Lines and Angles

CHAPTER 6

NCERT FOCUS

SOLUTIONS

EXERCISE - 6.1

1. Since, AB is a straight line.

$\therefore \quad \angle AOC + \angle COE + \angle EOB = 180°$

or $\quad (\angle AOC + \angle BOE) + \angle COE = 180°$

or $\quad 70° + \angle COE = 180°$ [$\because \angle AOC + \angle BOE = 70°$ (Given)]

or $\quad \angle COE = 180° - 70° = 110°$

$\therefore$ Reflex $\angle COE = 360° - 110° = 250°$

Now, as AB and CD intersect at O.

$\therefore \quad \angle COA = \angle BOD$ [Vertically opposite angles]

But $\angle BOD = 40°$ [Given]

$\therefore \quad \angle COA = 40°$

Also, $\angle AOC + \angle BOE = 70°$

$\therefore \quad 40° + \angle BOE = 70° \Rightarrow \angle BOE = 70° - 40° = 30°$

Thus, $\angle BOE = 30°$ and reflex $\angle COE = 250°$.

2. Since, XOY is a straight line.

$\therefore \quad b + a + \angle POY = 180°$

But $\angle POY = 90°$ [Given]

$\therefore \quad b + a = 180° - 90° = 90°$ ...(i)

Also, we have $a : b = 2 : 3$

$\therefore \quad a = \dfrac{2}{5}(a+b) = \dfrac{2}{5} \times 90° = 36°$

and $\quad b = \dfrac{3}{5} \times 90° = 54°$ (Using (i))

Since, XY and MN intersect at O.

$\therefore \quad c = a + \angle POY$ [Vertically opposite angles]

$\Rightarrow \quad c = 36° + 90° = 126°$

Thus, the required measure of $c = 126°$.

3. Since, ST is a straight line.

$\therefore \quad \angle PQR + \angle PQS = 180°$ ...(i) [Linear pair]

Similarly, $\angle PRT + \angle PRQ = 180°$ ...(ii) [Linear Pair]

From (i) and (ii), we have

$\angle PQR + \angle PQS = \angle PRT + \angle PRQ$

But $\angle PQR = \angle PRQ$ [Given]

$\therefore \quad \angle PQS = \angle PRT$

4. Since, sum of all the angles around a point = 360°

$\therefore \quad x + y + z + w = 360°$ or $(x + y) + (z + w) = 360°$

But $(x + y) = (z + w)$ [Given]

$\therefore \quad (x + y) + (x + y) = 360° \Rightarrow 2(x + y) = 360°$

$\Rightarrow \quad (x+y) = \dfrac{360°}{2} = 180°$

$\therefore \quad AOB$ is a straight line. [By linear pair axiom]

5. Since, POQ is a straight line. [Given]

$\therefore \quad \angle POS + \angle ROS + \angle ROQ = 180°$

But $OR \perp PQ \quad \therefore \angle ROQ = 90°$

$\Rightarrow \quad \angle POS + \angle ROS + 90° = 180°$

$\Rightarrow \quad \angle POS + \angle ROS = 90°$

$\Rightarrow \quad \angle ROS = 90° - \angle POS$ ...(i)

Also, we have $\angle ROS + \angle ROQ = \angle QOS$

$\Rightarrow \quad \angle ROS + 90° = \angle QOS$

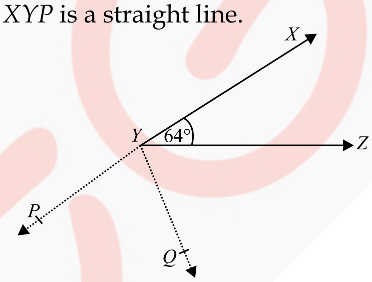
$\Rightarrow \quad \angle ROS = \angle QOS - 90°$ ...(ii)

Adding (i) and (ii), we have

$2\angle ROS = (\angle QOS - \angle POS)$

$\therefore \quad \angle ROS = \dfrac{1}{2}(\angle QOS - \angle POS)$

6. Since, XYP is a straight line.



$\therefore \quad \angle XYZ + \angle ZYQ + \angle QYP = 180°$

$\Rightarrow \quad 64° + \angle ZYQ + \angle QYP = 180°$ [$\because \angle XYZ = 64°$ (given)]

$\Rightarrow \quad 64° + 2\angle QYP = 180°$

[$\because$ YQ bisects $\angle ZYP$, so $\angle QYP = \angle ZYQ$]

$\Rightarrow \quad 2\angle QYP = 180° - 64° = 116°$

$\Rightarrow \quad \angle QYP = \dfrac{116°}{2} = 58°$

$\therefore$ Reflex $\angle QYP = 360° - 58° = 302°$

Since $\angle XYQ = \angle XYZ + \angle ZYQ$

$\Rightarrow \quad \angle XYQ = 64° + \angle QYP$

[$\because \angle XYZ = 64°$(given) and $\angle ZYQ = \angle QYP$]

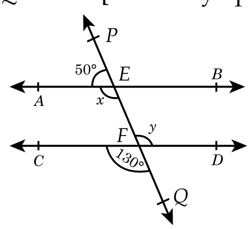
$\Rightarrow \quad \angle XYQ = 64° + 58° = 122°$ [$\because \angle QYP = 58°$]

Thus, $\angle XYQ = 122°$ and reflex $\angle QYP = 302°$

EXERCISE - 6.2

1. Let PQ be the given transversal intersecting AB and CD at E and F respectively as shown in the figure.

Clearly, $y = \angle CFQ = 130°$ [Vertically opposite angles]...(i)



Again, PQ is a straight line and EA stands on it.

$\therefore \quad \angle AEP + \angle AEQ = 180°$ [By linear pair axiom]

or $\quad 50° + x = 180°$

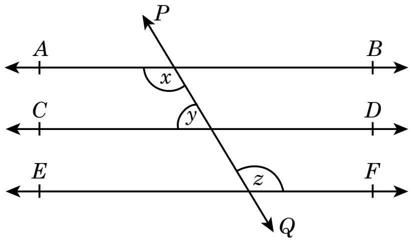
$\Rightarrow \quad x = 180° - 50° = 130°$ ... (ii)

From (i) and (ii), we get $x = y$,

which forms a pair of alternate interior angles.

$\therefore \quad AB \parallel CD$

2. Given, $AB \parallel CD$ and $CD \parallel EF$
Let PQ be the given transversal
Then, we have $AB \parallel EF$ and PQ is a transversal.
[$\because$ $AB \parallel CD$ and $CD \parallel EF \Rightarrow AB \parallel EF$]
$\therefore$ $x = z$ [Alternate interior angles] ...(i)

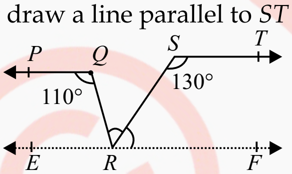


Again, $AB \parallel CD$ and PQ is a transversal
$\therefore$ $x + y = 180°$ [Co-interior angles]
$\Rightarrow$ $z + y = 180°$ [Using (i)]
But $y : z = 3 : 7$
$\therefore$ $z = \left[\frac{7}{(3+7)}\right] \times 180° = \frac{7}{10} \times 180° = 126°$ ...(ii)

From (i) and (ii), we get
$x = 126°$.

3. Since, $AB \parallel CD$ and GE is a transversal.
$\therefore$ $\angle AGE = \angle GED$ [Alternate interior angles]
But $\angle GED = 126°$ [Given]
$\therefore$ $\angle AGE = 126°$
Also, $\angle GEF + \angle FED = \angle GED$
$\Rightarrow$ $\angle GEF + 90° = 126°$ [$\because$ $EF \perp CD$ (Given)]
$\Rightarrow$ $\angle GEF = 126° - 90° = 36°$
Again, as $AB \parallel CD$ and GE is a transversal.
$\therefore$ $\angle FGE + \angle GED = 180°$ [Co-interior angles]
$\Rightarrow$ $\angle FGE + 126° = 180°$
$\Rightarrow$ $\angle FGE = 180° - 126° = 54°$
Thus, $\angle AGE = 126°$, $\angle GEF = 36°$ and $\angle FGE = 54°$.

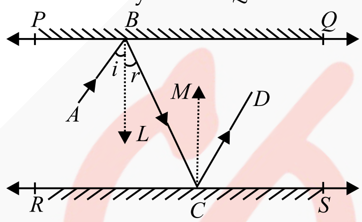
4. Let us first draw a line parallel to ST through R.



Since $PQ \parallel ST$ [Given]
and $EF \parallel ST$ [By construction]
$\therefore$ $PQ \parallel EF$ and QR is a transversal.
$\Rightarrow$ $\angle PQR = \angle QRF$ [Alternate interior angles]
But $\angle PQR = 110°$ [Given]
$\therefore$ $\angle QRF = \angle QRS + \angle SRF = 110°$ ...(i)
Again $ST \parallel EF$ and RS is a transversal
$\therefore$ $\angle RST + \angle SRF = 180°$ [Co-interior angles]
or $130° + \angle SRF = 180°$
$\Rightarrow$ $\angle SRF = 180° - 130° = 50°$ ...(ii)
Now, from (i) and (ii), we have
$\angle QRS + 50° = 110°$
$\Rightarrow$ $\angle QRS = 110° - 50° = 60°$
Thus, $\angle QRS = 60°$.

5. We have $AB \parallel CD$ and PQ is a transversal.
$\therefore$ $\angle APQ = \angle PQR$ [Alternate interior angles]
or $50° = x$ [$\because$ $\angle APQ = 50°$ (Given)]
Again, $AB \parallel CD$ and PR is a transversal.
$\therefore$ $\angle APR = \angle PRD$ [Alternate interior angles]
$\Rightarrow$ $\angle APR = 127°$ [$\because$ $\angle PRD = 127°$ (Given)]
$\Rightarrow$ $\angle APQ + \angle QPR = 127° \Rightarrow$ $50° + y = 127°$
[$\because$ $\angle APQ = 50°$ (Given)]
$\Rightarrow$ $y = 127° - 50° = 77°$
Thus, $x = 50°$ and $y = 77°$.

6. Let us first draw ray $BL \perp PQ$ and $CM \perp RS$



Since, $PQ \parallel RS$, therefore $BL \parallel CM$
Now, $BL \parallel CM$ and BC is a transversal.
$\therefore$ $\angle LBC = \angle MCB$ [Alternate interior angles] ...(i)
Since, angle of incidence = Angle of reflection
$\therefore$ $\angle ABL = \angle LBC$ and $\angle MCB = \angle MCD$
$\Rightarrow$ $\angle ABL = \angle MCD$ [Using (i)] ...(ii)
Now, Adding (i) and (ii), we get
$\therefore$ $\angle LBC + \angle ABL = \angle MCB + \angle MCD \Rightarrow \angle ABC = \angle BCD$
i.e., a pair of alternate interior angles are equal
Hence, $AB \parallel CD$.

EXERCISE - 6.3

1. We have, $\angle TQP + \angle PQR = 180°$
[By linear pair axiom]
$\Rightarrow$ $110° + \angle PQR = 180°$
$\Rightarrow$ $\angle PQR = 180° - 110° = 70°$
Since, the side QP of ΔPQR is produced to S.
$\therefore$ $\angle PQR + \angle PRQ = 135°$
[By exterior angle property of a triangle]
$\Rightarrow$ $70° + \angle PRQ = 135°$ [Using (i)]
$\Rightarrow$ $\angle PRQ = 135° - 70° \Rightarrow \angle PRQ = 65°$

2. In ΔXYZ, we have
$\angle XYZ + \angle YZX + \angle ZXY = 180°$
[By angle sum property of a triangle]
But $\angle XYZ = 54°$ and $\angle ZXY = 62°$
$\therefore$ $54° + \angle YZX + 62° = 180°$
$\Rightarrow$ $\angle YZX = 180° - 54° - 62° = 64°$
Now, as YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively.
$\therefore$ $\angle OYZ = \frac{1}{2}\angle XYZ = \frac{1}{2}(54°) = 27°$
and $\angle OZY = \frac{1}{2}\angle YZX = \frac{1}{2}(64°) = 32°$
Now, in ΔOYZ, we have
$\angle YOZ + \angle OYZ + \angle OZY = 180°$
[By angle sum property of a triangle]
$\Rightarrow$ $\angle YOZ + 27° + 32° = 180°$
$\Rightarrow$ $\angle YOZ = 180° - 27° - 32° = 121°$
Thus, $\angle OZY = 32°$ and $\angle YOZ = 121°$

3. Since, $AB \parallel DE$ and AE is a transversal.
So, $\angle BAC = \angle AED$ [Alternate interior angles]

But $\angle BAC = 35°$ [Given]

$\therefore \quad \angle AED = 35°$

Now, in ΔCDE, we have $\angle CDE + \angle DEC + \angle DCE = 180°$

[By angle sum property of a triangle]

$\therefore \quad 53° + 35° + \angle DCE = 180°$

[$\because \angle DEC = \angle AED = 35°$ and $\angle CDE = 53°$ (Given)]

$\Rightarrow \quad \angle DCE = 180° - 53° - 35° = 92°$

Thus, $\angle DCE = 92°$

4. In ΔPRT, we have $\angle P + \angle R + \angle PTR = 180°$

[By angle sum property of a triangle]

$\Rightarrow \quad 95° + 40° + \angle PTR = 180°$

[$\because \angle P = 95°, \angle R = 40°$ (Given)]

$\Rightarrow \quad \angle PTR = 180° - 95° - 40° = 45°$

Now, as PQ and RS intersect at T.

$\therefore \quad \angle PTR = \angle QTS$ [Vertically opposite angles]

$\Rightarrow \quad \angle QTS = 45°$ [$\because \angle PTR = 45°$]

Now, in ΔTQS, we have

$\angle TSQ + \angle STQ + \angle SQT = 180°$

[By angle sum property of a triangle]

$\Rightarrow \quad 75° + 45° + \angle SQT = 180°$

[$\because \angle TSQ = 75°$ and $\angle STQ = 45°$]

$\Rightarrow \quad \angle SQT = 180° - 75° - 45° = 60°$

Thus, $\angle SQT = 60°$

5. In ΔQRS, the side SR is produced to T.

$\therefore \quad \angle QRT = \angle RQS + \angle RSQ$

[By exterior angle property of a triangle]

But $\angle RQS = 28°$ and $\angle QRT = 65°$

So, $28° + \angle RSQ = 65°$

$\Rightarrow \quad \angle RSQ = 65° - 28° = 37°$

Since, $PQ \parallel SR$ and QS is a transversal.

$\therefore \quad \angle PQS = \angle RSQ = 37°$ [Alternate interior angles]

$\Rightarrow \quad x = 37°$

Again, $PQ \perp PS \Rightarrow \angle P = 90°$

Now, in ΔPQS, we have $\angle P + \angle PQS + \angle PSQ = 180°$

[By angle sum property of a triangle]

$\Rightarrow \quad 90° + 37° + y = 180°$

$\Rightarrow \quad y = 180° - 90° - 37° = 53°$

Thus, $x = 37°$ and $y = 53°$

6. In ΔPQR, side QR is produced to S, so by exterior angle property,

$\angle PRS = \angle P + \angle PQR$

$$\Rightarrow \frac{1}{2}\angle PRS = \frac{1}{2}\angle P + \frac{1}{2}\angle PQR \quad \text{...(i)}$$

$$\Rightarrow \angle TRS = \frac{1}{2}\angle P + \angle TQR \quad \text{(ii)}$$

[$\because QT$ and RT are bisectors of $\angle PQR$ and $\angle PRS$ respectively.]

Now, for the ΔQRT, we have

$\angle TRS = \angle TQR + \angle T$ ...(iii)

[By exterior angle property of a triangle]

From (ii) and (iii), we have

$$\angle TQR + \frac{1}{2}\angle P = \angle TQR + \angle T$$

$$\Rightarrow \frac{1}{2}\angle P = \angle T \Rightarrow \frac{1}{2}\angle QPR = \angle QTR$$

$$\Rightarrow \angle QTR = \frac{1}{2}\angle QPR$$