Areas Related to Circles

CHAPTER 12

EXAM DRILL

SOLUTIONS

1. **(c) :** Let the radius of the field be r.

Then, $\frac{\pi r^2}{2} = 3850$ [Given] $\Rightarrow \frac{1}{2}\times\frac{22}{7}\times r^2 = 3850$

$\Rightarrow r^2 = 3850 \times \frac{7}{11} = 2450 \Rightarrow r = 35\sqrt{2}$ m

Thus, perimeter of the field

$= \frac{22}{7}(35\sqrt{2}) + 2(35\sqrt{2}) = \sqrt{2}(110 + 70) = 180\sqrt{2}$ m

2. **(a) :** Area of circle, $A = \pi r^2 = \frac{r \times 2\pi r}{2} = \frac{rC}{2}$

$\Rightarrow 2A = rC$

3. **(d) :** Radius of a wheel = 14 cm [Given]

Circumference of the wheel $= 2\times\frac{22}{7}\times 14 = 88$ cm

Distance covered = Circumference of wheel × Number of rotations

= 88 × 50 = 4400 cm

4. **(d) :** Let a be the side of the square and R be the radius of the circle. Then, $a^2 = \pi R^2$ [Given]

$\Rightarrow \frac{R^2}{a^2} = \frac{1}{\pi} \Rightarrow \frac{R}{a} = \frac{1}{\sqrt{\pi}}$

Ratio of perimeters of circle and square $= \frac{2\pi R}{4a}$

$= \frac{R}{2}\times\frac{\pi}{a} = \frac{\pi}{2}\times\frac{1}{\sqrt{\pi}} = \frac{\sqrt{\pi}}{2}$

5. **(c) :** Let r be the radius of a circle.

Now, $2\pi r - 2r = 207$ [Given]

$\Rightarrow 2r(\pi - 1) = 207 \Rightarrow 2r\left(\frac{22}{7} - 1\right) = 207$

$\Rightarrow 2r \times \frac{15}{7} = 207 \Rightarrow r = \frac{207\times 7}{15\times 2} = 48.3$ cm

6. **(a) :** Since the minute hand rotates through 6° in one minute.

$\therefore$ Required area = Area of sector with sector angle 6° and radius 7 cm

$= \frac{6°}{360°}\times\frac{22}{7}\times 7\times 7 = \frac{1}{60}\times 22\times 7 = 2.57 \text{ cm}^2$

7. The distance travelled by a wheel in one revolution is equal to its circumference *i.e.*, πd cm = $\pi(2r)$cm = $2\pi r$ cm [$\because d = 2r$]

8. If $r > 2$, then numerical value of area of a circle is greater than numerical value of its circumference.

9. Perimeter of a semi-circular protractor = Perimeter of a semi-circle = $(2r + \pi r)$ cm

$\therefore 2r + \pi r = 36$ [Given] $\Rightarrow 2r(1 + \pi) = 36$

$\Rightarrow 2r = \frac{36\times 7}{29} = 8.68$ cm = Diameter of protractor

10. Circumference of circle = 44 cm [Given]

$\Rightarrow 2\pi r = 44 \Rightarrow r = \frac{44\times 7}{2\times 22} = 7$ cm

$\therefore$ Area of quadrant of circle $= \frac{1}{4}\times\frac{22}{7}\times 7\times 7 = 38.5 \text{ cm}^2$

11. Here, $\theta = x°$ and radius, $R = 2r$

Area of sector $= \frac{\theta}{360°}\times\pi R^2 = \frac{x°}{360°}\times\pi(2r)^2 = \frac{x°}{90°}\pi r^2$

12. No. Since, diameter of the circle is equal to the breadth of the rectangle.

$\therefore$ The area of the largest circle that can be drawn inside a rectangle is $\pi\left(\frac{b}{2}\right)^2$ cm^2, where $\left(\frac{b}{2}\right)$ is the radius of the circle.

13. Let radius of circle be r and length of arc be l.

Perimeter of a sector of a circle = 24.4 cm [Given]

$\Rightarrow 2r + l = 24.4$

$\Rightarrow 2 \times 4.3 + l = 24.4 \Rightarrow l = 24.4 - 8.6 = 15.8$ cm

Area of sector $= \frac{1}{2}\times lr = \frac{1}{2} \times 15.8 \times 4.3 = 33.97 \text{ cm}^2$

14. Let r be the radius of circle. It is given that

Area of sector of circle $= \frac{3}{18}$ × Area of the same circle

$\Rightarrow \frac{\theta}{360°}\times\pi r^2 = \frac{3}{18}\times\pi r^2 \Rightarrow \theta = \frac{3}{18}\times 360° = 60°$

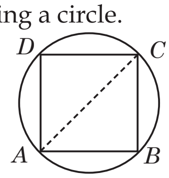
15. Let $ABCD$ be the square circumscribing a circle.

$AC = 2 \times$ Radius of circle $= 2r$ cm

In right ΔABC, $AB^2 + BC^2 = AC^2$

$\Rightarrow 2AB^2 = (2r)^2$ ($\because AB = BC$)

$\Rightarrow AB^2 = \frac{4r^2}{2} = 2r^2 \Rightarrow AB = \sqrt{2}r$ cm



$\therefore$ Perimeter of square $ABCD = 4 \times$ side $= 4\sqrt{2}\,r$ cm

16. Yes. If circumferences of two circles are equal, then their corresponding radii will be equal. So, their areas would be equal.

17. **(i)** **(a) :** Since BOC is the diameter and $\angle BAC = 90°$

$\therefore BC^2 = AB^2 + AC^2$

$= 7^2 + 24^2 = 625$

$\Rightarrow$ $BC = 25$ cm

$\therefore$ Radius of circle $= \frac{25}{2}$ cm $= 12.5$ cm

(ii) (c) : Area of circle $= \pi(12.5)^2 = \frac{22}{7} \times 12.5 \times 12.5$

$= 491.07$ cm^2

(iii) (b) : Clearly, $\angle COD = 90°$

[$\because$ $\angle COB = 180°$ and equal arcs subtends equal angles at the centre]

Area of region $COD = \frac{90°}{360°} \times \pi r^2$

$= \frac{1}{4}(491.07) = 122.76 \text{ cm}^2$

(iv) (d) : Area of $\Delta BAC = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times 7 \times 24 = 84 \text{ cm}^2$

(v) (b) : Area of the polluted region = Area of circle – Area of sector COD – Area of ΔABC

$= 491.07 - 122.76 - 84$

$= 284.31$ cm^2

18. (i) (b) : Let the radius of outer most circle be R.

Outer most circumference = 308 m [Given]

$\Rightarrow$ $2\pi R = 308 \Rightarrow 2 \times \frac{22}{7} \times R = 308$

$\Rightarrow$ $R = \frac{308 \times 7}{2 \times 22} = 49$ m

(ii) (c) : Let the radius of inner most circle be r

Inner most circumference = 264 m [Given]

$\Rightarrow$ $2\pi r = 264$

$\Rightarrow$ $2 \times \frac{22}{7} \times r = 264 \Rightarrow r = \frac{264 \times 7}{2 \times 22} = 42$ m

(iii) (a) : Width of the track = Radius of outer most track – Radius of inner most track = 49 – 42 = 7 m

(iv) (d) : Area of the race track = Area of outer circle – Area of inner circle

$= \pi(R^2 - r^2) = \pi [(49)^2 - (42)^2]$

$= \frac{22}{7}[2401 - 1764] = 2002$ m^2

(v) (b) : Cost of painting the whole race track $= ₹ (6 \times 2002) = ₹ 12012$.

19. (i) (a) : Area of sector $ODCO = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154$ cm^2

(ii) (b) : Area of $\Delta AOB = \frac{1}{2} \times OA \times OB = \frac{1}{2}(20 \times 20)$

$= 200$ cm^2

(iii) (a) : Area of region which is golden plated

= area of ΔOAB – area of sector $ODCO$.

$= 200 - 154 = 46$ cm^2

$\therefore$ Total cost of golden plating $= ₹ (6 \times 46) = ₹ 276$

(iv) (c) : Area of major sector = area of circle – area of minor sector

$= \pi r^2 - \frac{1}{4}\pi r^2 = \frac{3\pi r^2}{4} = \frac{3}{4} \times \frac{22}{7} \times 14 \times 14 = 462$ cm^2

(v) (d) : Length of arc $DC = \frac{90°}{360°} \times 2 \times \frac{22}{7} \times 14$

$= 22$ cm

20. (i) (c) : Area of the region containing blue colour

$= \frac{22}{7} \times 5 \times 5 \times \frac{80°}{360°} - \frac{22}{7} \times 3 \times 3 \times \frac{80°}{360°}$

$= \frac{22}{7} \times \frac{2}{9} \times [25 - 9] = \frac{44}{63}(16) = 11.17$ cm^2

(ii) (b) : Area of the region containing green colour

$= \frac{22}{7} \times \frac{60°}{360°}[5 \times 5 - 3 \times 3] = \frac{22}{7} \times \frac{1}{6} \times 16 = 8.38$ cm^2

(iii) (d) : Perimeter of the region containing red colour

= 2 + 2 + length of arc of sector having radius 3 cm + length of arc of sector having radius 5 cm

$= 4 + 2 \times \frac{22}{7} \times 3 \times \frac{20°}{360°} + 2 \times \frac{22}{7} \times 5 \times \frac{20°}{360°}$

$= 4 + \frac{44}{7} \times \frac{1}{18} \times 8 = 4 + \frac{176}{63} = 4 + 2.79 = 6.79$ cm

(iv) (a) : Required area $= \frac{22}{7} \times 3 \times 3 \times \frac{160°}{360°}$

$= \frac{88}{7} = 12.57$ cm^2

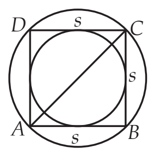
(v) (a) : Angle of given sector = 80° + 60° + 20° = 160°

Thus, the given region represents minor sector of a circle.

21. In ΔABC, by Pythagoras theorem

$AC^2 = AB^2 + BC^2 = s^2 + s^2 = 2s^2$

$\Rightarrow$ $AC = s\sqrt{2}$



$\therefore$ Radius of larger circle, $R = \frac{s\sqrt{2}}{2}$

$\therefore$ Area of larger circle $= \pi R^2 = \pi\left(\frac{s\sqrt{2}}{2}\right)^2 = \frac{\pi s^2}{2}$

Radius of smaller circle, $r = \frac{s}{2}$

Area of smaller circle $= \pi r^2 = \pi\left(\frac{s}{2}\right)^2 = \frac{\pi s^2}{4}$

$\therefore$ Ratio of areas $= \dfrac{\pi s^2/4}{\pi s^2/2} = \dfrac{1}{2} = 1:2$

22. We know, that number of revolutions made by wheel $= \dfrac{\text{Distance covered}}{\text{Circumference of wheel}}$

$\Rightarrow \quad 400 = \dfrac{16.72 \text{ km}}{\text{Circumference of wheel}}$

$\Rightarrow$ Circumference of wheel $= \dfrac{16.72}{400}$ km

$\Rightarrow \quad 2\pi r = \left(\dfrac{16.72}{400} \times 1000 \times 100\right)$ cm

$\Rightarrow \quad \dfrac{2 \times 22}{7} \times r = \dfrac{1672 \times 10}{4} \quad \Rightarrow 2r = 1330$

$\therefore$ Diameter of wheel = 1330 cm.

23. Let R and r be the radius of outer and inner circle respectively.

Given, $2\pi R = 88$ cm and $2\pi r = 66$ cm

$\Rightarrow \quad R = \dfrac{88}{2\pi}$ cm and $r = \dfrac{66}{2\pi}$ cm

Width of the ring $= R - r = \dfrac{88}{2\pi} - \dfrac{66}{2\pi}$

$= \dfrac{1}{2} \times \dfrac{7}{22} \times 22 = 3.5$ cm

24. Given, $r_1 = 4$ cm and $r_2 = 3$ cm

Let r be the radius of new circle.

Area of first circle, $A_1 = \pi r_1^2 = \pi(4)^2 = 16\,\pi$ cm^2

Area of second circle, $A_2 = \pi r_2^2 = \pi(3)^2 = 9\,\pi$ cm^2

Area of new circle $= A_1 + A_2$ [Given]

$\Rightarrow \quad \pi r^2 = 16\pi + 9\pi \quad \Rightarrow \quad \pi r^2 = 25\pi$

$\Rightarrow \quad r^2 = 25 \quad \Rightarrow \quad r = 5$ cm

[Since radius cannot be negative so, $r \neq -5$]

Hence, the radius of new circle is 5 cm.

25. Here, sector angle, $\theta = 90°$, $r = 14$ m

Area of the field graze by the horse, $A_1 = \dfrac{\theta}{360°} \times \pi r^2$

$= \dfrac{90°}{360°} \times 3.14 \times (14)^2$

$= \dfrac{3.14 \times 196}{4} = 153.86$ cm^2

Now, area of field $= (25)^2 = 625$ m^2

$\therefore$ Area of field not grazed by horse = Area of field $- A_1$

$= 625 - 153.86 = 471.14$ m^2

26. Since, ΔABC is an equilateral triangle.

$\therefore \angle A = \angle B = \angle C = 60°$ and $AB = BC = CA = 10$ cm

E, F and D are mid-points of the given sides.

$\therefore \quad AE = EC = CD = DB = BF = FA = 5$ cm

Radius of a sector $(r) = 5$ cm

Now, area of sector CDE $= \dfrac{60°}{360°} \times 3.14 \times (5)^2$

$= \dfrac{78.5}{6} = 13.0833$ cm^2

$\therefore$ Area of shaded region = 3 × (Area of sector CDE)

$= (3 \times 13.0833)$ cm^2 $= 39.25$ cm^2

27. Let R and r be the outer and inner radius of circle respectively.

Area of circular park = 1386 m^2 [Given]

$\Rightarrow \quad \pi r^2 = 1386 \quad \Rightarrow \quad r^2 = 441 \quad \Rightarrow \quad r = 21$ m

Now, cost to construct a track around a circular park at the rate of ₹ 3.50 per m^2 is ₹ 4440

$\therefore$ Area of track $= \dfrac{4440}{3.5}$ m^2

$\Rightarrow \quad \pi R^2 - \pi r^2 = \dfrac{4440}{3.5} \quad \Rightarrow \quad \dfrac{22}{7} \times R^2 - 1386 = \dfrac{4440}{3.5}$

$\Rightarrow \quad \dfrac{22}{7} \times R^2 = \dfrac{4440}{3.5} + 1386 = \dfrac{4440 + 4851}{3.5} = \dfrac{9291}{3.5}$

$\Rightarrow \quad R^2 = \dfrac{9291}{3.5} \times \dfrac{7}{22} = \dfrac{18582}{22} = 844.63$

$\Rightarrow \quad R = 29.06$ cm

$\therefore$ Width of track $= R - r = 29.06 - 21 = 8.06$ cm

28. Let $OB = r = AO$

Perimeter of APB + Perimeter of $AQOB$ = 40

$\Rightarrow \quad \pi r + \dfrac{1}{2}\pi r + r = 40$

$\Rightarrow r\left(\dfrac{3}{2}\pi + 1\right) = 40 \Rightarrow r\left(\dfrac{3}{2} \times \dfrac{22}{7} + 1\right) = 40$

$\Rightarrow \quad r\left(\dfrac{33}{7} + 1\right) = 40 \quad \Rightarrow \quad 40r = 280 \quad \Rightarrow \quad r = 7$ cm

$\therefore$ Shaded area $= \dfrac{\pi(r/2)^2}{2} + \dfrac{\pi r^2}{2}$

$= \dfrac{1}{2} \times \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2} + \dfrac{1}{2} \times \dfrac{22}{7} \times 7 \times 7$

$= \dfrac{77}{4} + 77 = \dfrac{77 + 308}{4} = \dfrac{385}{4} = 96.25$ cm^2

29. Area of region $ABDC$ = (Area of sector OAB) – (Area of sector OCD)

$= \dfrac{\pi(OA)^2 \times 60°}{360°} - \dfrac{\pi(OC)^2 \times 60°}{360°}$

$= \dfrac{\pi}{6} \times (42)^2 - \dfrac{\pi}{6} \times (21)^2 \quad [\because OA = 42 \text{ cm and } OC = 21 \text{ cm}]$

$= \dfrac{\pi}{6}(1764 - 441) = \dfrac{1323\pi}{6}$ cm^2

Area of circular ring $= \pi[(42)^2 - (21)^2]$

$= \pi(1764 - 441) = 1323\,\pi$ cm^2

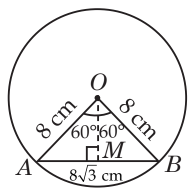
$\therefore$ Area of shaded region = Area of circular ring – Area of region $ABDC$

$= 1323\pi - \dfrac{1323\pi}{6} = 1323\pi\left(1 - \dfrac{1}{6}\right)$

$= 1323 \times \dfrac{22}{7} \times \dfrac{5}{6} = 3465$ cm^2

30. Given diameter of circle = 16 cm

$\therefore$ Radius of circle $= \dfrac{16}{2} = 8$ cm



Draw, $OM \perp AB$

Then, OM bisects the side AB and also bisects $\angle AOB$ $(\because OA = OB = 8 \text{ cm})$

$\Rightarrow AM = BM = \frac{1}{2}AB = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$ cm

In right ΔAOM, $\sin\angle AOM = \frac{AM}{OA} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \angle AOM = 60°$ $\quad \left(\because \sin 60° = \frac{\sqrt{3}}{2}\right)$

$\therefore \quad \angle AOB = 2\angle AOM = 2 \times 60° = 120°$

and $\cos 60° = \frac{OM}{OA} \Rightarrow \frac{1}{2} = \frac{OM}{8} \Rightarrow OM = 4$ cm

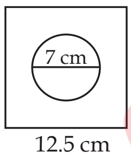
Area of minor segment = Area of sector AOB – Area of ΔAOB

$= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} \times AB \times OM$

$= \frac{120°}{360°} \times \frac{22}{7} \times 8 \times 8 - \frac{1}{2} \times 8\sqrt{3} \times 4$

$= 67.04 - 16 \times 1.732 = 39.328$ cm^2

31. Diameter of disc = 7 cm



$\therefore$ Radius of disc, $r = \frac{7}{2}$ cm

Area of disc = $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{11 \times 7}{2} = 38.5$ cm^2

Area of aluminium plate = (side)^2 = $(12.5)^2$ = 156.25 cm^2

Area of remaining part = Area of aluminium plate – Area of disc

= 156.25 – 38.5 = 117.75

Now, weight of 1 cm^2 plate = 0.8 g

$\therefore$ Weight of 117.75 cm^2 plate = 0.8 × 117.75 = 94.2 g

32. AB is the diameter of the circle, $\angle ACB = 90°$

[$\because$ Angle in a semi-circle is 90°]

In right ΔACB,

$AC^2 + BC^2 = AB^2$ [By Pythagoras theorem]

$\Rightarrow 15^2 + 8^2 = AB^2 \Rightarrow 225 + 64 = AB^2$

$\Rightarrow AB = \sqrt{289} = 17$ cm

$\therefore$ Radius of circle, $r = \frac{AB}{2} = \frac{17}{2}$ cm

Area of $\Delta ACB = \frac{1}{2} \times BC \times AC = \frac{1}{2} \times 8 \times 15 = 60$ cm^2

Area of ΔABD= 60 cm^2 ($\because \Delta ACB$ is congruent to ΔABD)

Area of circle = $\pi r^2 = 3.14 \times \frac{17}{2} \times \frac{17}{2} = 226.865$ cm^2

$\therefore$ Area of shaded region = Area of the circle – (Area of ΔACB + Area of ΔABD)

= 226.865 – 2 × 60 = 106.87 cm^2

OR

Here, $\angle AOB = 90°$

$\therefore$ ΔAOB is a right angled triangle.

$OB = OA = 10.5 + 3.5 = 14$ cm

Area of $\Delta AOB = \frac{1}{2} \times OB \times OA = \frac{1}{2} \times 14 \times 14 = 98$ cm^2

Area of sector $ODC = \frac{\theta}{360°} \times \pi r^2$

$= \frac{90°}{360°} \times \frac{22}{7} \times 10.5 \times 10.5$

$= \frac{1}{4} \times 22 \times 1.5 \times 10.5 = 86.625$ cm^2

Area of shaded part = Area of ΔAOB – Area of sector ODC

= 98 – 86.625 = 11.375 cm^2

Cost of silver plated of 1 cm^2 shaded part = ₹ 50

$\therefore$ Cost of silver plated of 11.375 cm^2 shaded part = ₹ (50 × 11.375) = ₹ 568.75

Total cost awarded by school = 15 × 568.75 = ₹ 8531.25

33. Let height of trapezium = h cm

Area of trapezium = $\frac{1}{2}$(Sum of parallel sides) × height

$\therefore \frac{1}{2}(10+4) \times h = 24.5$ [Given]

$\Rightarrow (14)h = 49 \Rightarrow h = \frac{49}{14} = 3.5$ cm

$\therefore$ Radius of sector ABE = 3.5 cm

Area of sector $ABE = \frac{\theta}{360°} \times \pi r^2$

$= \frac{90°}{360°} \times \frac{22}{7} \times 3.5 \times 3.5 = \frac{1}{4} \times 22 \times 0.5 \times 3.5 = 9.625$ cm^2

$\therefore$ Area of shaded region = Area of trapezium – Area of sector ABE

= (24.5 – 9.625) cm^2 = 14.875 cm^2

34. We know that tangent to a circle is perpendicular to radius.

$\therefore \quad OA \perp AP \Rightarrow \angle OAP = 90°$

In right ΔOAP,

$AP^2 = OP^2 - OA^2$ [By Pythagoras theorem]

$\Rightarrow (AP)^2 = (10)^2 - (5)^2 = 100 - 25 = 75$

$\Rightarrow AP = 5\sqrt{3}$ cm

$\therefore$ Area of ΔAOP

$= \frac{1}{2} \times OA \times AP = \frac{1}{2} \times (5) \times (5\sqrt{3}) = \frac{25\sqrt{3}}{2}$ cm^2

In ΔAOP and ΔBOP,

$AO = BO$ [Radii of circle]

$\angle OAP = \angle OBP$ [Each 90°]

$OP = OP$ [Common]

$\therefore \quad \Delta AOP \cong \Delta BOP$ [By RHS congruency criteria]

$\Rightarrow \quad \angle AOP = \angle BOP$ [By CPCT] ...(i)

$\Rightarrow$ Area of $\Delta BOP = \frac{25\sqrt{3}}{2}$ cm^2

In right ΔAOP, $\cos\theta = \frac{OA}{OP}$

$\Rightarrow \cos\theta = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = 60° = \angle BOP$ [From (i)]

$\therefore \quad \angle AOB = 120°$

Hence, area of sector $OACB$

$= \frac{120°}{360°} \times 3.14 \times 5 \times 5 = 26.17 \text{ cm}^2$

$\therefore$ Area of shaded region = Area of $(\Delta AOP + \Delta BOP)$ – Area of sector $OACB$

$= (25\sqrt{3} - 26.17) = (25 \times 1.73 - 26.17) = 17.08 \text{ cm}^2$

Length of belt which is still in contact with the pulley

$= \frac{240°}{360°} \times 2\pi(5)$ [$\because$ Sector angle = 360° – 120° = 240°]

$= \frac{2}{3} \times 2 \times 3.14 \times 5 = 20.93$ cm

35. Length of arc $PA = \frac{\theta}{360°} \times 2\pi r$

In ΔOAB, $\tan\theta = \frac{AB}{OA} \Rightarrow AB = r\tan\theta$ ...(i) $(\because OA = r)$

Now, $\sec\theta = \frac{BO}{r} \Rightarrow BO = r\sec\theta$

Length of $BP = OB - OP = r\sec\theta - r$

So, perimeter of shaded region = Length of arc $PA + AB + BP$

$= \frac{\theta}{360°} \times 2\pi r + r\tan\theta + r\sec\theta - r = r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180°} - 1\right]$

Area of sector OPA with sector angle, $\theta = \frac{\theta}{360°} \times \pi r^2$

Area of $\Delta OAB = \frac{1}{2} \times OA \times AB$

$= \frac{1}{2} \times r \times r\tan\theta = \frac{1}{2}r^2\tan\theta$

Area of shaded region = Area of ΔOAB – Area of sector OPA

$= \frac{1}{2}r^2\tan\theta - \frac{\theta}{360°}\pi r^2$

$= \frac{1}{2}r^2\left(\tan\theta - \frac{\pi\theta}{180°}\right)$

OR

Area of the square lawn $ABCD = (56 \times 56) \text{ m}^2$ ...(i)

Let $OA = OB = x$ m

In ΔAOB,

$x^2 + x^2 = 56^2$ [By Pythagoras theorem]

$\Rightarrow 2x^2 = 56 \times 56 \Rightarrow x^2 = 28 \times 56$ ...(ii)

Now, area of sector $OAB = \frac{90°}{360°} \times \pi x^2 = \frac{1}{4} \times \pi x^2$

$= \left(\frac{1}{4} \times \frac{22}{7} \times 28 \times 56\right) \text{ m}^2$ [From (ii)] ...(iii)

Also, area of $\Delta OAB = \left(\frac{1}{2} \times 28 \times 56\right) \text{ m}^2$ $(\because \angle AOB = 90°)$ ...(iv)

So, area of flower bed AB

$= \left(\frac{1}{4} \times \frac{22}{7} \times 28 \times 56 - \frac{1}{2} \times 28 \times 56\right)\text{m}^2$ [From (iii) and (iv)]

$= \frac{1}{4} \times 28 \times 56\left(\frac{22}{7} - 2\right) = \left(\frac{1}{4} \times 28 \times 56 \times \frac{8}{7}\right)\text{m}^2$ ...(v)

Similarly, area of the other flower bed

$= \left(\frac{1}{4} \times 28 \times 56 \times \frac{8}{7}\right)\text{m}^2$ ...(vi)

Therefore, total area

$= \left(56 \times 56 + \frac{1}{4} \times 28 \times 56 \times \frac{8}{7} + \frac{1}{4} \times 28 \times 56 \times \frac{8}{7}\right)\text{m}^2$

[From (i), (v) and (vi)]

$= \left[28 \times 56\left(2 + \frac{2}{7} + \frac{2}{7}\right)\right]\text{m}^2 = 4032 \text{ m}^2$

36. Perimeter of the shaded region = Length of $\overparen{APB}$ + Length of $\overparen{ARC}$ + Length of $\overparen{CQD}$ + Length of $\overparen{DSB}$

Now, perimeter of $\overparen{APB}$ = Perimeter of $\overparen{CQD}$

$= \frac{1}{2} \times 2\pi\left(\frac{7}{2}\right) = \frac{22}{7} \times \frac{7}{2} = 11$ cm

Perimeter of $\overparen{ARC}$ = Perimeter of $\overparen{DSB}$

$= \frac{1}{2} \times 2\pi(7) = \frac{22}{7} \times 7 = 22$ cm

Thus, perimeter of the shaded region

$= 2 \times (11) + 2 \times (22) = 66$ cm

OR

In right ΔABC, $AC^2 = AB^2 + BC^2$

$\Rightarrow AC = \sqrt{(6)^2 + (6)^2} = 6\sqrt{2}$ cm

$\therefore$ Radius of the circle, $\frac{6\sqrt{2}}{2} = 3\sqrt{2}$ cm

(i) The sum of the areas of the two designed segments made by the chords AB and BC

= (Area of the semi-circle in which ΔABC is inscribed) – (Area of ΔABC)

$= \left(\frac{1}{2}\pi \times (3\sqrt{2})^2\right) - \left(\frac{1}{2} \times 6 \times 6\right)$

$= \left(\frac{1}{2} \times 3.14 \times 18\right) - 18 = 28.26 - 18 = 10.26 \text{ cm}^2$

(ii) The area of the designed segment made by the chord PQ = Area of sector OPQ – Area of ΔOPQ

$= \frac{90°}{360°} \times \pi r^2 - \frac{1}{2} \times OP \times OQ$

$= \frac{1}{4} \times 3.14 \times (3\sqrt{2})^2 - \frac{1}{2} \times 3\sqrt{2} \times 3\sqrt{2}$

$= 14.13 - 9 = 5.13 \text{ cm}^2$

(iii) Now, area of the total designed part

$= (10.26 + 5.13) \text{ cm}^2 = 15.39 \text{ cm}^2$

$\therefore$ Total cost of making the designs at the rate of ₹ 10.25 per cm^2 = ₹ (10.25 × 15.39) = ₹ 157.75