}{10} = \frac{2\times 22\times 3\times 63\times 63}{1000}$

$= \frac{523908}{1000} = 523.9\text{m}^3$ (approx.)

Thus, the required volume of air inside the dome is 523.9 m^3 (approx).

9. (i) Radius of small sphere = r

$\therefore$ Its volume $= \frac{4}{3}\pi r^3$

Volume of 27 small spheres $= 27\times\left(\frac{4}{3}\pi r^3\right)$

Radius of the new sphere = r'

$\therefore$ Volume of the new sphere $= \frac{4}{3}\pi(r')^3$

Since, $\frac{4}{3}\pi(r')^3 = 27\times\frac{4}{3}\pi r^3 \Rightarrow (r')^3 = \frac{27\times\frac{4}{3}\pi r^3}{\frac{4}{3}\pi} = 27r^3$

$\Rightarrow (r')^3 = (3r)^3 \Rightarrow r' = 3r$

(ii) Surface area of small sphere = $4\pi r^2$

$\Rightarrow S = 4\pi r^2$ and $S' = 4\pi(3r)^2$ $\quad [\because r' = 3r]$

Now, $\frac{S}{S'} = \frac{4\pi r^2}{4\pi(3r)^2} = \frac{4\pi r^2}{4\pi(9r^2)} = \frac{1}{9}$

Thus, $S : S' = 1 : 9$

10. Diameter of the spherical capsule = 3.5 mm

$\Rightarrow$ Radius (r) $= \frac{3.5}{2}$ mm

$\therefore$ Volume of the spherical capsule

$= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times\left(\frac{35}{20}\right)^3$

$= \frac{4}{3}\times\frac{22}{7}\times\frac{35}{20}\times\frac{35}{20}\times\frac{35}{20} = \frac{22\times 35\times 35}{3\times 20\times 20}$

$= \frac{26950}{1200}$ = 22.45833 mm^3 = 22.46 mm^3 (approx.)

Thus, the required quantity of medicine is 22.46 mm^3 (approx.).

EXERCISE - 13.9

1. Here, length (l) = 85 cm, breadth (b) = 25 cm and height (h) = 110 cm

External surface area = Area of four faces + Area of back + Area of front beading

= [2 × (110 + 85) × 25 + 110 × 85 + (110 × 5 × 2) + (75 × 5) × 4] = 21700 cm^2

$\therefore$ Cost of polishing external faces $= ₹\left(21700\times\frac{20}{100}\right) = ₹4340$

Internal surface area = Area of five faces of 3 cuboids each of dimensions 75 cm × 30 cm × 20 cm

= Total surface area of 3 cuboids of dimensions 75 cm × 30 cm × 20 cm – Area of bases of 3 cuboids of dimensions 75 cm × 30 cm × 20 cm

{2(75 × 30 + 30 × 20 + 75 × 20)} cm^2 – {3 × (75 × 30)} cm^2

= 6(2250 + 600 + 1500) cm^2 – 6750 cm^2 = 19350 cm^2

$\therefore$ Cost of painting inner faces $= ₹\left(19350\times\frac{10}{100}\right) = ₹1935$

Hence, total expenses = ₹ (4340 + 1935) = ₹ 6275

2. Here, diameter of sphere = 21 cm

$\Rightarrow$ Radius of sphere (r) $= \frac{21}{2}$ cm

$\therefore$ Surface area of sphere = $4\pi r^2$

$\therefore$ Surface area of 8 spheres $= 8\times 4\times\frac{22}{7}\times\left(\frac{21}{2}\right)^2$

= 8 × 22 × 3 × 21 = 11088 cm^2

Now, radius of cylinder (r_1) = 1.5 cm

Height of cylinder (h) = 7 cm

Curved surface area of a cylinder = $2\pi r_1 h$

$\therefore$ Curved surface area of 8 cylinders

$= 8\times 2\times\frac{22}{7}\times 1.5\times 7$ = 528 cm^2

Surface area of the top of 8 cylinders = $8\pi r^2$

$= 8\times\frac{22}{7}\times(1.5)^2$ = 56.57 cm^2 (approx.)

$\therefore$ Cost of painting

$= ₹\left[(11088-56.57)\times\frac{25}{100}+528\times\frac{5}{100}\right]$

$= ₹\left[(11031.43)\times\frac{25}{100}+\frac{2640}{100}\right] = ₹\left[\frac{275785.75}{100}+\frac{2640}{100}\right]$

$= ₹\left[\frac{278425.75}{100}\right]$ = ₹ 2784. 25

Hence, the cost of paint required = ₹ 2784.25 (approx.)

3. Let the diameter of a sphere be d.

After decreasing, diameter of the sphere

$= d-\frac{25}{100}\times d = d-\frac{1}{4}d = \frac{3}{4}d$

Since, surface area of a sphere = $4\pi r^2 = \pi(2r)^2 = \pi d^2$

Surface area of sphere, when diameter of the sphere is

$\frac{3}{4}d = \pi\left(\frac{3}{4}d\right)^2 = \frac{9\pi}{16}d^2$

Now, percentage decrease in curved surface area

$= \frac{\pi d^2 - \frac{9\pi}{16}d^2}{\pi d^2}\times 100 = \frac{16-9}{16}\times 100 = \frac{7}{16}\times 100 = 43.75\%$