CHAPTER 13

Surface Areas and Volumes

TRY YOURSELF SOLUTIONS

1. Given, length (l) = 10 m, breadth (b) = 10 m and height (h) = 5 m

Now, length of the longest pole = diagonal of a cuboid

$= \sqrt{l^2+b^2+h^2} = \sqrt{(10)^2+(10)^2+(5)^2}$

$= \sqrt{100+100+25} = \sqrt{225} = 15$ m

2. Since, we know that

Total surface area = 2(Surface area of base) + (Lateral surface area)

$\Rightarrow$ 20 = 2(Surace area of base) + 16

$\Rightarrow$ 4 = 2 (Surface area of base)

$\Rightarrow$ Surface area of base $= \frac{4}{2} = 2\text{m}^2$

3. Given, height of the room (h) = 5 m

Let the length (l) and the breadth (b) of the room be $4x$ m and x m respectively.

$\therefore$ Area of the four walls $= 2(l+b)h = 2(4x+x)\times 5 = 50x\text{ m}^2$

Now, cost of papering the four walls at the rate of ₹ 0.70 per square metre = ₹ $(50x \times 0.70)$ = ₹ $35x$

$\Rightarrow$ $35x = 157.50$ (Given) $\Rightarrow x = \frac{157.50}{35} = 4.5$

Hence, length of the room = $4x$ m = (4×4.5)m = 18m

Breadth of the room = x m = 4.5 m

4. Since, Mary wants to paste the paper on the outer surface of the box. So, the quantity of paper required would be equal to the surface area of the box which is of the shape of a cuboid. The dimensions of the box are Length (l) = 80 cm, Breadth (b) = 40 cm, Height (h) = 20 cm.

Surface area of the box = $2(lb + bh + hl)$

= 2[(80 × 40) + (40 × 20) + (20 × 80)]

= 2[3200 + 800 + 1600] = 2 × 5600 = 11200 cm^2

Area of each sheet of the paper = 40 × 40 = 1600 cm^2

$\therefore$ Number of sheets required

$= \frac{\text{Surface area of box}}{\text{Area of one sheet of paper}} = \frac{11200}{1600} = 7$

So, she would require 7 sheets.

5. Let the length (l), breadth (b) and height (h) (in metres) of the box be $2x$, $3x$ and $4x$ respectively.

$\therefore$ TSA of the box = $2(lb + bh + hl)$

$= 2[2x \times 3x + 3x \times 4x + 4x \times 2x]$

$= 2[6x^2 + 12x^2 + 8x^2] = 52x^2\text{ m}^2$

According to the question, we have,

$9.50 \times 52x^2 - 8 \times 52x^2 = 1248 \Rightarrow 52x^2 \times (9.50 - 8) = 1248$

$\Rightarrow 52x^2 \times 1.50 = 1248 \Rightarrow 78x^2 = 1248 \Rightarrow x^2 = 16 \Rightarrow x = 4$

[$\because x$ can't be negative]

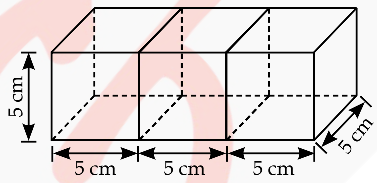
So, length = 2 × 4 = 8 m, breadth = 3 × 4 = 12 m and height = 4 × 4 = 16 m.

6. Given, length of edge (a) = 10 m

$\therefore$ Surface area of the cube $= 6a^2 = 6 \times (10)^2 = (6 \times 100)$

$= 600\text{ m}^2$

7. The dimensions of the cuboid so formed are length (l) = 5 + 5 + 5 = 15 cm, breadth (b) = 5 cm and height (h) = 5 cm.



So, surface area of the cuboid

$= 2(lb + bh + hl)$

$= 2(15 \times 5 + 5 \times 5 + 15 \times 5) = 350\text{ cm}^2$

8. Length of side of cube (a) = 8 m

$\therefore$ Required tarpaulin = area covered by tarpaulin

$= 5a^2 = 5 \times 8^2 = 5 \times 64 = 320\text{ m}^2$

9. Curved surface area = circumference of base × height

$= 2\pi rh = 180 \times 30 = 5400\text{ cm}^2$

10. Radius of the base of the cylindrical kaleidoscope, (r) = 3.5 cm.

Height (length) of kaleidoscope (h) = 25 cm

Area of chart paper required = curved surface area of the kaleidoscope $= 2\pi rh = 2\times\frac{22}{7}\times 3.5\times 25 = 550\,\text{cm}^2$

11. Curved surface area of a cylinder = $2\pi rh$

$\Rightarrow 8.8 = 2\times\frac{22}{7}\times 0.7\times h \Rightarrow h = \frac{8.8\times 7}{44\times 0.7} = 2$ m

Hence, the height of the cylinder is 2 m.

Total surface area $= 2\pi r(h + r) = 2\times\frac{22}{7}\times 0.7\,(2+0.7)$

$= 2\times\frac{22}{7}\times 0.7\times 2.7 = 11.88\text{ m}^2$

12. Here, radius $(r) = 1\frac{3}{4}\text{cm} = \frac{7}{4}\text{cm}$

Height (h) = 3 cm

Now, area of cardboard used = Curved surface area of cylinder

$= 2\pi rh = 2\times\frac{22}{7}\times\frac{7}{4}\times 3 = 33\text{ cm}^2$

13. External radius, $R = \frac{25}{2} = 12.5$ cm

Thickness of pipe = 1 cm [Given]

$\therefore$ Internal radius, r = (External radius – thickness)

= (12.5 – 1) cm = 11.5 cm

Given, height of the pipe = 20 m = 2000 cm

Total surface area of the pipe = External curved surface area + internal curved surface area + 2 (surface area of each base)

$= 2\pi(R + r)h + 2\pi(R^2 - r^2)$

$= 2\times\frac{22}{7}(12.5 + 11.5)2000 + 2\times\frac{22}{7}[(12.5)^2 - (11.5)^2]$

$= 2\times\frac{22}{7}\times 24\times 2000 + 2\times\frac{22}{7}\times 24 = 2\times\frac{22}{7}\times 24\times(2000+1)$

$= 2\times\frac{22}{7}\times 24\times 2001 = 301865.14\text{ cm}^2$

14. Here external radius $(R) = 8$ cm
Internal radius $(r) = 6$ cm
and height $(h) = 2.1$ m $= (2.1 \times 100)$ cm $= 210$ cm
Now, CSA of hollow cylinder (pipe) $= 2\pi h(R + r)$

$= 2\times\frac{22}{7}\times 210(8+6) = 18480\text{ cm}^2 = \frac{18480}{10000}\text{m}^2 = 1.848\text{ m}^2$

15. Here, radius $(r) = 7$ cm and
Height $(h) = 13\sqrt{2}$ cm
Let l be the slant height of the cone.
$\therefore\ l^2 = r^2 + h^2 = 7^2 + (13\sqrt{2})^2$
$\Rightarrow\ l = \sqrt{49+338} = \sqrt{387} = 3\sqrt{43}$ cm

16. We have, $r = 3$ cm and $h = 4$ cm
Let l be the slant height of the cone. Then,
$l^2 = r^2 + h^2$
$\Rightarrow\ l^2 = 3^2 + 4^2 \Rightarrow l = \sqrt{25} = 5$ cm
$\therefore$ Area of the curved surface $= \pi rl$
$= \left(\frac{22}{7}\times 3\times 5\right)\text{cm}^2 = 47.14\text{ cm}^2$

17. Here, radius $(r) = 21$ cm and
Slant height $(l) = 60$ cm
$\therefore$ CSA of cone $= \pi rl = \frac{22}{7}\times 21\times 60 = 3960\text{ cm}^2$

18. We have, radius $(r) = 6$ cm height $(h) = 8$ cm
$\therefore$ Slant height $(l) = \sqrt{r^2+h^2} = \sqrt{6^2+8^2} = \sqrt{100} = 10$ cm
So, TSA of cone $= \pi r(r + l) = \frac{22}{7}\times 6(6+10) = \frac{22}{7}\times 6\times 16$
$= 301.71\text{ cm}^2$

19. Height of conical tent $(h) = 3.5$ m and radius of the base of conical tent $(r) = 12$ m

Slant height $(l) = \sqrt{h^2+r^2} = \sqrt{(3.5)^2+(12)^2}$
$= \sqrt{12.25+144} = \sqrt{156.25}\text{ m} = 12.5$ m

$\therefore$ Canvas required = Curved surface area of the conical tent $= \pi rl = \left(\frac{22}{7}\times 12\times 12.5\right)\text{m}^2 = 471.42\text{ m}^2$

Hence, the canvas required to make the conical tent is 471.42 m^2.

20. let r be the radius of the sphere.
$\therefore$ Surface area of sphere $= 4\pi r^2 = 11880$ [Given]

$\Rightarrow r^2 = \frac{11880\times 7}{4\times 22} = 945 \Rightarrow r = 3\sqrt{105}$ cm

Now, diameter of sphere $= 2r = 2\times 3\sqrt{105} = 6\sqrt{105}$ cm

21. Let r_1 and r_2 be the radii of two spheres.

Then, $\frac{\text{Surface area of first sphere}}{\text{Surface area of second sphere}} = \frac{16}{49}$ (Given)

$\Rightarrow\ \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{16}{49} \Rightarrow \frac{r_1^2}{r_2^2} = \frac{16}{49} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{16}{49}} = \frac{4}{7}$

$\therefore$ Required ratio $= 4 : 7$

22. Let r units be the radius of sphere.
$\therefore$ Total surface area of sphere $= 4\pi r^2$ sq. units
Radius of hemisphere $(r_1) = 4r$ units
$\therefore$ Curved surface area of hemisphere $= 2\pi r_1^2$
$= 2\pi(4r)^2 = 32\pi r^2$ sq. units

$\therefore$ Required ratio $= \frac{4\pi r^2}{32\pi r^2} = 1:8$

23. Let r cm be the radius of hemispherical dome.
Then, circumference $= 2\pi r$
$\Rightarrow\ 2\pi r = 17.6$
$\Rightarrow\ 2\times\frac{22}{7}\times r = 17.6 \Rightarrow r = \frac{17.6\times 7}{2\times 22} = 2.8$ m

Surface area of inside of dome $= 2\pi r^2$
$= 2\times\frac{22}{7}\times(2.8)^2 = 49.28\text{ m}^2 = 49.28 \times 10000\text{ cm}^2$
$= 492800\text{ cm}^2$
Cost of painting 100 cm^2 = ₹ 5
Cost of painting 1 cm^2 = ₹ $\left(\frac{5}{100}\right)$
Cost of painting 492800 cm^2 = ₹ $\left(\frac{5}{100}\times 492800\right)$ = ₹ 24,640

24. For water tank, $l = 48$ m, $b = 36$ m, $h = 28$ m
$\therefore$ Volume of the tank $= lbh = 48 \times 36 \times 28 = 48384\text{ m}^3$

25. Here, dimensions of the plank are length $(l) = 4$ m,
breadth $(b) = 50$ cm $= \frac{50}{100}$ m $= 0.5$ m

and height $(h) = 20$ cm $= \frac{20}{100}$ m $= 0.2$ m

$\therefore$ Volume of 1 plank $= l \times b \times h = 4 \times 0.5 \times 0.2 = 0.4\text{ m}^3$
Also, given that dimensions of the pit are
length $(l) = 16$ m, breadth $(b) = 12$ m and height $(h) = 4$ m
$\therefore$ Volume of the pit $= l \times b \times h = (16 \times 12 \times 4)\text{ m}^3$

Now, number of planks $= \frac{\text{Volume of the pit}}{\text{Volume of 1 plank}}$

$= \frac{16\times 12\times 4}{0.4} = 16 \times 12 \times 10 = 1920$

Hence, the number of planks that can be stored in the pit is 1920.

26. Let length, breadth and height of the cuboid be l, b and h respectively. Then,
$p = lb,\ q = bh,\ r = hl$
$\therefore\ pqr = (lb)(bh)(hl) = l^2b^2h^2$ ...(i)
Again, $v = lbh$
$\therefore\ v^2 = (lbh)^2 = l^2b^2h^2$ ...(ii)
From (i) and (ii) we have, $v^2 = pqr$

27. Let a be the side of cube.
Total surface area of cube = $96\,cm^2$ [Given]
$\Rightarrow$ $6(a)^2 = 96 \Rightarrow a^2 = 16 \Rightarrow a = 4$ cm
$\therefore$ Volume of cube = $(a)^3 = (4)^3 = 64\ cm^3$

28. Let the length of each edge of the cube be a.
Total surface area of the cube $= 32\frac{2}{3}m^3$ [Given]
$$\Rightarrow \quad 6a^2 = \frac{98}{3} \Rightarrow a^2 = \frac{98}{3\times 6}$$
$$\Rightarrow \quad a^2 = \frac{49}{9} \Rightarrow a = \sqrt{\frac{49}{9}} \Rightarrow a = \frac{7}{3}\text{m}$$
$\therefore$ Volume of the cube = a^3
$$= \left(\frac{7}{3}\right)^3 = \frac{343}{27}\text{m}^3 = 12.70\ \text{m}^3$$

29. Volume of each cube = $(\text{side})^3$
$= (3)^3 = 27\ cm^3$
Now, number of cubes in the structure = 15
Therefore, volume of the structure = $27 \times 15\ cm^3 = 405\ cm^3$

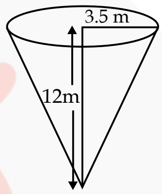
30. Here, height (h) = 1.5 m = 150 cm
Radius (r) = $\frac{70}{2}$ cm
$\therefore$ Volume of cylindrical rod
$$= \pi r^2 h = \frac{22}{7}\times\frac{70}{2}\times\frac{70}{2}\times 150 = 577500\ cm^3$$

31. Radius of cylindrical vessel (r) = 20 cm
Quantity of rain water in the vessel = $105600\ cm^3$
Let h be the height of water in the vessel.
Quantity of rain water in the vessel = volume of rainfall
$\therefore$ $\pi r^2 h = 105600$
$$\Rightarrow \quad \frac{22}{7}\times 20\times 20\times h = 105600$$
$$\Rightarrow \quad h = \frac{105600\times 7}{22\times 20\times 20} = 84\text{ cm}$$

32. Let r and h be the radius and height of cylinder.
Circumference of base of cylinder = 220 cm [Given]
$\Rightarrow$ $2\pi r = 220$
And C.S.A. = $2200\ cm^2$
$\Rightarrow$ $2\pi rh = 2200$
$\Rightarrow$ $220 \times h = 2200$ [$\because$ $2\pi r = 220$]
$\Rightarrow$ $h = 10$ cm
Now, $2\pi r = 220$
$$\Rightarrow \quad 2\times\frac{22}{7}\times r = 220 \Rightarrow r = 35\text{ cm}$$
$\therefore$ Volume of cylinder = $\pi r^2 h$
$$= \frac{22}{7}\times 35\times 35\times 10 = 38500\ cm^3$$

33. External diameter of the pipe = 2.4 cm
External radius of the pipe, (R) = $\frac{2.4}{2}$ cm = 1.2 cm
Thickness of the pipe = 3 mm = 0.3 cm
Internal radius, (r) = External radius – thickness
= 1.2 cm – 0.3 cm = 0.9 cm
Length of the pipe (h) = 3.5 m = 350 cm
Volume of lead = $\pi(R^2 - r^2)h$
$$= \frac{22}{7}\times[(1.2)^2 - (0.9)^2]\times 350 = \frac{22}{7}\times 0.63\times 350 = 693\ cm^3$$
$\because$ Mass = volume × density
= 693 × 12 [$\because$ Weight of 1 cm^3 = 12 g (Given)]
$$= 8316\text{ g} = \frac{8316}{1000}\text{ kg} = 8.316\text{ kg}$$

34. Given, diameter = 7 m $\Rightarrow$ r = 3.5 m and h = 12 m



Capacity = Volume $= \frac{1}{3}\pi r^2 h$
$$= \frac{1}{3}\times\frac{22}{7}\times 3.5\times 3.5\times 12$$
= $154\ m^3$
= 154000 l [1 m^3 = 1000 l]

35. Radius of base of conical granary (r) = 8.4 m
Height of base of conical granary (h) = 87.5 m
$\therefore$ Volume of conical granary $= \frac{1}{3}\pi r^2 h$
$$= \frac{1}{3}\times\frac{22}{7}\times(8.4)^2\times 87.5$$
$$= \frac{1}{3}\times\frac{22}{7}\times 8.4\times 8.4\times 87.5 = 6468\ m^3$$
Volume of rice in each bag = $196\ m^3$ [Given]
$\therefore$ Required number of bags
$$= \frac{\text{Volume of conical granary}}{\text{Volume of each bag}} = \frac{6468}{196} = 33$$

36. Let r and h be the radius and height of the original right circular cone.
$\therefore$ Volume of original cone $= \frac{1}{3}\pi r^2 h = V$ (Given)
If radius is halved and height is doubled, then volume of new cone $= \frac{1}{3}\pi\times\left(\frac{r}{2}\right)^2\times(2h)$
$$= \frac{1}{3}\pi\times\frac{r^2}{4}\times 2h = \frac{1}{2}\left(\frac{1}{3}\pi r^2 h\right) = \frac{1}{2}V$$

37. Radius of bigger sphere (R) = 2.1 cm
$\therefore$ Volume of bigger sphere $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\times(2.1)^3\ cm^3$
Let radius of smaller sphere be x cm.
$\therefore$ Volume of small sphere $= \frac{4}{3}\pi x^3\ cm^3$
Also, it is given that
Volume of bigger sphere = 8 × volume of small sphere
$$\Rightarrow \quad \frac{4}{3}\pi(2.1)^3 = 8\times\frac{4}{3}\pi x^3$$
$$\Rightarrow \quad x^3 = \frac{(2.1)^3}{8} = \left(\frac{2.1}{2}\right)^3 \Rightarrow x = \frac{2.1}{2} = 1.05\text{ cm}$$

38. Original radius of sphere (r_1) = 10 cm
$\therefore$ Original volume of sphere $= \frac{4}{3}\pi r_1^3$
$$= \frac{4}{3}\times\frac{22}{7}\times 10\times 10\times 10 = 4190.5\ cm^3 \text{ (approx.)}$$
New radius (r_2) = 10 + 2 = 12 cm

$\therefore$ New volume of sphere $= \frac{4}{3}\pi r_2^3 = \frac{4}{3}\times\frac{22}{7}\times 12\times 12\times 12$
$= 7241.1\text{ cm}^3$ (approx.)

$\therefore$ Percentage increase in volume

$= \frac{7241.1 - 4190.5}{4190.5}\times 100 = 72.8\%$ (approx.)

39. Cost of painting 100 m^2 = ₹ 20

Cost of painting 1 m^2 = ₹ $\frac{20}{100}$

Total cost of painting inner surface of hemispherical bowl = ₹ 30.80 [Given]

$\therefore$ Inner surface area of bowl

$= \frac{\text{Total cost}}{\text{Cost per m}^2} = \frac{30.80}{20/100} = 154\text{ m}^2$

$\Rightarrow$ $2\pi r^2 = 154$ (r is radius of hemisphere)

$\Rightarrow$ $2\times\frac{22}{7}\times r^2 = 154$

$\Rightarrow$ $r^2 = \frac{154\times 7}{2\times 22} = 24.5$

$\Rightarrow$ $r = 4.9$ m (approx.)

Now, volume of air inside the bowl $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\times\frac{22}{7}\times(4.9)^3 = \frac{2}{3}\times\frac{22}{7}\times 4.9\times 4.9\times 4.9$

$= 246.5\text{ m}^3$ (approx.)