Areas of Parallelograms and Triangles

CHAPTER 9

NCERT FOCUS

SOLUTIONS

EXERCISE - 9.1

1. The figures (i), (iii) and (v) lie on the same base and between the same parallels.

	Common base	Two parallels
Fig. (i)	*DC*	*DC* and *AB*
Fig. (iii)	*QR*	*QR* and *PS*
Fig. (v)	*AD*	*AD* and *BQ*

EXERCISE - 9.2

1. We have, $AB = 16$ cm

$\because$ $AB = CD$ [Opposite sides of parallelogram]

$\therefore$ $CD = 16$ cm

Now, area of parallelogram $ABCD = CD \times AE$

$= (16 \times 8)\text{ cm}^2 = 128\text{ cm}^2$

Since, $CF \perp AD$

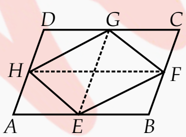
$\therefore$ Area of parallelogram $ABCD = AD \times CF$

$\Rightarrow$ $AD \times CF = 128 \Rightarrow AD \times 10 = 128$ [$\because CF = 10$ cm]

$\Rightarrow$ $AD = \frac{128}{10} = 12.8$

Thus, the required length of *AD* is 12.8 cm.

2. Join *GE* and *HF*, then $GE \parallel BC \parallel DA$ and $HF \parallel AB \parallel DC$.



$\because$ $AB = DC \Rightarrow AE = DG$

and $AB \parallel CD$

$\Rightarrow$ $AE \parallel DG$.

Thus, *AEGD* is a parallelogram.

Similarly, *ABFH*, *DCFH* and *EBCG* are parallelograms.

We know that, if a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

Now, ΔEFG and parallelogram *EBCG* are on the same base *EG* and between the same parallels *EG* and *BC*.

$\therefore$ $ar(\Delta EFG) = \frac{1}{2} ar(\|^{gm}\ EBCG)$ ... (i)

Similarly, $ar(\Delta EHG) = \frac{1}{2} ar(\|^{gm}\ AEGD)$ ... (ii)

Adding (i) and (ii), we get

$ar(\Delta EFG) + ar(\Delta EHG) = \frac{1}{2}[ar(\|^{gm} EBCG) + ar(\|^{gm}\ AEGD)]$

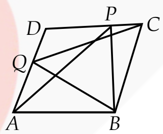
$\Rightarrow$ $ar(EFGH) = \frac{1}{2}\ ar(\|^{gm} ABCD)$

Hence proved.

3. Since, ΔAPB and parallelogram *ABCD* are on the same base *AB* and between the same parallels *AB* and *CD*.

$\therefore$ $ar(\Delta APB) = \frac{1}{2} ar(\|^{gm}\ ABCD)$ ...(i)

Also, ΔBQC and parallelogram *ABCD* are on the same base *BC* and between the same parallels *BC* and *AD*.



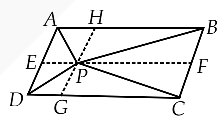
$\therefore$ $ar(\Delta BQC) = \frac{1}{2} ar(\|^{gm}\ ABCD)$ ...(ii)

From (i) and (ii), we have

$ar(\Delta APB) = ar(\Delta BQC)$.

Hence proved.

4. We have a parallelogram *ABCD*, *i.e.*, $AB \parallel CD$ and $BC \parallel AD$. Let us draw $EF \parallel AB$ and $HG \parallel AD$ through *P*.



Clearly, *AEFB*, *CDEF*, *ADGH* and *BCGH*, all are parallelograms.

(i) Since, ΔAPB and parallelogram *AEFB* are on the same base *AB* and between the same parallels *AB* and *EF*.

$\therefore$ $ar(\Delta APB) = \frac{1}{2} ar(\|^{gm}\ AEFB)$ ...(1)

Also, ΔPCD and parallelogram *CDEF* are on the same base *CD* and between the same parallels *CD* and *EF*.

$\therefore$ $ar(\Delta PCD) = \frac{1}{2} ar(\|^{gm}\ CDEF)$ ...(2)

Adding (1) and (2), we get

$ar(\Delta APB) + ar(\Delta PCD) = \frac{1}{2} ar(\|^{gm}\ AEFB) + \frac{1}{2} ar(\|^{gm}\ CDEF)$

$\Rightarrow$ $ar(\Delta APB) + ar(\Delta PCD) = \frac{1}{2} ar(\|^{gm}\ ABCD)$ ...(3)

(ii) ΔAPD and parallelogram *ADGH* are on the same base *AD* and between the same parallels *AD* and *GH*.

$\therefore$ $ar(\Delta APD) = \frac{1}{2} ar(\|^{gm}\ ADGH)$ ...(4)

Similarly, $ar(\Delta PBC) = \frac{1}{2} ar(\|^{gm}\ BCGH)$ ...(5)

Adding (4) and (5), we get

$ar(\Delta APD) + ar(\Delta PBC) =$

$\frac{1}{2} ar(\|^{gm}\ ADGH) + \frac{1}{2} ar(\|^{gm}\ BCGH)$

$\Rightarrow$ $ar(\Delta APD) + ar(\Delta PBC) = \frac{1}{2} ar(\|^{gm}\ ABCD)$ ...(6)

From (3) and (6), we get

$ar(\Delta APD) + ar(\Delta PBC) = ar(\Delta APB) + ar(\Delta PCD)$

5. (i) Parallelogram $PQRS$ and parallelogram $ABRS$ are on the same base RS and between the same parallels RS and PB.

$\therefore \quad ar(\|^{gm} PQRS) = ar(\|^{gm} ABRS)$

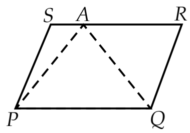
(ii) ΔAXS and parallelogram $ABRS$ are on the same base AS and between the same parallels AS and BR.

$\therefore \quad ar(\Delta AXS) = \frac{1}{2}\, ar(\|^{gm} ABRS)$ ...(1)

But $ar(\|^{gm} PQRS) = ar(\|^{gm} ABRS)$ [Proved in (i) part] ...(2)

$\therefore$ From (1) and (2), we get $ar(\Delta AXS) = \frac{1}{2} ar(PQRS)$

6. Given, the farmer is having a field in the form of parallelogram $PQRS$ and a point A is situated on RS. Join AP and AQ.



Clearly, the field is divided into three parts *i.e.*, ΔAPS, ΔPAQ and ΔQAR.

Now, as ΔPAQ and parallelogram $PQRS$ are on the same base PQ and between the same parallels PQ and RS.

$\therefore \quad ar(\Delta PAQ) = \frac{1}{2}\, ar(\|^{gm} PQRS)$ ...(i)

Now, $ar(\|^{gm} PQRS) - ar(\Delta PAQ)$

$= ar(\|^{gm} PQRS) - \frac{1}{2}\, ar(\|^{gm} PQRS)$ [From (i)]

$\Rightarrow \quad ar(\Delta APS) + ar(\Delta QAR) = \frac{1}{2}\, ar\,(\|^{gm} PQRS)$ ...(ii)

From (i) and (ii), we have

$ar(\Delta PAQ) = ar[(\Delta APS) + (\Delta QAR)]$

Thus, the farmer can sow wheat in ΔPAQ and pulses in ΔAPS and ΔQAR or wheat in ΔAPS and ΔQAR and pulses in ΔPAQ.

EXERCISE - 9.3

1. In ΔABC, AD is a median.

$\therefore \quad ar(\Delta ABD) = ar(\Delta ADC)$ ...(i)

[$\because$ A median divides the triangle into two triangles of equal area]

Similarly, in ΔBEC, we have

$ar(\Delta BED) = ar(\Delta DEC)$ ...(ii)

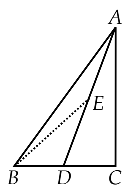
Subtracting (ii) from (i), we get

$ar(\Delta ABD) - ar(\Delta BED) = ar(\Delta ADC) - ar(\Delta DEC)$

$\Rightarrow \quad ar(\Delta ABE) = ar(\Delta ACE)$.

2. In a ΔABC, AD is a median.

Since, a median divides the triangle into two triangles of equal area.



$\therefore \quad ar(\Delta ABD) = \frac{1}{2} ar(\Delta ABC)$ ...(i)

Join B and E.

Now, in ΔABD, BE is a median.

[$\because$ E is the mid-point of AD]

$\therefore \quad ar(\Delta BED) = \frac{1}{2} ar(\Delta ABD)$ ...(ii)

From (i) and (ii), we get

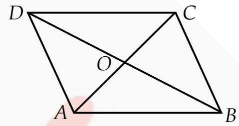
$$ar(\Delta BED) = \frac{1}{2}\left[\frac{1}{2} ar(\Delta ABC)\right]$$

$\Rightarrow \quad ar(\Delta BED) = \frac{1}{4} ar(\Delta ABC)$

3. Let $ABCD$ be a parallelogram such that its diagonals intersect at O.

$\because$ Diagonals of a parallelogram bisect each other.

$\therefore \quad AO = OC$ and $BO = OD$



Since, a median of a triangle divides it into two triangles of equal area.

$\therefore \quad ar(\Delta BOC) = ar(\Delta DOC)$ ...(i)

Similarly, $ar(\Delta AOD) = ar(\Delta DOC)$ ...(ii)

and $ar(\Delta AOB) = ar(\Delta BOC)$ ...(iii)

From (i), (ii) and (iii), we get

$ar(\Delta AOB) = ar(\Delta BOC) = ar(\Delta COD) = ar(\Delta AOD)$

Thus, the diagonals of a parallelogram divide it into four triangles of equal area.

4. We have, ΔABC and ΔABD are on the same base AB.

$\because \quad CD$ is bisected at O. [Given]

$\therefore \quad CO = DO$

Since, a median of a triangle divides it into two triangles of equal area.

$\therefore \quad ar(\Delta OAC) = ar(\Delta OAD)$ ...(i)

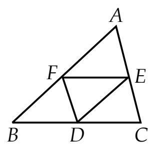
Similarly, $ar(\Delta OBC) = ar(\Delta OBD)$ ...(ii)

Adding (i) and (ii), we get

$ar(\Delta OAC) + ar(\Delta OBC) = ar(\Delta OAD) + ar(\Delta OBD)$

$\Rightarrow \quad ar(\Delta ABC) = ar(\Delta ABD)$

5. We have, ΔABC such that D, E and F are the mid-points of BC, CA and AB respectively.



(i) In ΔABC, E and F are the mid-points of AC and AB respectively.

$\therefore \quad EF \parallel BC$ and $EF = \frac{1}{2} BC$ [By mid-point theorem]

$\Rightarrow \quad EF \parallel BD$ and $EF = BD$ [$\because$ D is the mid-point of BC]

Now, $BDEF$ is a quadrilateral whose one pair of opposite sides is parallel and of equal lengths.

$\therefore \quad BDEF$ is a parallelogram.

(ii) We have proved that $BDEF$ is a parallelogram.

Similarly, $DCEF$ is a parallelogram and $DEAF$ is a parallelogram.

Now, as diagonal of a parallelogram divides it into two triangles of equal area

$\therefore \quad ar(\Delta BDF) = ar(\Delta DEF)$ ...(1)

[$\because$ FD is a diagonal of parallelogram $BDEF$]

Similarly, $ar(\Delta CDE) = ar(\Delta DEF)$ ...(2)

and $ar(\Delta AEF) = ar(\Delta DEF)$ ...(3)

From (1), (2) and (3), we have

$ar(\Delta AEF) = ar(\Delta FBD) = ar(\Delta CDE) = ar(\Delta DEF)$

Thus, $ar(\Delta ABC) = ar(\Delta AEF) + ar(\Delta FBD)$

$+ ar(\Delta DEF) + ar(\Delta CDE) = 4\, ar(\Delta DEF)$

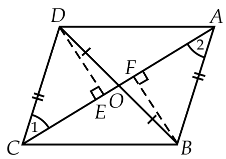
$\Rightarrow \quad ar(\Delta DEF) = \frac{1}{4} ar(\Delta ABC)$

(iii) We have, $ar(\|^{gm}\ BDEF) = ar(\Delta BDF) + ar(\Delta DEF)$
$= ar(\Delta DEF) + ar(\Delta DEF)$ $[\because\ ar(\Delta DEF) = ar(\Delta BDF)]$
$= 2ar(\Delta DEF)$

$$= 2\left[\frac{1}{4}ar(\Delta ABC)\right] = \frac{1}{2}ar(\Delta ABC)$$

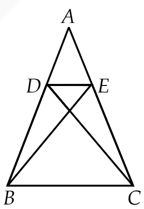
Thus, $ar(\|^{gm}\ BDEF) = \frac{1}{2}ar(\Delta ABC)$.

6. We have a quadrilateral $ABCD$ whose diagonals AC and BD intersect at O such that $OB = OD$.
Also, we have $AB = CD$
Let us draw $DE \perp AC$ and $BF \perp AC$.

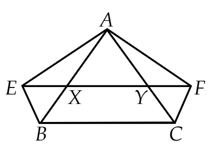


(i) In ΔDEO and ΔBFO, we have
$DO = BO$ [Given]
$\angle DOE = \angle BOF$ [Vertically opposite angles]
$\angle DEO = \angle BFO$ [Each 90°]
$\therefore$ $\Delta DEO \cong \Delta BFO$ [By AAS congruency criterion]
$\Rightarrow$ $DE = BF$ [By CPCT]
and $ar(\Delta DEO) = ar(\Delta BFO)$ ...(1)
Now, in ΔDEC and ΔBFA, we have
$\angle DEC = \angle BFA$ [Each 90°]
$DE = BF$ [Proved above]
$DC = BA$ [Given]
$\therefore$ $\Delta DEC \cong \Delta BFA$ [By RHS congruency criterion]
$\Rightarrow$ $ar(\Delta DEC) = ar(\Delta BFA)$ ...(2)
and $\angle 1 = \angle 2$ [By CPCT] ...(3)
$\therefore$ Adding (1) and (2), we get
$ar(\Delta DEO) + ar(\Delta DEC) = ar(\Delta BFO) + ar(\Delta BFA)$
$\Rightarrow$ $ar(\Delta DOC) = ar(\Delta AOB)$
(ii) Since, $ar(\Delta DOC) = ar(\Delta AOB)$ [Proved above]
Adding $ar(\Delta BOC)$ on both sides, we get
$ar(\Delta DOC) + ar(\Delta BOC) = ar(\Delta AOB) + ar(\Delta BOC)$
$\Rightarrow$ $ar(\Delta DCB) = ar(\Delta ACB)$
(iii) Since, ΔDCB and ΔACB are on the same base CB and having equal areas.
$\therefore$ They must lie between the same parallels.
$\Rightarrow$ $CB \parallel DA$
Also, $\angle 1 = \angle 2$, [From (3)]
which form a pair of alternate interior angles.
So, $AB \parallel CD$
Hence, $ABCD$ is a parallelogram.

7. We have, a ΔABC and points D and E on AB and AC respectively are such that $ar(\Delta DBC) = ar(\Delta EBC)$.
Since, ΔDBC and ΔEBC are on the same base BC and having same area.
$\therefore$ They must lie between the same parallels.
Hence, $DE \parallel BC$
Hence proved.



8. We have, a ΔABC such that $XY \parallel BC$, $BE \parallel AC$ and $CF \parallel AB$.
Since, $XY \parallel BC$ and $BE \parallel CY$.
$\therefore$ $BCYE$ is a parallelogram.
Now, the parallelogram $BCYE$ and ΔABE are on the same base BE and between the same parallels BE and AC.



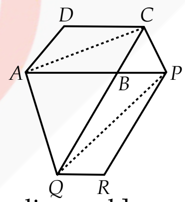
$$\therefore\quad ar(\Delta ABE) = \frac{1}{2}ar(\|^{gm}\ BCYE) \quad ...(i)$$

Again, $XY \parallel BC$ and $CF \parallel AB$ [Given]
$\Rightarrow$ $XF \parallel BC$ and $CF \parallel XB$
$\therefore$ $BCFX$ is a parallelogram.
Now, ΔACF and parallelogram $BCFX$ are on the same base CF and between the same parallels AB and FC.

$$\therefore\quad ar(\Delta ACF) = \frac{1}{2}ar(\|^{gm}\ BCFX) \quad ...(ii)$$

Also parallelogram $BCFX$ and parallelogram $BCYE$ are on the same base BC and between the same parallels BC and EF.
$\therefore$ $ar(\|^{gm}\ BCFX) = ar(\|^{gm}\ BCYE)$ ...(iii)
From (i), (ii) and (iii), we get
$ar(\Delta ABE) = ar(\Delta ACF)$

9. Join AC and PQ.
We know that diagonal of a parallelogram divides it into two triangles of equal area.



$$\therefore\quad ar(\Delta ABC) = \frac{1}{2}ar(\|^{gm}\ ABCD) \quad ...(i)$$

$[\because ABCD$ is a parallelogram and AC is its diagonal.]
Again, $PBQR$ is a parallelogram and QP is its diagonal.

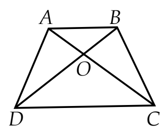
$$\therefore\quad ar(\Delta BPQ) = \frac{1}{2}ar(\|^{gm}\ PBQR) \quad ...(ii)$$

Since, ΔACQ and ΔAPQ are on the same base AQ and between the same parallels AQ and CP.
$\therefore$ $ar(\Delta ACQ) = ar(\Delta APQ)$
$\Rightarrow$ $ar(\Delta ACQ) - ar(\Delta ABQ) = ar(\Delta APQ) - ar(\Delta ABQ)$
[Subtracting $ar(\Delta ABQ)$ from both sides]
$\Rightarrow$ $ar(\Delta ABC) = ar(\Delta BPQ)$ ...(iii)
From (i), (ii) and (iii), we get

$$\frac{1}{2}ar(\|^{gm}\ ABCD) = \frac{1}{2}ar(\|^{gm}\ PBQR)$$

$\Rightarrow$ $ar(\|^{gm}\ ABCD) = ar(\|^{gm}\ PBQR)$

10. We have, a trapezium $ABCD$ with $AB \parallel CD$ and its diagonals AC and BD intersect at O.
Since, triangles on the same base and between the same parallels have equal areas and here ΔABD and ΔABC are on the same base AB and between the same parallels AB and DC.

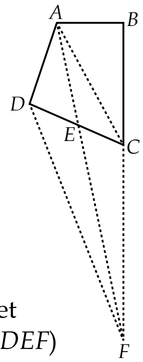


$\therefore$ $ar(\Delta ABD) = ar(\Delta ABC)$
Subtracting $ar(\Delta AOB)$ from both sides, we get
$ar(\Delta ABD) - ar(\Delta AOB) = ar(\Delta ABC) - ar(\Delta AOB)$
$\Rightarrow$ $ar(\Delta AOD) = ar(\Delta BOC)$

11. We have, a pentagon $ABCDE$ and a line BF such that $BF \parallel AC$ and intersect DC produced at F.
(i) Since, the triangles between the same parallels and on the same base are equal in area and here ΔACB and ΔACF are on the same base AC and between the same parallels AC and BF.
$\therefore$ $ar(\Delta ACB) = ar(\Delta ACF)$

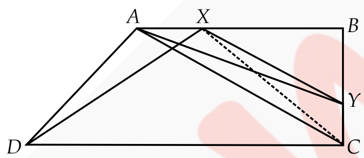
(ii) Since, $ar(\Delta ACB) = ar(\Delta ACF)$ [Proved above]
Adding ar(quad. $AEDC$) on both sides, we get
$ar(\Delta ACB) + ar(\text{quad. } AEDC) = ar(\Delta ACF) + ar(\text{quad. } AEDC)$
$\therefore$ $ar(ABCDE) = ar(AEDF)$

12. Let the plot be in the form of a quadrilateral $ABCD$.



Join AC.
Draw $DF \parallel AC$ such that it meets BC produced at F and join A and F.
Now, ΔDAF and ΔDCF are on the same base DF and between the same parallels AC and DF.
$\therefore$ $ar(\Delta DAF) = ar(\Delta DCF)$
Subtracting $ar(\Delta DEF)$ from both sides, we get
$ar(\Delta DAF) - ar(\Delta DEF) = ar(\Delta DCF) - ar(\Delta DEF)$
$\Rightarrow$ $ar(\Delta ADE) = ar(\Delta CEF)$
Thus, the portion of ΔADE can be taken over by the Gram Panchyat by adding the land (ΔCEF) to his (Itwaari) land so as to form a triangular plot *i.e.*, ΔABF.
Note that $ar(\Delta ABF) = ar$(quad. $ABCD$).
[$\because$ $ar(\Delta CEF) = ar(\Delta ADE)$ [Proved above]
$\therefore$ Adding ar(quad. $ABCE$) on both sides, we get
$ar(\Delta CEF) + ar(\text{quad. } ABCE)$
$= ar(\Delta ADE) + ar(\text{quad. } ABCE)$
$\Rightarrow$ $ar(\Delta ABF) = ar(\text{quad. } ABCD)$]

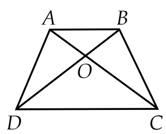
13. We have, a trapezium $ABCD$ with $AB \parallel DC$ and a line XY such that $XY \parallel AC$ meets AB at X and BC at Y.
Join CX.



$\because$ ΔADX and ΔACX are on the same base AX and between the same parallels AB and DC.
$\therefore$ $ar(\Delta ADX) = ar(\Delta ACX)$ ...(i)
$\because$ ΔACX and ΔACY are on the same base AC and between the same parallels AC and XY.
$\therefore$ $ar(\Delta ACX) = ar(\Delta ACY)$ ...(ii)
From (i) and (ii), we have $ar(\Delta ADX) = ar(\Delta ACY)$
Hence proved.

14. We have, $AP \parallel BQ \parallel CR$.
$\because$ ΔBCQ and ΔBQR are on the same base BQ and between the same parallels BQ and CR.
$\therefore$ $ar(\Delta BCQ) = ar(\Delta BQR)$ ...(i)
Similarly, ΔABQ and ΔPBQ are on the same base BQ and between the same parallels AP and BQ.
$\therefore$ $ar(\Delta ABQ) = ar(\Delta PBQ)$ ...(ii)
Adding (i) and (ii), we get
$ar(\Delta BCQ) + ar(\Delta ABQ) = ar(\Delta BQR) + ar(\Delta PBQ)$
$\Rightarrow$ $ar(\Delta AQC) = ar(\Delta PBR)$
Hence proved.

15. We have, a quadrilateral $ABCD$ and its diagonals AC and BD intersect at O such that $ar(\Delta AOD) = ar(\Delta BOC)$

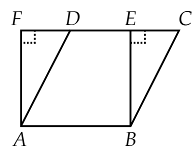


Adding $ar(\Delta AOB)$ on both sides, we get
$ar(\Delta AOD) + ar(\Delta AOB) = ar(\Delta BOC) + ar(\Delta AOB)$
$\Rightarrow$ $ar(\Delta ABD) = ar(\Delta ABC)$
Also, they lie on the same base AB.
$\therefore$ They must lie between the same parallels.
$\therefore$ $AB \parallel DC$
Now, $ABCD$ is a quadrilateral having a pair of opposite sides parallel. So, $ABCD$ is a trapezium.
Hence proved.

16. We have, $ar(\Delta DRC) = ar(\Delta DPC)$ [Given]
Also, they lie on the same base DC.
$\therefore$ ΔDRC and ΔDPC must lie between the same parallels.
So, $DC \parallel RP$. Thus, a pair of opposite sides of quadrilateral $DCPR$ is parallel.
$\therefore$ Quadrilateral $DCPR$ is a trapezium.
Again, we have
$ar(\Delta BDP) = ar(\Delta ARC)$ ...(i)
Also, we have $ar(\Delta DPC) = ar(\Delta DRC)$ ...(ii)
Subtracting (ii) from (i), we get
$ar(\Delta BDP) - ar(\Delta DPC) = ar(\Delta ARC) - ar(\Delta DRC)$
$\Rightarrow$ $ar(\Delta BDC) = ar(\Delta ADC)$
Also, they lie on the same base DC.
$\therefore$ ΔBDC and ΔADC must lie between the same parallels.
So, $AB \parallel DC$ and hence quadrilateral $ABCD$ is a trapezium.

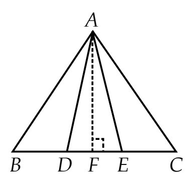
EXERCISE - 9.4

1. We have, a parallelogram $ABCD$ and a rectangle $ABEF$ such that



$ar(\parallel^{gm} ABCD) = ar(\text{rect. } ABEF)$
Now, $AB = CD$
[Opposite sides of parallelogram]
and $AB = EF$ [Opposite sides of a rectangle]
$\Rightarrow$ $CD = EF$
$\Rightarrow$ $AB + CD = AB + EF$ ...(i)
Now, $BE < BC$ and $AF < AD$
[$\because$ In a right triangle, hypotenuse is the longest side]
$\Rightarrow$ $(BC + AD) > (BE + AF)$ ...(ii)
From (i) and (ii), we get
$(AB + CD) + (BC + AD) > (AB + EF) + (BE + AF)$
$\Rightarrow$ $(AB + BC + CD + DA) > (AB + BE + EF + FA)$
$\Rightarrow$ Perimeter of parallelogram $ABCD$ >
Perimeter of rectangle $ABEF$

2. Draw AF perpendicular to BC.
Then, AF will be the height of ΔABD, ΔADE and ΔAEC.



Area of a triangle $= \frac{1}{2} \times$ base $\times$ height

$\therefore$ $ar(\Delta ABD) = \frac{1}{2} \times BD \times AF$

Similarly, $ar(\Delta ADE) = \frac{1}{2} \times DE \times AF$ and

$ar(\Delta AEC) = \frac{1}{2} \times EC \times AF$

Since, $BD = DE = EC$

$$\therefore \quad \left(\frac{1}{2}\times BD\times AF\right)=\left(\frac{1}{2}\times DE\times AF\right)=\left(\frac{1}{2}\times EC\times AF\right)$$

$\Rightarrow \quad ar(\Delta ABD) = ar(\Delta ADE) = ar(\Delta AEC)$.

3. We have, parallelograms $ABCD$, $DCFE$ and $ABFE$.
Since, opposite sides of parallelogram are equal.

$\therefore \quad AD = BC,\ DE = CF$ and $AE = BF$ ...(i)

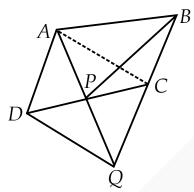
In ΔADE and ΔBCF, we have

$AD = BC,\ DE = CF$ and $AE = BF$ (From (i))

$\therefore \quad \Delta ADE \cong \Delta BCF$ (By SSS congruence criterion)

$\Rightarrow \quad ar(\Delta ADE) = ar(\Delta BCF)$

4. We have, a parallelogram $ABCD$ and $AD = CQ$.
Let us join AC.



We know that triangles on the same base and between the same parallels are equal in area. Since ΔQAC and ΔQDC are on the same base QC and between the same parallels AD and BQ.

$\therefore \quad ar(\Delta QAC) = ar(\Delta QDC)$

Subtracting $ar(\Delta QPC)$from both sides, we get

$ar(\Delta QAC) - ar(\Delta QPC) = ar(\Delta QDC) - ar(\Delta QPC)$

$\Rightarrow \quad ar(\Delta PAC) = ar(\Delta QDP)$ ...(i)

Since, ΔPAC and ΔPBC are on the same base PC and between the same parallels AB and CD

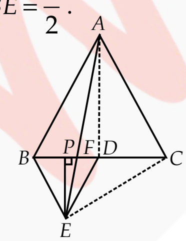
$\therefore \quad ar(\Delta PAC) = ar(\Delta PBC)$ ...(ii)

From (i) and (ii), we get

$ar(\Delta PBC) = ar(\Delta QDP)$

5. Join EC and AD. Draw $EP \perp BC$.

Let $AB = BC = CA = a$, then $BD = DE = BE = \frac{a}{2}$.



(i) Clearly, $ar(\Delta ABC) = \frac{\sqrt{3}}{4}a^2$ and

$$ar(\Delta BDE) = \frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 = \frac{\sqrt{3}}{16}a^2$$

$$\Rightarrow \quad ar(\Delta BDE) = \frac{1}{4}\ ar\ (\Delta ABC)$$

(ii) Since, ΔABC and ΔBED are equilateral triangles.

$\therefore \quad \angle ACB = \angle DBE = 60°$

$\Rightarrow BE \parallel AC$ [$\because$ A pair of alternate interior angles is equal.]

Now, as ΔBAE and ΔBEC are on the same base BE and between the same parallels BE and AC.

$\therefore \quad ar(\Delta BAE) = ar(\Delta BEC)$

$\Rightarrow \quad ar(\Delta BAE) = 2\ ar(\Delta BDE)$

[$\because DE$ is median of ΔBEC $\therefore$ $ar\ (\Delta BEC) = 2\ ar(\Delta BDE)$]

$$\Rightarrow \quad ar(\Delta BDE) = \frac{1}{2}\ ar(\Delta BAE)$$

(iii) Since, $ar(\Delta ABC) = 4\ ar(\Delta BDE)$ [Proved in (i) part]

$ar(\Delta BEC) = 2\ ar(\Delta BDE)$ [$\because DE$ is median of ΔBEC]

$\Rightarrow \quad ar(\Delta ABC) = 2\ ar(\Delta BEC)$

(iv) Since ΔABC and ΔBDE are equilateral triangles.

$\therefore \quad \angle ABC = \angle BDE = 60°$

$\Rightarrow \quad AB \parallel DE$

[$\because$ A pair of alternate interior angles is equal]

Now, as ΔBED and ΔAED are on the same base ED and between same parallels AB and DE.

$\therefore \quad ar(\Delta BED) = ar(\Delta AED)$

Subtracting $ar(\Delta EFD)$ from both sides, we get

$\Rightarrow \quad ar(\Delta BED) - ar(\Delta EFD) = ar(\Delta AED) - ar(\Delta EFD)$

$\Rightarrow \quad ar(\Delta BFE) = ar(\Delta AFD)$

(v) Since, ABC is an equilateral triangle, therefore AD will be perpendicular to BC also.

In right angled ΔABD, we have $AD^2 = AB^2 - BD^2$

$$\Rightarrow \quad AD^2 = a^2 - \frac{a^2}{4} = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4} \Rightarrow AD = \frac{\sqrt{3}a}{2}$$

In right angled ΔPED, $EP^2 = DE^2 - DP^2$

$$\Rightarrow \quad EP^2 = \left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2 = \frac{a^2}{4} - \frac{a^2}{16} = \frac{3a^2}{16} \Rightarrow EP = \frac{\sqrt{3}a}{4}$$

$$\therefore \quad ar(\Delta AFD) = \frac{1}{2}\times FD\times AD = \frac{1}{2}\times FD\times\frac{\sqrt{3}}{2}a \quad ...(1)$$

$$\text{and } ar(\Delta EFD) = \frac{1}{2}\times FD\times EP = \frac{1}{2}\times FD\times\frac{\sqrt{3}}{4}a \quad ...(2)$$

From (1) and (2), we get $ar(\Delta AFD) = 2\ ar(\Delta FED)$

Also, we have $ar(\Delta AFD) = ar(\Delta BFE)$ [From (iv) part]

$\Rightarrow \quad ar(\Delta BFE) = 2\ ar(\Delta FED)$

(vi) $ar(\Delta AFC) = ar(\Delta AFD) + ar(\Delta ADC)$

$= ar(\Delta BFE) + \frac{1}{2}ar(\Delta ABC)$ [From (iv) part and DE is the median of ΔABC]

$= ar(\Delta BFE) + \frac{1}{2} \times 4 \times ar(\Delta BDE)$ [From (i) part]

$= ar(\Delta BFE) + 2ar(\Delta BDE)$

$= 2ar(\Delta FED) + 2[ar(\Delta BFE) + ar(\Delta FED)]$ [From (v) part]

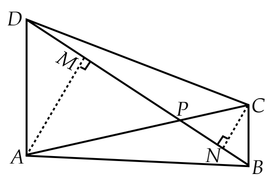
$= 2ar(\Delta FED) + 2[2ar(\Delta FED) + ar(\Delta FED)]$ [From (v) part]

$= 2ar(\Delta FED) + 2[3ar(\Delta FED)]$

$= 2ar(\Delta FED) + 6ar(\Delta FED) = 8ar(\Delta FED)$

$$\therefore \quad \frac{1}{8}ar(\Delta AFC) = ar(\Delta FED)$$

6. We have a quadrilateral $ABCD$ such that its diagonals AC and BD intersect at P.
Draw $AM \perp BD$ and $CN \perp BD$.



Now, $ar(\Delta APB) = \frac{1}{2}\times BP\times AM$

and $ar(\Delta CPD) = \frac{1}{2}\times DP\times CN$

$$\therefore \quad ar(\Delta APB)\times ar(\Delta CPD) = \left(\frac{1}{2}\times BP\times AM\right) \times\left(\frac{1}{2}\times DP\times CN\right)$$

$$= \frac{1}{4}\times BP\times DP\times AM\times CN \quad ...(i)$$

Similarly, $ar(\Delta APD) \times ar(\Delta BPC)$

$$= \left(\frac{1}{2}\times DP\times AM\right)\times\left(\frac{1}{2}\times BP\times CN\right)$$

$$= \frac{1}{4}\times BP\times DP\times AM\times CN \quad ...(ii)$$

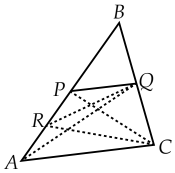
From (i) and (ii), we get

$ar(\Delta APB) \times ar(\Delta CPD) = ar(\Delta APD) \times ar(\Delta BPC)$

7. We have, a ΔABC such that P is the mid-point of AB and Q is the mid-point of BC.

Also, R is the mid-point of AP.

Join AQ and PC.



(i) In ΔAPQ, R is the mid-point of AP. [Given]

$\therefore$ RQ is a median of ΔAPQ.

So, $ar(\Delta PRQ) = \frac{1}{2} ar(\Delta APQ)$ ...(1)

In ΔABQ, P is the mid-point of AB.

$\therefore$ QP is a median of ABQ.

So, $ar(\Delta APQ) = \frac{1}{2} ar(\Delta ABQ)$ ...(2)

From (1) and (2), we get

$ar(\Delta PRQ) = \frac{1}{2} \times \frac{1}{2} ar(\Delta ABQ)$

$= \frac{1}{4} ar(\Delta ABQ) = \frac{1}{4} \times \frac{1}{2} ar(\Delta ABC)$

[$\because$ AQ is a median of ΔABC]

$= \frac{1}{8} ar(\Delta ABC)$ ...(3)

Now, $ar(\Delta ARC) = \frac{1}{2} ar(\Delta APC)$

[$\because$ CR is a median of ΔAPC]

$= \frac{1}{2} \times \frac{1}{2} ar(\Delta ABC)$ [$\because$ CP is a median of ΔABC]

$= \frac{1}{4} (ar \Delta ABC)$ ...(4)

From (3) and (4), we get

$ar(\Delta PRQ) = \frac{1}{8} ar(\Delta ABC) = \frac{1}{2} \times \left(\frac{1}{4} ar\ \Delta ABC\right) = \frac{1}{2} ar(\Delta ARC)$

Thus, $ar(\Delta PRQ) = \frac{1}{2} ar(\Delta ARC)$

(ii) In ΔRBC, RQ is a median.

$\therefore$ $ar(\Delta RQC) = ar(\Delta RBQ) = ar(\Delta PRQ) + ar(\Delta BPQ)$

$= \frac{1}{8} ar(\Delta ABC) + ar(\Delta BPQ)$ [From (3)]

$= \frac{1}{8} ar(\Delta ABC) + \frac{1}{2} ar(\Delta PBC)$

[$\because$ PQ is the median of ΔBPC]

$= \frac{1}{8} ar(\Delta ABC) + \frac{1}{2}\left(\frac{1}{2} ar(\Delta ABC)\right)$

[$\because$ CP is the median of ΔABC]

$= \frac{1}{8} ar(\Delta ABC) + \frac{1}{4} ar(\Delta ABC)$

$= \left(\frac{1}{8} + \frac{1}{4}\right) ar(\Delta ABC) = \frac{3}{8} ar(\Delta ABC)$

(iii) QP is a median of ΔABQ.

$\therefore$ $ar(\Delta PBQ) = \frac{1}{2}(\Delta ABQ) = \frac{1}{2} \times \frac{1}{2} ar(\Delta ABC)$

[$\because$ AQ is the median of ΔABC]

$= \frac{1}{4} ar(\Delta ABC) = ar(\Delta ARC)$ [From (4)]

Thus, $ar(\Delta PBQ) = ar(\Delta ARC)$

8. We have, a right ΔABC such that $BCED$, $ACFG$ and $ABMN$ are squares on its sides BC, CA and AB respectively. Also, line segment $AX \perp DE$ is drawn such that it meets BC at Y.

(i) In ΔABD and ΔMBC, we have

$AB = MB$, $BD = BC$ [Sides of a square]

$\angle CBD = \angle MBA$ [Each 90°]

$\Rightarrow$ $\angle CBD + \angle ABC = \angle MBA + \angle ABC$

[By adding $\angle ABC$ on both sides]

$\Rightarrow$ $\angle ABD = \angle MBC$

$\therefore$ $\Delta ABD \cong \Delta MBC$ [By SAS congruency criterion]

(ii) Since, $AX \perp DE$ therefore $AX \parallel BD$ and $AX \parallel CE$.

Thus, $BYXD$ and $CYXE$ both are parallelograms.

Now, as parallelogram $BYXD$ and ΔABD are on the same base BD and between the same parallels BD and AX.

$\therefore$ $ar(\Delta ABD) = \frac{1}{2} ar(\|^{gm}\ BYXD)$

But $ar(\Delta ABD) = ar(\Delta MBC)$

[Congruent triangles have equal areas]

$\Rightarrow$ $ar(\Delta MBC) = \frac{1}{2} ar(\|^{gm}\ BYXD)$

$\Rightarrow$ $ar(\|^{gm}\ BYXD) = 2ar(\Delta MBC)$

(iii) Since, $ar(BYXD) = 2ar(\Delta MBC)$ ...(1)

[From (ii) part]

Also, $ar(\text{square } ABMN) = 2ar(\Delta MBC)$ ...(2)

[$\because$ $\|^{gm}$ $ABMN$ and ΔMBC are on the same base MB and between the same parallels MB and NC]

From (1) and (2), we have

$ar(BYXD) = ar(ABMN)$

(iv) In ΔFCB and ΔACE, we have

$FC = AC$ [Sides of a square]

$CB = CE$ [Sides of a square]

$\angle FCA = \angle BCE$ [Each 90°]

$\Rightarrow$ $\angle FCA + \angle ACB = \angle BCE + \angle ACB$

[By adding $\angle ACB$ on both sides.]

$\Rightarrow$ $\angle FCB = \angle ACE$

$\therefore$ $\Delta FCB \cong \Delta ACE$ [By SAS congruency criterion]

(v) Since $\|^{gm}$ $CYXE$ and ΔACE are on the same base CE and between the same parallels CE and AX.

$\therefore$ $ar(\|^{gm}\ CYXE) = 2ar(\Delta ACE)$

But $\Delta ACE \cong \Delta FCB$ [From (iv) part]

$\therefore$ $ar(\Delta ACE) = ar(\Delta FCB)$

[$\because$ Congruent triangles are equal in area.]

$\therefore$ $ar(\|^{gm}\ CYXE) = 2ar(\Delta FCB)$

(vi) Since, $ar(\|^{gm}\ CYXE) = 2ar(\Delta FCB)$ [From (v) part] ...(3)

Also, ($\|^{gm}$ $ACFG$) and ΔFCB are on the same base FC and between the same parallels FC and BG.

$\therefore$ $ar(\|^{gm}\ ACFG) = 2ar(\Delta FCB)$ ...(4)

From (3) and (4), we get

$ar(\|^{gm}\ CYXE) = ar(\|^{gm}\ ACFG)$ ...(5)

(vii) Now, $ar(\|^{gm}\ BCED)$

$= ar(\|^{gm}\ CYXE) + ar(\|^{gm}\ BYXD)$

$= ar(\|^{gm}\ ACFG) + ar(\|^{gm}\ ABMN)$

[From (iii), (vi) parts]

Thus, $ar(\|^{gm}\ BCED) = ar(\|^{gm}\ ABMN) + ar(\|^{gm}\ ACFG)$