Triangles

CHAPTER 7

NCERT FOCUS

SOLUTIONS

EXERCISE - 7.1

1. In quadrilateral $ACBD$, we have $AC = AD$ and AB being the bisector of $\angle A$.
Now, in ΔABC and ΔABD,
$AC = AD$ [Given]
$\angle CAB = \angle DAB$ [$\because$ AB bisects $\angle CAD$]
$AB = AB$ [Common]
$\therefore$ $\Delta ABC \cong \Delta ABD$ [By SAS congruence]
So, $BC = BD$ [By C.P.C.T.]

2. In quadrilateral $ABCD$, we have
$AD = BC$ and $\angle DAB = \angle CBA$
In ΔABD and ΔBAC,
$AD = BC$ [Given]
$\angle DAB = \angle CBA$ [Given]
$AB = AB$ [Common]
$\therefore$ $\Delta ABD \cong \Delta BAC$ [By SAS congruence]
$\therefore$ $BD = AC$ [By C.P.C.T.]
and $\angle ABD = \angle BAC$ [By C.P.C.T.]

3. In ΔOBC and ΔOAD, we have
$\angle OBC = \angle OAD$ [Each equals 90°]
$BC = AD$ [Given]
$\angle BOC = \angle AOD$ [Vertically opposite angles]
$\therefore$ $\Delta OBC \cong \Delta OAD$ [By AAS congruence]
$\Rightarrow$ $OB = OA$ [By C.P.C.T.]
i.e., O is the mid-point of AB.
Thus, CD bisects AB.

4. $\because$ $p \parallel q$ and AC is a transversal.
$\therefore$ $\angle BAC = \angle DCA$ ...(i)
[Alternate interior angles]
Also $l \parallel m$ and AC is a transversal.
$\therefore$ $\angle BCA = \angle DAC$ ...(ii)
[Alternate interior angles]
Now, in ΔABC and ΔCDA, we have
$\angle BAC = \angle DCA$ [From (i)]
$AC = AC$ [Common]
$\angle BCA = \angle DAC$ [From (ii)]
$\therefore$ $\Delta ABC \cong \Delta CDA$ [By ASA congruence]

5. We have, l is the bisector of $\angle QAP$.
$\therefore$ $\angle QAB = \angle PAB$ and
$\angle Q = \angle P$ [Each equals 90°]
$\Rightarrow$ $\angle ABQ = \angle ABP$ ...(i)
[By angle sum property of triangle]
(i) Now, in ΔAPB and ΔAQB, we have
$\angle ABP = \angle ABQ$ [From (i)]
$AB = AB$ [Common]
$\angle PAB = \angle QAB$ [Given]
$\therefore$ $\Delta APB \cong \Delta AQB$ [By ASA congruence]
(ii) Since $\Delta APB \cong \Delta AQB$
$\Rightarrow$ $BP = BQ$ [By C.P.C.T.]
or Perpendicular distance of B from AP
= Perpendicular distance of B from AQ
Thus, the point B is equidistant from the arms of $\angle A$.

6. We have, $\angle BAD = \angle EAC$
Adding $\angle DAC$ on both sides, we have
$\angle BAD + \angle DAC = \angle EAC + \angle DAC$
$\Rightarrow$ $\angle BAC = \angle DAE$
Now, in ΔABC and ΔADE, we have
$\angle BAC = \angle DAE$ [Proved above]
$AB = AD$ [Given]
$AC = AE$ [Given]
$\therefore$ $\Delta ABC \cong \Delta ADE$ [By SAS congruence]
$\Rightarrow$ $BC = DE$ [By C.P.C.T.]

7. We have, P is the mid-point of AB.
$\therefore$ $AP = BP$
$\angle EPA = \angle DPB$ [Given]
Adding $\angle EPD$ on both sides, we get
$\angle EPA + \angle EPD = \angle DPB + \angle EPD$
$\Rightarrow$ $\angle APD = \angle BPE$
(i) Now, in ΔDAP and ΔEBP, we have
$\angle PAD = \angle PBE$ [$\because$ $\angle BAD = \angle ABE$]
$AP = BP$ [Proved above]
$\angle DPA = \angle EPB$ [Proved above]
$\therefore$ $\Delta DAP \cong \Delta EBP$ [By ASA congruence]
(ii) Since, $\Delta DAP \cong \Delta EBP$
$\Rightarrow$ $AD = BE$ [By C.P.C.T.]

8. Since M is the mid-point of AB. [Given]
$\therefore$ $BM = AM$
(i) In ΔAMC and ΔBMD, we have
$CM = DM$ [Given]
$\angle AMC = \angle BMD$ [Vertically opposite angles]
$AM = BM$ [Proved above]
$\therefore$ $\Delta AMC \cong \Delta BMD$ [By SAS congruence]
(ii) Since $\Delta AMC \cong \Delta BMD$
$\Rightarrow$ $\angle MAC = \angle MBD$ [By C.P.C.T.]
But they form a pair of alternate interior angles.
$\therefore$ $AC \parallel DB$

Now, BC is a transversal which intersects parallel lines AC and DB.

$\therefore \quad \angle BCA + \angle DBC = 180°$ [Co-interior angles]

But $\angle BCA = 90°$ [$\because \Delta ABC$ is right angled at C]

$\therefore \quad 90° + \angle DBC = 180°$

$\Rightarrow \quad \angle DBC = 90°$

(iii) Since, $\Delta AMC \cong \Delta BMD$

$\therefore \quad AC = BD$ [By C.P.C.T.]

Now, in ΔDBC and ΔACB, we have

$BD = CA$ [Proved above]

$\angle DBC = \angle ACB$ [Each equals 90°]

$BC = CB$ [Common]

$\therefore \quad \Delta DBC \cong \Delta ACB$ [By SAS congruence]

(iv) As $\Delta DBC \cong \Delta ACB$

$\Rightarrow \quad DC = AB$ [By C.P.C.T.]

But $DM = CM$ [Given]

$\therefore \quad CM = \frac{1}{2}DC = \frac{1}{2}AB$

EXERCISE - 7.2

1. (i) Let BD and CE are the bisectors of $\angle B$ and $\angle C$ respectively.

In ΔABC, we have $AC = AB$

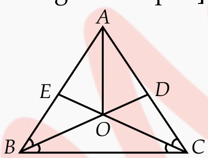
$\therefore \quad \angle ABC = \angle ACB$

[Angles opposite to equal sides of a triangle are equal]

$\Rightarrow \quad \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

$\Rightarrow \quad \angle OBC = \angle OCB$

$\Rightarrow \quad OC = OB$



[Sides opposite to equal angles of a triangle are equal]

(ii) In ΔABO and ΔACO, we have

$AB = AC$ [Given]

$\angle OBA = \angle OCA$ $\left[\because \frac{1}{2}\angle B = \frac{1}{2}\angle C\right]$

$OB = OC$ [Proved above]

$\therefore \quad \Delta ABO \cong \Delta ACO$ [By SAS congruence]

$\Rightarrow \quad \angle OAB = \angle OAC$ [By C.P.C.T.]

$\Rightarrow \quad AO$ bisects $\angle A$.

2. Since AD is bisector of BC.

$\therefore \quad BD = CD$

Now, in ΔABD and ΔACD, we have

$AD = AD$ [Common]

$\angle ADB = \angle ADC$ [Each equals 90°]

$BD = CD$ [Proved above]

$\therefore \quad \Delta ABD \cong \Delta ACD$ [By SAS congruence]

$\Rightarrow \quad AB = AC$ [By C.P.C.T.]

Thus, ABC is an isosceles triangle.

3. Given, ΔABC is an isosceles triangle with $AB = AC$

$\Rightarrow \quad \angle ACB = \angle ABC$

[Angles opposite to equal sides of a triangle are equal]

$\Rightarrow \quad \angle BCE = \angle CBF$

Now, in ΔBEC and ΔCFB, we have

$\angle BCE = \angle CBF$ [Proved above]

$\angle BEC = \angle CFB$ [Each equals 90°]

$BC = CB$ [Common]

$\therefore \quad \Delta BEC \cong \Delta CFB$ [By AAS congruence]

So, $BE = CF$ [By C.P.C.T.]

4. (i) In ΔABE and ΔACF, we have

$\angle AEB = \angle AFC$ [Each 90° as $BE \perp AC$ and $CF \perp AB$]

$\angle A = \angle A$ [Common]

$BE = CF$ [Given]

$\therefore \quad \Delta ABE \cong \Delta ACF$ [By AAS congruence]

(ii) Since, $\Delta ABE \cong \Delta ACF$

$\therefore \quad AB = AC$ [By C.P.C.T.]

5. In ΔABC, we have

$AB = AC$ [$\because ABC$ is an isosceles triangle]

$\therefore \quad \angle ABC = \angle ACB$ ...(i)

[Angles opposite to equal sides of a triangle are equal]

Again, in ΔBDC, we have

$BD = CD$ [$\because BDC$ is an isosceles triangle]

$\therefore \quad \angle CBD = \angle BCD$ ...(ii)

[Angles opposite to equal sides of a triangle are equal]

Adding (i) and (ii), we have

$\angle ABC + \angle CBD = \angle ACB + \angle BCD$

$\Rightarrow \quad \angle ABD = \angle ACD$

6. Given, $AB = AC$ and $AB = AD$

$\therefore \quad AC = AD$

Now, in ΔABC, we have

$\angle B + \angle ACB + \angle BAC = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad 2\angle ACB + \angle BAC = 180°$ ...(i)

[$\because \angle B = \angle ACB$ (Angles opposite to equal sides of a triangle are equal)]

Similarly, in ΔACD,

$\angle D + \angle ACD + \angle CAD = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad 2\angle ACD + \angle CAD = 180°$ ...(ii)

[$\because \angle D = \angle ACD$ (Angles opposite to equal sides of a triangle are equal)]

Adding (i) and (ii), we have

$2\angle ACB + \angle BAC + 2\angle ACD + \angle CAD = 180° + 180°$

$\Rightarrow \quad 2[\angle ACB + \angle ACD] + [\angle BAC + \angle CAD] = 360°$

$\Rightarrow \quad 2\angle BCD + 180° = 360°$

[$\because \angle BAC$ and $\angle CAD$ form a linear pair]

$\Rightarrow \quad 2\angle BCD = 360° - 180° = 180°$

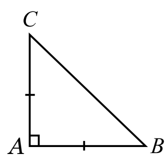
$\Rightarrow \quad \angle BCD = \frac{180°}{2} = 90°$

7. In ΔABC, we have

$AB = AC$ [Given]

$\therefore \quad \angle ACB = \angle ABC$

[Angles opposite to equal sides of a triangle are equal]



Now, $\angle A + \angle B + \angle C = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad 90° + \angle B + \angle C = 180°$ [$\angle A = 90°$(Given)]

$\Rightarrow \quad \angle B + \angle C = 180° - 90° = 90°$

But $\angle ABC = \angle ACB$, *i.e.*, $\angle B = \angle C$

$\therefore \quad \angle B = \angle C = \frac{90°}{2} = 45°$

Thus, $\angle B = 45°$ and $\angle C = 45°$

8. In ΔABC, we have

$AB = BC = CA$ [$\because ABC$ is an equilateral triangle]

Now, $AB = BC \Rightarrow \angle A = \angle C$ ...(i)

[Angles opposite to equal sides of a triangle are equal]

Similarly, $AC = BC \Rightarrow \angle A = \angle B$ ...(ii)

From (i) and (ii), we have $\angle A = \angle B = \angle C$

Let $\angle A = \angle B = \angle C = x$

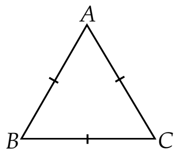
Since, $\angle A + \angle B + \angle C = 180°$

[Angle sum property of a triangle]

$\therefore$ $x + x + x = 180° \Rightarrow 3x = 180°$

$\Rightarrow$ $x = 60°$

$\therefore$ $\angle A = \angle B = \angle C = 60°$



Thus, the angles of an equilateral triangle are 60° each.

EXERCISE - 7.3

1. (i) In ΔABD and ΔACD, we have

$AB = AC$ [Given]

$AD = AD$ [Common]

$BD = CD$ [Given]

$\therefore$ $\Delta ABD \cong \Delta ACD$ [By SSS congruence]

$\Rightarrow$ $\angle BAD = \angle CAD$ [By C.P.C.T.]

$\Rightarrow$ $\angle BAP = \angle CAP$ ...(1)

(ii) In ΔABP and ΔACP, we have

$AB = AC$ [Given]

$\angle BAP = \angle CAP$ [From (1)]

$AP = AP$ [Common]

$\therefore$ $\Delta ABP \cong \Delta ACP$ [By SAS congruence]

(iii) Since, $\angle BAP = \angle CAP$ [From (1)]

$\therefore$ AP is the bisector of $\angle A$.

Again, in ΔBDP and ΔCDP, we have

$BD = CD$ [Given]

$DP = DP$ [Common]

$BP = CP$ [$\because \Delta ABP \cong \Delta ACP$]

$\therefore$ $\Delta BDP \cong \Delta CDP$ [By SSS congruence]

$\Rightarrow$ $\angle BDP = \angle CDP$ [By C.P.C.T.]

$\Rightarrow$ DP (or AP) is the bisector of $\angle BDC$

$\therefore$ AP is the bisector of $\angle A$ as well as $\angle D$.

(iv) As, $\Delta ABP \cong \Delta ACP$

$\Rightarrow$ $\angle APB = \angle APC$ and $BP = CP$ [By C.P.C.T.]

But $\angle APB + \angle APC = 180°$ [Linear pair]

$\therefore$ $\angle APB = \angle APC = 90°$

$\Rightarrow$ $AP \perp BC$.

Hence, AP is the perpendicular bisector of BC.

2. (i) In right ΔABD and ΔACD,

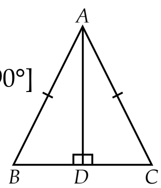
$AB = AC$ [Given]

$\angle ADB = \angle ADC$ [Each equals 90°]

$AD = AD$ [Common]

$\therefore$ $\Delta ABD \cong \Delta ACD$ [By RHS congruence]

So, $BD = CD$ [By C.P.C.T.]



$\Rightarrow$ D is the mid-point of BC or AD bisects BC.

(ii) Since, $\Delta ABD \cong \Delta ACD$,

$\Rightarrow$ $\angle BAD = \angle CAD$ [By C.P.C.T.]

So, AD bisects $\angle A$.

3. In ΔABC, AM is the median [Given]

$\therefore$ $BM = \frac{1}{2}BC$ ...(1)

In ΔPQR, PN is the median [Given]

$\therefore$ $QN = \frac{1}{2}QR$ ...(2)

Also, $BC = QR \Rightarrow \frac{1}{2}BC = \frac{1}{2}QR$

$\Rightarrow$ $BM = QN$ [From (1) and (2)] ...(3)

(i) In ΔABM and ΔPQN, we have

$AB = PQ$ [Given]

$AM = PN$ [Given]

$BM = QN$ [From (3)]

$\therefore$ $\Delta ABM \cong \Delta PQN$ [By SSS congruence]

(ii) Since $\Delta ABM \cong \Delta PQN$

$\Rightarrow$ $\angle B = \angle Q$ [By C.P.C.T.] ...(4)

Now, in ΔABC and ΔPQR, we have

$\angle B = \angle Q$ [From (4)]

$AB = PQ$ [Given]

$BC = QR$ [Given]

$\therefore$ $\Delta ABC \cong \Delta PQR$ [By SAS congruence]

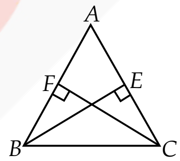
4. Since, $BE \perp AC$ [Given]

$\therefore$ ΔBEC is a right triangle such that

$\angle BEC = 90°$

Similarly, $\angle CFB = 90°$

Now, in right ΔBEC and right ΔCFB, we have



$BE = CF$ [Given]

$BC = CB$ [Common]

$\angle BEC = \angle CFB = 90°$

$\therefore$ $\Delta BEC \cong \Delta CFB$ [By RHS congruence]

So, $\angle BCE = \angle CBF$ [By C.P.C.T.]

or $\angle BCA = \angle CBA$

Now, in ΔABC, $\angle BCA = \angle CBA \Rightarrow AB = AC$

[Sides opposite to equal angles of a triangle are equal]

$\therefore$ ABC is an isosceles triangle.

5. We have, $AP \perp BC$

$\therefore$ $\angle APB = 90°$ and $\angle APC = 90°$

In ΔABP and ΔACP, we have

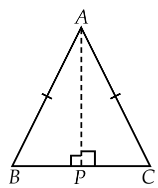
$\angle APB = \angle APC$ [Each equals 90°]

$AB = AC$ [Given]

$AP = AP$ [Common]

$\therefore$ $\Delta ABP \cong \Delta ACP$ [By RHS congruence]

So, $\angle B = \angle C$ [By C.P.C.T.]



EXERCISE - 7.4

1. Let us consider ΔABC such that $\angle B = 90°$.

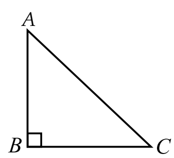
$\therefore$ $\angle A + \angle B + \angle C = 180°$

$\Rightarrow$ $[\angle A + \angle C] + \angle B = 180°$

$\Rightarrow$ $\angle A + \angle C = 90°$

$\Rightarrow$ $\angle A + \angle C = \angle B$

$\therefore$ $\angle B > \angle A$ and $\angle B > \angle C$



$\Rightarrow$ Side opposite to $\angle B$ is longer than the side opposite to $\angle A$. *i.e.*, $AC > BC$. Similarly, $AC > AB$.

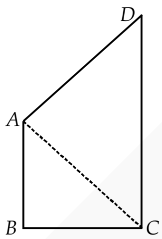
Therefore, we get AC is the longest side. But AC is the hypotenuse of the triangle.

Thus, hypotenuse is the longest side.

2. $\angle ABC + \angle PBC = 180°$ [Linear pair]
and $\angle ACB + \angle QCB = 180°$ [Linear pair]
But $\angle PBC < \angle QCB$ [Given]
$\Rightarrow$ $180° - \angle PBC > 180° - \angle QCB$
$\Rightarrow$ $\angle ABC > \angle ACB$
$\Rightarrow$ $AC > AB$ [Side opposite to greater angle is longer]

3. Since, $\angle B < \angle A$ [Given]
$\Rightarrow$ $\angle A > \angle B$
$\therefore$ $OB > OA$ ...(i)
Similarly, $OC > OD$ ...(ii)
Adding (i) and (ii), we have
$OB + OC > OA + OD$
$\Rightarrow$ $BC > AD$ $\Rightarrow$ $AD < BC$

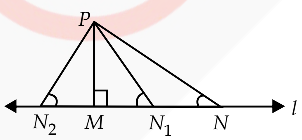
4. Let us join AC.



Now, in ΔABC, $BC > AB$
[$\because$ AB is the smallest side of the quadrilateral $ABCD$]
$\Rightarrow$ $\angle BAC > \angle BCA$ ...(i)
[Angle opposite to longer side is larger]
Again, in ΔACD, $CD > AD$
[$\because$ CD is the longest side of the quadrilateral $ABCD$]
$\Rightarrow$ $\angle CAD > \angle ACD$ ...(ii)
[Angle opposite to longer side is larger]
Adding (i) and (ii), we get
$\angle BAC + \angle CAD > \angle BCA + \angle ACD$
$\Rightarrow$ $\angle A > \angle C$
Similarly, by joining BD, we get $\angle B > \angle D$.

5. In ΔPQR, PS bisects $\angle QPR$ [Given]
$\therefore$ $\angle QPS = \angle RPS$ and $PR > PQ$ [Given]
$\Rightarrow$ $\angle PQS > \angle PRS$
[Angle opposite to longer side is larger]
$\Rightarrow$ $\angle PQS + \angle QPS > \angle PRS + \angle RPS$ ...(i)
[$\angle QPS = \angle RPS$]
$\because$ Exterior $\angle PSR = [\angle PQS + \angle QPS]$ and
Exterior $\angle PSQ = [\angle PRS + \angle RPS]$
[An exterior angle is equal to the sum of interior opposite angles]
Now, from (i), we have $\angle PSR > \angle PSQ$.

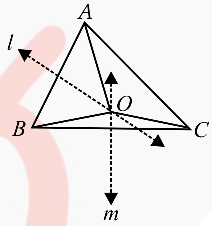
6. Let us consider the ΔPMN such that $\angle M = 90°$.
Since, $\angle PMN + \angle N + \angle MPN = 180°$ [Sum of angles of a triangle]
and $\angle M = 90°$ [$\because$ $PM \perp l$]



So, $\angle N + \angle MPN = \angle PMN \Rightarrow \angle N < \angle PMN$
$\Rightarrow$ $PM < PN$ ...(i)
Similarly, $PM < PN_1$ ...(ii)
and $PM < PN_2$ ...(iii)
From (i), (ii) and (iii), we have PM is the smallest line segment drawn from P on the line l. Thus, the perpendicular line segment is the shortest line segment drawn on a line from a point not on it.

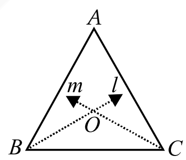
EXERCISE - 7.5

1. Let us consider a ΔABC.
Draw l, the perpendicular bisector of AB.
Draw m, the perpendicular bisector of BC.
Let the two perpendicular bisectors l and m meet at O.
O is the required point which is equidistant from all the vertices A, B and C.

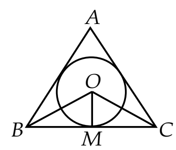


Note: If we draw a circle with centre O and radius OA, OB or OC, then it will pass through A, B and C. The point O is called circumcentre of the triangle.

2. Let us consider a ΔABC.
Draw l, the bisector of $\angle B$.
Draw m, the bisector of $\angle C$.
Let the two bisectors l and m meet at O.
Thus, O is the required point which is equidistant from the sides of ΔABC.

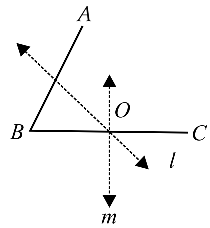


Note: If we draw $OM \perp BC$ and draw a circle with O as centre and OM as radius, then the circle will touch the sides of the triangle.



Point O is called the incentre of the triangle.

3. Let us join A and B, and draw l, the perpendicular bisector of AB.
Now, join B and C, and draw m, the perpendicular bisector of BC.
Let the perpendicular bisectors l and m meet at O. The point O is the required point where the ice cream parlour be set up.



Note: If we join A and C and draw the perpendicular bisectors, then it will also meet (or pass through) the point O.

4. It is an activity. We require 150 equilateral triangles of side 1 cm in the figure (i) and 300 equilateral triangles in the figure (ii).
$\therefore$ The figure (ii) has more triangles.