Triangles

TRY YOURSELF — SOLUTIONS

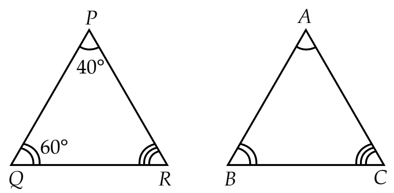
1. No. We have, $\Delta ABC \cong \Delta PQR$

As we know, two triangles are congruent, if the sides and angles of one triangle are equal to the corresponding sides and angles of other triangle.

$\therefore$ $\angle ABC = \angle PQR$, $\angle BCA = \angle QRP$, $\angle CAB = \angle RPQ$

Thus, it is not true to say that $\angle BCA = \angle PQR$.

2.



We have, $\Delta PQR \cong \Delta ABC$

$\therefore$ $\angle R = \angle C$ [By C.P.C.T.]

$\Rightarrow$ $\angle R = 180° - \angle P - \angle Q$

[By angle sum property of ΔPQR]

$\Rightarrow$ $\angle C = 180° - 40° - 60° = 80°$ [$\because$ $\angle P = 40°$, $\angle Q = 60°$]

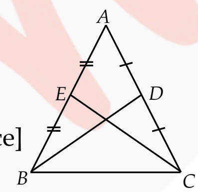
$\Rightarrow$ $\angle C = 80°$

3. In ΔABC, $AB = AC$ [Given]

$\Rightarrow$ $\frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow AE = AD$ [$\because$ D is the mid-point of AC and E is the mid-point of AB]

Now, in ΔABD and ΔACE,



$AB = AC$ [Given]

$\angle A = \angle A$ [Common]

$AD = AE$ [Proved above]

$\therefore$ $\Delta ABD \cong \Delta ACE$ [By SAS congruence]

$\Rightarrow$ $BD = CE$ [By C.P.C.T.]

4. We have, $\angle ABX = \angle PQY$

$\Rightarrow$ $180° - \angle ABC = 180° - \angle PQR$

[$\because$ $\angle ABX + \angle ABC = 180°$ and $\angle PQY + \angle PQR = 180°$ (Linear Pair)]

$\Rightarrow$ $\angle ABC = \angle PQR$ ...(i)

In ΔABC and ΔPQR,

$AB = PQ$ [Given]

$\angle ABC = \angle PQR$ [From (i)]

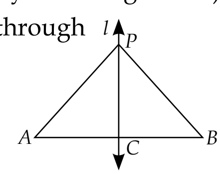
$BC = QR$ [Given]

$\therefore$ $\Delta ABC \cong \Delta PQR$ (By SAS congruence)

5. Given a line $l \perp AB$. Let it passes through C which is the mid-point of AB.

To show that $PA = PB$.

Consider ΔPCA and ΔPCB.



We have, $AC = BC$

[$\because$ C is the mid-point of AB]

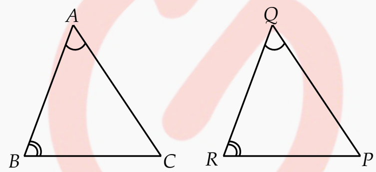
$\angle PCA = \angle PCB = 90°$ [$\because$ $l \perp AB$]

$PC = PC$ [Common]

$\therefore$ $\Delta PCA \cong \Delta PCB$ [By SAS congruence]

$\Rightarrow$ $PA = PB$ [By C.P.C.T.]

6. We have given,



In ΔABC and ΔPQR, $\angle A = \angle Q$ and $\angle B = \angle R$

Since, the sides AB and QR are included between equal angles. Thus, the side QR of ΔPQR should be equal to side AB of ΔABC such that $\Delta ABC \cong \Delta QRP$, by ASA congruence rule.

7. In ΔABD and ΔCBE

$\angle A = \angle C$ [Given]

$AB = BC$ [Given]

$\angle B = \angle B$ [Common]

$\therefore$ $\Delta ABD \cong \Delta CBE$ [By ASA congruence]

8. Given AC is bisector of $\angle A$ and $\angle C$.

$\therefore$ $\angle BAC = \angle DAC$ and $\angle BCA = \angle DCA$ ....(i)

In ΔABC and ΔADC,

$\angle BAC = \angle DAC$ [From (i)]

$\angle BCA = \angle DCA$ [From (i)]

$AC = AC$ [Common]

$\therefore$ $\Delta ABC \cong \Delta ADC$ [By ASA congruence]

$\Rightarrow$ $AB = AD$ [By C.P.C.T.]

$\therefore$ $AB = 5$ cm [$\because$ $AD = 5$ cm]

9. Let each of the base angle of an isosceles triangle be $x°$.

We have vertical angle = 100°

So, $100° + x° + x° = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $2x° = 180° - 100° = 80°$

$\Rightarrow$ $x = 40°$

$\therefore$ Each base angle measures 40°.

10. Since, AE is the bisector of $\angle CAD$

$\therefore$ $\angle 1 = \angle 2$ ...(i)

Since, $AE \parallel BC$ and AC is the transversal

$\therefore$ $\angle 1 = \angle C$ [Alternate angles] ...(ii)

Since, $AE \parallel BC$ and AB is the transversal

$\therefore$ $\angle 2 = \angle B$ [Corresponding angles] ...(iii)

From (i), (ii) and (iii), we get

$\angle B = \angle C$

$\Rightarrow$ $AC = AB$ [Sides opposite to equal angles of a triangle are equal]

$\therefore$ ΔABC is isosceles.

11. In ΔABD and ΔACD,
$\angle BAD = \angle CAD$ [$\because$ AD is the bisector of $\angle A$]
$AD = AD$ [Common]
$\angle ADB = \angle ADC = 90°$ [Given]
So, $\Delta ABD \cong \Delta ACD$ [By ASA congruence]
$\Rightarrow$ $AB = AC$ [By C.P.C.T.]
$\therefore$ ΔABC is an isosceles triangle.

12. In ΔADC and ΔCBA
$CD = AB$ [Given]
$AD = CB$ [Given]
$CA = CA$ [Common]
$\therefore$ $\Delta ADC \cong \Delta CBA$ [By SSS congruence]

13. Given, $PA = PB$ and $QA = QB$
In ΔPAQ and ΔPBQ
$AP = BP$ [Given]
$AQ = BQ$ [Given]
$PQ = PQ$ [Common]
So, $\Delta PAQ \cong \Delta PBQ$ [By SSS congruence]
Therefore, $\angle APQ = \angle BPQ$ [By C.P.C.T.]
Now, in ΔPAC and ΔPBC,
$AP = BP$ [Given]
$\angle APC = \angle BPC$ [$\because$ $\angle APQ = \angle BPQ$ (Proved above)]
$PC = PC$ [Common]
So, $\Delta PAC \cong \Delta PBC$ [By SAS congruence]
Therefore, $AC = BC$ [By C.P.C.T.] ...(i)
and $\angle ACP = \angle BCP$ [By C.P.C.T.]
Also, $\angle ACP + \angle BCP = 180°$ [Linear pair]
$\Rightarrow$ $2\angle ACP = 180°$ $\Rightarrow$ $\angle ACP = 90°$ ...(ii)
From (i) and (ii), PQ is the perpendicular bisector of AB.

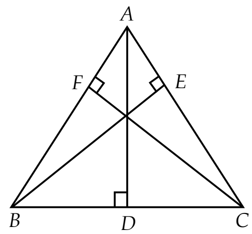
14. In ΔQLM and ΔRNM, we have
$QM = RM$ [Given]
$LM = NM$ [Given]
$\angle QLM = \angle RNM$ [Each equals 90°]
$\therefore$ $\Delta QLM \cong \Delta RNM$ [By RHS congruence]
$\Rightarrow$ $\angle Q = \angle R$ [By C.P.C.T.]
$\Rightarrow$ $PR = PQ$
[Sides opposite to equal angles of a triangle are equal]

15. In ΔABC and ΔQPR,
$\angle ACB = \angle QRP$ [Each equals 90°]
$AB = PQ$ [Given]
$BC = PR$ [Given]
$\therefore$ $\Delta ABC \cong \Delta QPR$ [By RHS congruence]
$\Rightarrow$ $AC = QR$ [By C.P.C.T.] ....(i)
Also, $BC = PR$ [Given]
$\Rightarrow$ $BC + CR = PR + CR$ $\Rightarrow$ $BR = CP$ ...(ii)
Now, in ΔACP and ΔQRB,
$AC = QR$ [From (i)]
$\angle ACP = \angle QRB$ [Each equals 90°]
$CP = RB$ [From (ii)]
$\therefore$ $\Delta ACP \cong \Delta QRB$ [By SAS congruence]

16. In ΔDAC,
$AD = AC$ [Given]
So, $\angle ACD = \angle ADC$ [Angles opposite to equal sides of a triangle are equal]
Now, $\angle ADC$ is an exterior angle for ΔABD.
So, $\angle ADC > \angle ABD$
$\Rightarrow$ $\angle ACD > \angle ABD$
or $\angle ACB > \angle ABC$
So, $AB > AC$ [Side opposite to larger angle is longer]
or $AB > AD$ [$\because$ $AD = AC$]

17. In ΔPQR, we have
$PQ > PR$ [Given]
$\Rightarrow$ $\angle PRQ > PQR$
[Angle opposite to longer side of a triangle is greater]
$\Rightarrow$ $\frac{1}{2}\angle PRQ > \frac{1}{2}\angle PQR$
$\Rightarrow$ $\angle SRQ > \angle SQR$
[RS and QS are bisectors of $\angle PRQ$ and $\angle PQR$ respectively]
$\Rightarrow$ $SQ > SR$ [Side opposite to greater angle is longer]

18. Let in ΔABC, AD, BE and CF are three altitudes.



In right angled ΔABD,
AB is hypotenuse.
$\Rightarrow$ $AB > AD$ ...(i)
Similarly, in right angled ΔBEC and ΔCFA, we have
$BC > BE$ ...(ii)
and $CA > CF$ ...(iii)
Adding (i), (ii) and (iii), we get
$AB + BC + CA > AD + BE + CF$
i.e., Perimeter of $\Delta ABC > (AD + BE + CF)$.