CHAPTER 11

Constructions

EXAM DRILL

SOLUTIONS

1. **(c) :** In the given figure, $B_3P \parallel B_5C$ and $QP \parallel AC$.

$\therefore$ $\Delta QBP \sim \Delta ABC$ and $\Delta BPB_3 \sim \Delta BCB_5$.

$$\Rightarrow \frac{BQ}{AB} = \frac{BP}{BC} = \frac{QP}{AC} \qquad \text{...(i)}$$

$$\text{and } \frac{BP}{BC} = \frac{B_3P}{B_5C} = \frac{BB_3}{BB_5} = \frac{3}{5} \qquad \text{...(ii)}$$

From (i) and (ii), we have

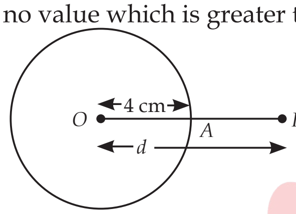
$$\frac{BQ}{AB} = \frac{BP}{BC} = \frac{QP}{AC} = \frac{3}{5} \quad \therefore \quad \frac{x}{y} = \frac{3}{5}$$

2. **(b) :** Two tangents can be drawn to a circle from a point lying outside the circle.

3. **(a) :** We cannot construct a pair of parallel tangents to a circle from an external point to the circle.

4. **(d) :** Here, radius is 4 cm. So, $d > 4$.

Since, there is no value which is greater than 4.



So, a pair of tangents can't be constructed from P for all the given values.

5. **(d) :** Pair of tangents inclined at a reflex angle can't be constructed to a circle.

6. In the given figure, A_3P is parallel to A_5B such that $\angle AA_3P = \angle AA_5B$.

$$\therefore \frac{AP}{PB} = \frac{AA_3}{A_3A_5} = \frac{3}{2}$$

Hence, $AP : PB = 3 : 2$.

7. In the given figure, $A_5C \parallel A_6B$

$$\therefore \frac{AC}{CB} = \frac{AA_5}{A_5A_6} = \frac{5}{1}$$

$\therefore$ The line segment AB is divided into two parts in the ratio 5 : 1.

8. From the given figure, it is clear that there are three points at equal distances on AX and there are four points at equal distances on BY. Here, P divides AB on joining A_3B_4. So, P divides AB internally in the ratio 3 : 4.

9. The given figure is the construction of division of a line segment internally in the ratio 2 : 6.

So, AX and BY are parallel.

$\Rightarrow$ $\angle ABY = \angle BAX = 30°$.

10. The next step is : Join B_8 to C.

11. Since, 3 < 3.5 (radius of a circle). So, the point P lies inside the circle.

$\therefore$ It is not possible to construct a pair of tangents from a point P situated at a distance of 3 cm from the centre of a circle of radius 3.5 cm.

12. True. Since, the angle between the pair of tangents is always greater than 0° but less than 180°. Hence, we can draw a pair of tangents to a circle inclined at an angle of 170°.

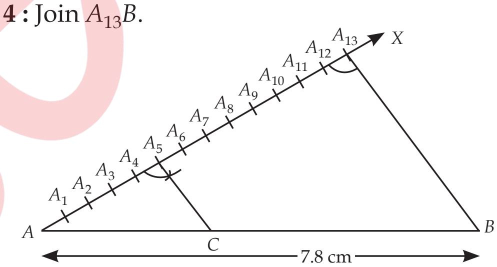
13. **Steps of Construction :**

Step 1 : Draw AB = 7.8 cm.

Step 2 : Draw a ray AX, making an acute angle with AB.

Step 3 : Locate (5 + 8 =) 13 points *i.e.,* $A_1, A_2, A_3, ..., A_{13}$ on AX, so that $AA_1 = A_1A_2 = A_2A_3 = ... = A_{12}A_{13}$.

Step 4 : Join $A_{13}B$.



Step 5 : Through A_5, draw A_5C parallel to $A_{13}B$, meeting AB at C.

Thus C divides AB in the ratio 5 : 8

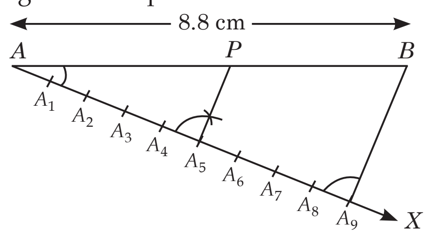
14. **Steps of Construction**

Step 1 : Draw AB of length 8.8 cm and draw a ray AX making an acute angle with AB.

Step 2 : Locate 9 points $A_1, A_2, A_3, ..., A_9$ on AX such that $AA_1 = A_1A_2 = A_2A_3 = ... = A_8A_9$.

Step 3 : Join A_9B.

Step 4 : Through A_5, draw a line A_5P parallel to A_9B intersecting AB at the point P.



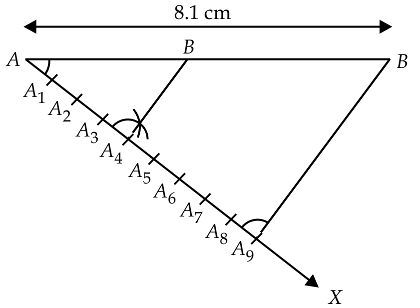
Thus, P is a point on AB which divides the line segment AB in the ratio 5 : 4.

15. **Steps of Construction**

Step 1 : Draw a line segment AB = 8.1 cm.

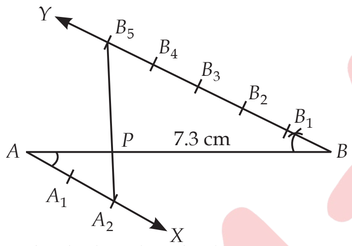
Step 2 : Draw a ray AX below the line segment AB making an acute angle $\angle BAX$.

Step 3 : Mark 9 points A_1, A_2, ... , A_9 on AX such that $AA_1 = A_1A_2 = A_2A_3$ and so on.
Step 4 : Join A_9B.
Step 5 : From point A_4 draw a line A_4B' parallel to A_9B which intersects AB at B'.



Thus, AB is divided internally in the ratio 4 : 5.

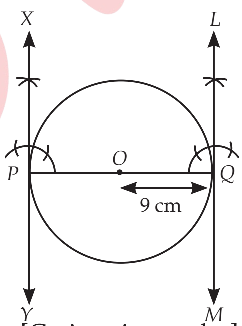
16. Steps of Construction
Step 1 : Draw a line segment AB of length 7.3 cm.
Step 2 : Draw a ray AX making an acute angle with AB. Draw another ray BY parallel to AX by making $\angle ABY$ equal to $\angle BAX$.
Step 3 : Locate 2 points A_1, A_2 on AX and 5 points B_1, B_2, B_3, B_4 and B_5 on BY such that $AA_1 = A_1A_2 = BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
Step 4 : Join A_2B_5 which intersects AB at P.



Thus, P is the point on AB which divides the line segment AB in the ratio 2 : 5.

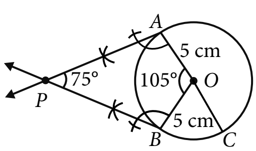
17. The given figure is the construction of division of a line segment internally.
$\therefore \quad AX \parallel BY$
Now, $\angle AOA_5 = \angle BOB_2 = 115°$ [Vertically opposite angles]
$\angle OBB_2 = \angle OAA_5 = 22°$ (Alternate angles)
In ΔBOB_2, $\angle OB_2B = 180° - (115° + 22°) = 43°$.

18. Steps of Construction
Step 1 : Draw a circle with centre O and radius 9 cm.
Step 2 : Draw the diameter POQ.
Step 3 : Construct $XY \perp OP$ at P and $LM \perp OQ$ at Q.



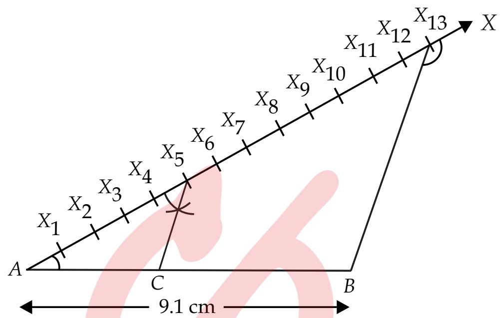
Thus, XPY and LQM are the two tangents at P and Q, respectively, to the circle.
Yes, lines $XY \parallel LM$
Since, $\angle OPX + \angle LQO = 180°$ [Co-interior angles]

19. Steps of Construction
Step 1 : Draw a circle of radius 5 cm with centre O.
Step 2 : At O, construct radii OA and OB such that $\angle AOB = 105°$ $(180° - 75°)$.



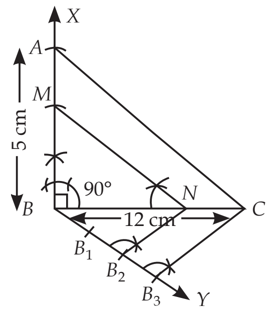
Step 3 : Draw perpendiculars at A and B such that these perpendiculars intersect at P.
Hence, PA and PB are required tangents.

20. Steps of Construction
Step 1 : Draw a line segment $AB = 9.1$ cm.
Step 2 : Draw a ray AX making an acute angle with AB.
Step 3 : Mark 13 (= 8 + 5) equal points on AX, such that $AX_1 = X_1X_2 = ... = X_{12}X_{13}$.
Step 4 : Join points X_{13} and B.
Step 5 : From point X_5, draw $X_5C \parallel X_{13}B$, which meets AB at C.



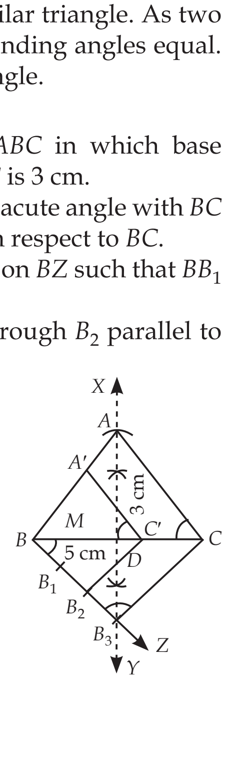
Thus, C divides AB in the ratio 5 : 8.
On measuring the two parts, we get $AC = 3.5$ cm and $CB = 5.6$ cm.

21. Steps of Construction
Step 1 : Construct a ΔABC in which $BC = 12$ cm, $AB = 5$ cm and $\angle B = 90°$
Step 2 : From B, draw an acute $\angle CBY$ downwards.
Step 3 : On ray BY, mark three points, B_1, B_2 and B_3, such that $BB_1 = B_1B_2 = B_2B_3$.



Step 4 : Join B_3C.
Step 5 : From point B_2, draw $B_2N \parallel B_3C$ intersecting BC at N.
Step 6 : From point N, draw $NM \parallel CA$ intersecting BA at M. Thus, ΔMBN is the required similar triangle. As two similar triangles have their corresponding angles equal. So, ΔMBN is also a right angled triangle.

22. Steps of Construction
Step 1 : Construct an isosceles ΔABC in which base $BC = 5$ cm and altitude from A to BC is 3 cm.
Step 2 : Draw any ray BZ making an acute angle with BC on the side opposite to vertex A with respect to BC.
Step 3 : Locate 3 points B_1, B_2 and B_3 on BZ such that $BB_1 = B_1B_2 = B_2B_3$.
Step 4 : Join B_3C and draw a line through B_2 parallel to B_3C to intersect BC at C'.
Step 5 : Draw a line through C' parallel to the line CA to intersect BA at A'.
Thus, $\Delta A'BC'$ is the required similar triangle.
Thus, $\Delta A'BC'$ is the required similar triangle whose sides are $\frac{2}{3}$ of the corresponding sides of triangle ABC.

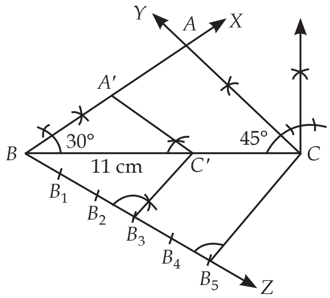


OR

Steps of Construction

Step 1 : Construct a ΔABC in which BC = 11 cm, $\angle B$ = 30° and $\angle A$ = 105°.

Step 2 : Draw a ray BZ making an acute angle with BC on the side opposite to vertex A with respect to BC.



Step 3 : Mark-off five points B_1, B_2, B_3, B_4 and B_5 on BZ such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.

Step 4 : Join B_5 to C and draw a line B_3C' parallel to B_5C, intersecting the line segment BC at C'.

Step 5 : Draw a line through C' parallel to CA intersecting the line segment AB at A'.

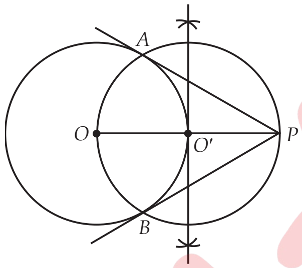
Thus, $\Delta A'BC'$ is the required similar triangle.

23. Steps of Construction

Step 1 : Draw a circle of radius 5 cm. Mark its centre as O.

Step 2 : Take a point P such that OP = 10 cm. Join OP.

Step 3 : Draw perpendicular bisector of OP and let O' be its mid-point.



Step 4 : Taking O' as center and $O'O$ or $O'P$ as radius, draw a circle which intersect the given circle at point, A and B.

Step 5 : Join PA and PB.

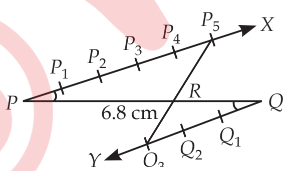
Thus, PA and PB are the required tangents of the circle from the point P.

OR

Steps of Construction

Step 1 : Draw PQ of length 6.8 cm.

Step 2 : Draw a ray PX making an acute angle with PQ.



Step 3 : Draw a ray QY parallel to PX by making $\angle PQY$ equal to $\angle QPX$.

Step 4 : Locate 5 points P_1, P_2, P_3, P_4 and P_5 on PX and 3 points Q_1, Q_2 and Q_3 on QY such that $PP_1 = P_1P_2 = P_2P_3 = P_3P_4 = P_4P_5 = QQ_1 = Q_1Q_2 = Q_2Q_3$.

Step 5 : Join P_5Q_3 which intersects PQ at R.

Thus, $PR : RQ = 5 : 3$.

Justification :

In ΔPRP_5 and ΔQRQ_3,

$\angle PRP_5 = \angle QRQ_3$ [Vertically opposite angles]

$\angle P_5PR = \angle Q_3QR$ [By construction]

$\therefore \quad \Delta PRP_5 \sim \Delta QRQ_3$ [By AA similarity]

$\therefore \quad \dfrac{PR}{RQ} = \dfrac{PP_5}{QQ_3}$

$\therefore \quad \dfrac{PR}{RQ} = \dfrac{5}{3} \qquad \left[\because \dfrac{PP_5}{QQ_3} = \dfrac{5}{3}\right]$

24. Steps of Construction

Step 1 : Consruct a ΔABC in which BC = 8 cm, AB = 7 cm and $\angle ABC$ = 75°.

Step 2 : Below BC, draw a ray BX such that $\angle CBX$ is an acute angle.

Step 3 : Along BX, mark off seven points B_1, B_2, B_3, B_4, B_5, B_6 and B_7 such that
$BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.

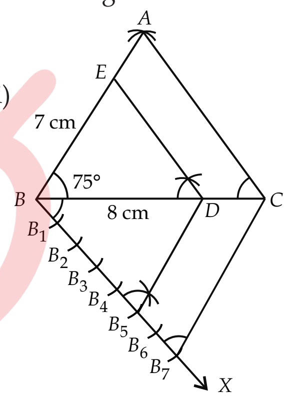
Step 4 : Join B_7C.

Step 5 : From B_5, draw $B_5D \parallel B_7C$, meeting BC at D.

Step 6 : From D, draw $ED \parallel AC$, meeting BA at E.

Thus, ΔEBD is the required similar triangle.

Justification : Since $ED \parallel AC$



$\therefore \quad \Delta EBD \sim \Delta ABC$

$\Rightarrow \quad \dfrac{EB}{AB} = \dfrac{BD}{BC} = \dfrac{ED}{AC}$ ...(i)

Now, $B_5D \parallel B_7C$

$\therefore \quad \Delta BDB_5 \sim \Delta BCB_7$

$\Rightarrow \quad \dfrac{BD}{BC} = \dfrac{B_5D}{B_7C} = \dfrac{BB_5}{BB_7} = \dfrac{5}{7}$ ...(ii)

From (i) and (ii), we have

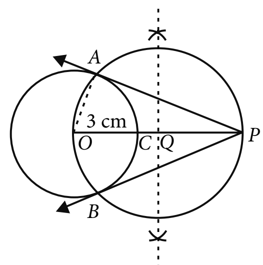
$\dfrac{EB}{AB} = \dfrac{BD}{BC} = \dfrac{ED}{AC} = \dfrac{5}{7}$

25. Steps of Construction

Step 1 : Draw a circle of radius 3 cm, taking O as centre and OC be its radius.

Step 2 : Produce OC to P such that OP = 7 cm.

Step 3 : Draw perpendicular bisector of OP which intersects OP at Q.



Step 4 : Taking Q as centre and radius QP or OQ, draw a circle which intersects previous circle at points A and B.

Step 5 : Join PA and PB. Thus, PA and PB are the required tangents.

Now, join OA to find PA.

In ΔAOP, $\angle OAP = 90°$ [Angle in semi-circle]

$\therefore \quad AP^2 = OP^2 - OA^2 = 7^2 - 3^2 = 40$

$\Rightarrow \quad AP$ = 6.3 cm (approx.)

$\therefore \quad$ Length of each tangent = 6.3 cm.

OR

Steps of Construction

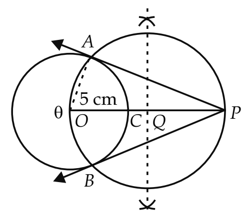
Step 1 : Draw a circle of radius 5 cm, taking O as centre and OC as its radius.

Step 2 : Produce OC to P such that OP = 13 cm.

Step 3 : Draw perpendicular bisector of OP which intersects OP at Q.

Step 4 : Taking Q as centre and radius QP or OQ, draw a circle which intersect previous circle at points A and B.

Step 5 : Join PA and PB. Thus, PA and PB are the required tangents.



Now, join OA to find PA.

In ΔAOP, $\angle OAP = 90°$ [Angle in semi-circle]

$\therefore AP^2 = OP^2 - OA^2 = 13^2 - 5^2 = 144 \Rightarrow AP = 12$ cm

$\therefore$ Length of each tangent = 12 cm.

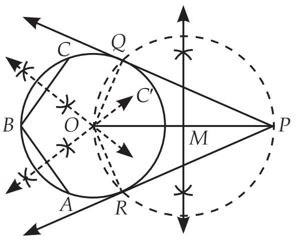
26. Steps of Construction

Step 1 : Draw a circle with the help of a circular ring. For finding the centre, draw two chords AB and BC. Draw perpendicular bisector of AB and BC, intersecting each other at point O, 'O' is centre of the circle.

Step 2 : Take a point P outside the circle. Join OP.

Step 3 : Draw perpendicular bisector of OP, which intersects OP at point M.

Step 4 : Taking M as the centre and OM as radius, draw a circle intersecting the given circle at points R and Q.



Step 5 : Join PQ and PR.

Thus, PQ and PR are the required pair of tangents.

27. Steps of Construction

Here diameter = 8 cm = Radius = 4 cm

Step 1 : Draw a circle of radius 4 cm, taking O as centre and OC be its radius.

Step 2 : Produce OC to P such that OP = 11 cm.

Step 3 : Draw perpendicular bisector of OP which intersects OP at Q.

Step 4 : Taking Q as centre and radius QP, draw a circle which intersect previous circle at points A and B.

Step 5 : Join PA and PB. Thus, PA and PB are the required tangents.

Now, join OA.

In ΔAOP, $\angle OAP = 90°$

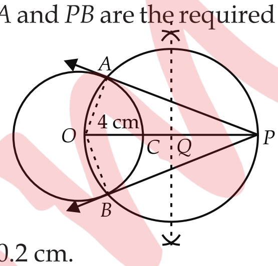
[Angle in semi-circle]

$\therefore \quad AP^2 = OP^2 - OA^2$

$= 11^2 - 4^2 = 105$

$\Rightarrow \quad AP = 10.2$ cm. (Approx.)

$\therefore$ Length of each tangent = 10.2 cm.



Justification :

Since, OP is a diameter.

$\therefore \quad \angle OAP = 90° = \angle OBP$ [Angles in semi-circle]

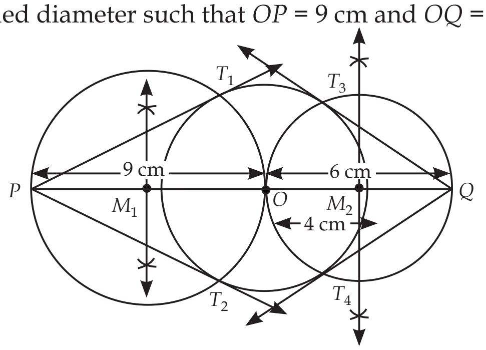
Also, OA and OB are radii of the same circle.

$\therefore \quad AP$ and PB are tangents to the circle.

28. Steps of Construction

Step 1 : Taking a point O as centre, draw a circle of radius 4 cm.

Step 2 : Take two points P and Q on either side of the extended diameter such that OP = 9 cm and OQ = 6 cm.



Step 3 : Bisect OP and OQ at M_1 and M_2 respectively.

Step 4 : Draw a circle with M_1 as centre and M_1P as radius to intersect the given circle at T_1 and T_2.

Step 5 : Join PT_1 and PT_2.

Step 6 : Draw a circle with M_2 as centre and M_2Q as radius to intersect the given circle at T_3 and T_4.

Step 7 : Join QT_3 and QT_4.

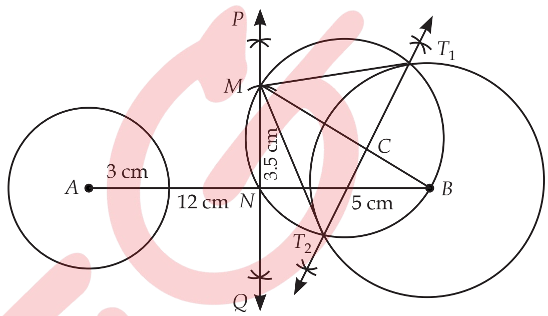
Thus, PT_1, PT_2, QT_3 and QT_4 are the required tangents.

29. Steps of Construction

Step 1 : Draw a circle of radius 3 cm with centre A.

Step 2 : Mark a point B at a distance of 12 cm from the centre A and join AB.

Step 3 : With B as centre, draw a circle of radius 5 cm.



Step 4 : Construct the perpendicular bisector, PQ of the line segment AB which bisects AB at N.

Step 5 : Along NP, cut off MN = 3.5 cm.

Step 6 : Join B and M.

Step 7 : Draw a perpendicular bisector of BM intersecting BM at C.

Step 8 : Draw a circle with C as centre and radius CM or CB, that intersects the bigger circle at T_1 and T_2.

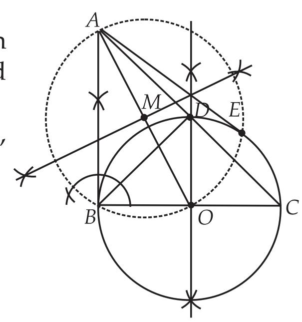
Step 9 : Join MT_1 and MT_2.

Thus, MT_1 and MT_2 are the required tangents.

30. Steps of Construction

Step 1 : Draw a ΔABC such that AB = 5 cm, BC = 12 cm and $\angle B = 90°$.

Step 2 : Draw $BD \perp AC$. Now, bisect BC and let its mid-point be O. With O as centre and OD as radius, draw the circle passing through B, C and D.



Step 3 : Join AO.

Step 4 : Bisect AO. Let M be the mid-point of AO.

Step 5 : Taking M as centre and MA as radius, draw a circle intersecting the given circle at B and E.

Step 6 : Join AB and AE.

Thus, AB and AE are the required two tangents to the given circle from A.

Justification:

Join OE, then $\angle AEO = 90°$ [Angle in a semi-circle]

$\therefore \quad AE \perp OE$

But OE is a radius of the given circle.

$\Rightarrow \quad AE$ has to be a tangent to the circle.

Similarly, AB is also a tangent to the given circle.