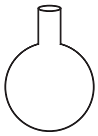
Surface Areas and Volumes

CHAPTER 13

EXAM DRILL

SOLUTIONS

1. (a) : The shape of Surahi is as



, which is combination of sphere and cylinder.

2. (b) : Let the radius of the cone be r cm.

Since, the height and diameter of the base of the largest right circular cone = Edge of the cube.

$\therefore \quad 2r = 8$ cm $\Rightarrow r = 4$ cm

3. (a) : The radius of the greatest sphere that can be cut off from the cylinder = 1 cm

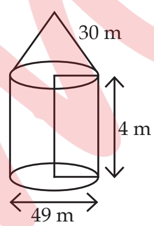
$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi \text{ cm}^3$

4. (c) : Volume of a cone : Volume of a hemisphere : Volume of a cylinder

$= \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h = \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 \quad (\because r = h)$

$= 1 : 2 : 3$

5. (a) : Area of canvas required = Curved surface area of cylinder + Curved surface area of cone



$= 2\pi rh + \pi rl$

$= \pi r[2h + l] = \frac{22}{7} \times \frac{98}{2}[2 \times 4 + 30]$

$= 5852 \text{ m}^2$

6. (b) : Given, diameter of cylinder = 16 cm

$\therefore$ Radius of cylinder (r) = 8 cm

Height of cylinder (h) = 2 cm

Volume of the cylinder $= \pi r^2 h = \pi \cdot (8)^2 \cdot 2 = 128\pi \text{ cm}^3$

Let R be the radius of sphere.

Volume of one sphere $= \frac{\text{Volume of the cylinder}}{12}$

$\Rightarrow \frac{4}{3}\pi R^3 = \frac{128}{12}\pi \Rightarrow R^3 = 8 \Rightarrow R = 2$ cm

Hence, radius of each sphere is 2 cm.

7. When, we join two solid hemispheres along their bases of radius r, we get a solid sphere. Also, curved surface area of a hemisphere is $2\pi r^2$.

Hence, the curved surface area of new solid $= 2\pi r^2 + 2\pi r^2 = 4\pi r^2$.

8. Since sphere is recast into right circular cylinder.

$\therefore$ Volume of cylinder = Volume of sphere

$\Rightarrow \quad \pi r^2 h = \pi$

where r and h are radius of base and height of cylinder

$\Rightarrow \quad (0.5)^2 h = 1 \qquad [\because r = 0.5 \text{ cm (Given)}]$

$\Rightarrow \quad \left(\frac{1}{2}\right)^2 h = 1 \Rightarrow h = 4$ cm

9. Given, radius of spherical bullet = 1 mm = $\frac{1}{10}$ cm

Radius of cylinder (r) = 4 cm

Height of cylinder (h) = 6 cm

Number of spherical bullets

$= \frac{\text{Volume of cylinder}}{\text{Volume of a spherical bullet}} = \frac{\pi(4)^2 \times 6}{\frac{4}{3}\pi\left(\frac{1}{10}\right)^3}$

$= 4 \times 3 \times 6 \times (10)^3 = 72000$

10. Here, radius of two circular ends of frustum are $r_1 = 20$ cm, $r_2 = 17$ cm and height $h = 4$ cm

Slant height of frustum (l) $= \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{4^2 + (20-17)^2} = \sqrt{16+9} = \sqrt{25} = 5$ cm

11. Let the rainfall be x m.

Now, volume of water on roof = volume of cone

$\Rightarrow \quad 44 \times 10 \times x = \frac{1}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 7$

$\Rightarrow \quad x = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{1}{44} \times \frac{1}{10}$

$\Rightarrow \quad x = \frac{1}{240} \text{ m} = \frac{1}{240} \times 100 \text{ cm} = \frac{5}{12} \text{ cm}$

Hence, required rainfall is 5/12 cm.

12. Let r be radius of sphere.

Since, cone is recast into a sphere.

$\therefore$ Volume of sphere = Volume of cone

$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{1}{3}\pi \times 7 \times 7 \times 28 \quad \Rightarrow \quad r^3 = \frac{7 \times 7 \times 28}{4}$

$\Rightarrow \quad r^3 = 343 \Rightarrow r = 7$ cm

$\therefore$ Radius of the sphere = 7 cm

13. (i) (b) : Lateral surface area of *Hermika* which is cubical in shape $= 4a^2 = 4 \times (8)^2 = 256 \text{ m}^2$

(ii) (a) : Diameter of cylindrical base = 42 m

$\therefore$ Radius of cylindrical base (r) = 21 m

Height of cylindrical base (h) = 12 m

$\therefore$ Number of bricks used $= \frac{\frac{22}{7} \times 21 \times 21 \times 12}{0.01}$

$= 1663200$

(iii) (c) : Given, diameter of *Anda* which is hemispherical in shape = 42 m

$\Rightarrow$ Radius of *Anda* (r) = 21 m

$\therefore$ Volume of *Anda* $= \frac{2}{3}\pi r^3 = \frac{2}{3}\times\frac{22}{7}\times 21\times 21\times 21$

$= 44 \times 21 \times 21 = 19404 \text{ m}^3$

(iv) (d) : Given, radius of *Pradakshina Path* (r) = 25 m

$\therefore$ Perimeter of *path* = $2\pi r$

$= \left(2\times\frac{22}{7}\times 25\right)$ m

$\therefore$ Distance covered by priest $= 14\times 2\times\frac{22}{7}\times 25$

= 2200 m

(v) (b) : $\because$ Radius of *Anda* (r) = 21 m

$\therefore$ Curved surface area of *Anda* = $2\pi r^2$

$= 2\times\frac{22}{7}\times 21\times 21 = 2772 \text{ m}^2$

14. We have, radius of each coin = 3.5 cm

$= \frac{35}{10}\text{cm} = \frac{7}{2}\text{cm}$

Thickness of each coin = 0.5 cm $= \frac{1}{2}\text{cm}$

So, height of cylinder made by Meera(h_1) $= 12\times\frac{1}{2} = 6$ cm

and height of cylinder made by Dhara (h_2)

$= 8\times\frac{1}{2} = 4$ cm

(i) (b) : Curved surface area of cylinder made by Meera

$= 2\times\frac{22}{7}\times\frac{7}{2}\times 6 = 132 \text{ cm}^2$

(ii) (b) : Required ratio

$= \frac{\text{Curved surface area of cylinder made by Meera}}{\text{Curved surface area of cylinder made by Dhara}}$

$= \frac{2\pi r h_1}{2\pi r h_2} = \frac{h_1}{h_2} = \frac{6}{4} = \frac{3}{2}$ *i.e.*, 3 : 2

(iii) (a) : Volume of cylinder made by Dhara = $\pi r^2 h_2$

$= \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 4 = 154 \text{ cm}^3$

(iv) (c) : Required ratio

$= \frac{\text{Volume of cylinder made by Meera}}{\text{Volume of cylinder made by Dhara}}$

$= \frac{\pi r^2 h_1}{\pi r^2 h_2} = \frac{h_1}{h_2} = \frac{6}{4} = \frac{3}{2}$ *i.e.*, 3 : 2

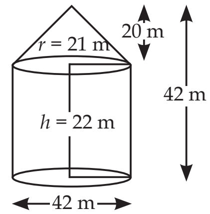
(v) (a) : When two coins are shifted from Meera's cylinder to Dhara's cylinder, then length of both cylinders become equal.
So, volume of both cylinders become equal.

15. (i) (c) : Required area of canvas = Curved surface area of cone + Curved surface area of cylinder

$= \pi r l + 2\pi r h = \pi r(l + 2h)$

$= \frac{22}{7}\times 21\,(29 + 44) = 4818 \text{ m}^2$

$\left[\because\ l = \sqrt{r^2 + h_1^2} = \sqrt{(21)^2 + (20)^2} = \sqrt{841} = 29 \text{ m}\right]$



(ii) (b): Area of floor = πr^2

$= \frac{22}{7}\times 21\times 21 = 1386 \text{ m}^2$

Number of persons that can be accommodated in the tent $= \frac{1386}{126} = 11$

(iii) (d): Since, cost of 100 m^2 of canvas = ₹ 425

$\therefore$ Cost of 1 m^2 of canvas = ₹ 4.25

Thus, cost of 4818 m^2 of canvas = ₹ 20476.50

(iv) (c) : Volume of tent = Volume of cone + Volume of cylinder $= \frac{1}{3}\pi r^2 h_1 + \pi r^2 h = \pi r^2\left(\frac{1}{3}h_1 + h\right)$

$= \frac{22}{7}\times(21)^2\left[\frac{20}{3} + 22\right] = \frac{9702}{7}\times\frac{86}{3} = 39732 \text{ m}^3$

(v) (a) : Required number of persons

$= \frac{\text{Volume of tent}}{\text{Space required by one person}} = \frac{39732}{1892} = 21$

16. (i) (b): Curved surface area of two identical cylindrical parts = $2\times 2\pi r h$ = $2\times 2\times\frac{22}{7}\times\frac{2.5}{2}\times 5$

$= 78.57 \text{ cm}^2$

(ii) (a) : Volume of big cylindrical part = $\pi r^2 h$

$= \frac{22}{7}\times\frac{4.5}{2}\times\frac{4.5}{2}\times 12 = 190.93 \text{ cm}^3$

(iii) (b): Volume of two hemispherical ends $= 2\times\frac{2}{3}\pi r^3$

$= \frac{2\times 2}{3}\times\frac{22}{7}\times\left(\frac{2.5}{2}\right)^3 = 8.18 \text{ cm}^3$

(iv) (c) : Curved surface area of two hemispherical ends

$= 2\times 2\pi r^2 = 2\times 2\times\frac{22}{7}\times\frac{2.5}{2}\times\frac{2.5}{2} = 19.64 \text{ cm}^2$

(v) (b): Difference of volume of bigger cylinder to two small hemispherical ends = 190.93 – 8.18 = 182.75 cm^3

17. Radius of the sphere (r) $= \frac{18}{2}\text{ cm} = 9$ cm

Radius of the cylinder (R) $= \frac{36}{2}\text{ cm} = 18$ cm

Let us assume that the water level in the cylinder rises by h cm.

After the sphere is completely submerged,

Volume of the sphere = Volume of water raised in the cylinder

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi R^2 h \Rightarrow \frac{4}{3}\pi(9)^3 = \pi(18)^2 \times h$$

$$\Rightarrow h = \frac{4\times9\times9\times9}{3\times18\times18} \Rightarrow h = 3$$

Thus, the water level in the cylinder rises by 3 cm.

18. Let h be the height of the frustum of the cone.

Let r_1 and r_2 be radii of the ends of the frustum.

$\therefore$ $r_1 - r_2 = 4$ cm

Slant height (l) = 5 cm

$\because$ The slant height of the frustum of a cone is given by,

$l^2 = h^2 + (r_1 - r_2)^2$

$\Rightarrow$ $h^2 = l^2 - (r_1 - r_2)^2$

$\Rightarrow$ $h^2 = (5)^2 - (4)^2 = 9$

$\Rightarrow$ $h = 3$ cm

Hence, height of frustum is 3 cm.

19. Volume of one cube = 64 cm^3

$\Rightarrow$ (Edge)^3 = 64 cm^3 $\Rightarrow$ Edge = 4 cm

Length of the cuboid (l) = 5 × Edge = 5 × 4 = 20 cm

breadth (b) = 4 cm and height (h) = 4 cm

$\therefore$ Surface area of cuboid = $2(lb + bh + hl)$

= 2[20 × 4 + 4 × 4 + 4 × 20] = 2 × 176 = 352 cm^2

Volume of the cuboid = $l \times b \times h$

= 20 × 4 × 4 = 320 cm^3

20. Let height of cylinder be h cm and radius be r cm.

Given, $h + r = 37$ cm

Total surface area of cylinder = $2\pi rh + 2\pi r^2$

$\Rightarrow$ $2\pi r(h + r) = 1628 \Rightarrow 2\pi r(37) = 1628$

$$\Rightarrow r = \frac{1628\times7}{2\times22\times37} = 7$$

$\therefore$ $h + r = 37 \Rightarrow h + 7 = 37 \Rightarrow h = 30$

Hence, volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7}\times7\times7\times30 = 4620 \text{ cm}^3$$

21. Here, radius of conical vessel (r) = 5 cm

Height of conical vessel (h) = 24 cm

Radius of the cylindrical vessel (R) = 10 cm

Let H cm be the height to which water rises in cylindrical vessel.

According to the question,

Volume of water in cylinder = Volume of water in cone

$$\Rightarrow \pi R^2 H = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \pi\times10\times10\times H = \frac{1}{3}\pi\times5^2\times24$$

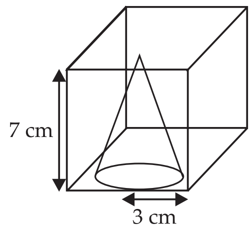
$$\therefore H = \frac{25\times8}{10\times10} = 2 \text{ cm}$$

Hence, the height to which the water will rise in the cylindrical vessel is 2 cm.

22. Given that, side of a solid cube (a)= 7 cm

Height of conical cavity (h) = 7 cm

Radius of conical cavity (r) = 3 cm



Now, volume of cube

= $a^3 = (7)^3 = 343$ cm^3

Volume of conical cavity

$$= \frac{1}{3}\pi\times r^2\times h = \frac{1}{3}\times\frac{22}{7}\times3\times3\times7$$

= 66 cm^3

$\therefore$ Volume of remaining solid

= Volume of cube – Volume of conical cavity

= 343 – 66 = 277 cm^3

23. Here, radius of cylindrical portion (r) = Radius of conical portion (r) $= \frac{105}{2}$ m

Height of cylindrical portion (h) = 3 m

Slant height of conical portion (l) = 53 m

Total canvas used in making the tent

= Curved surface area of cylindrical portion + Curved surface area of conical portion

$$= 2\pi rh + \pi rl = 2\times\frac{22}{7}\times\frac{105}{2}\times3 + \frac{22}{7}\times\frac{105}{2}\times53$$

= 990 + 8745 = 9735 m^2

24. Radius of the hemispherical bowl (r) = 9 cm

$\therefore$ Volume of the water in hemispherical bowl

$$= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(9)^3 \text{ cm}^3$$

Let height of water in the cylindrical vessel be h cm.

Also, radius of the cylinder (R) = 6 cm

$\therefore$ Volume of water in the cylindrical vessel = $\pi R^2 h$

= $\pi(6)^2 h$ cm^3

$\because$ Volume of water in cylindrical vessel = Volume of the water in hemispherical bowl

$$\Rightarrow \pi(6)^2 h = \frac{2}{3}\pi(9)^3 \Rightarrow h = \frac{2\times(9)^3}{3\times(6)^2}$$

$$\Rightarrow h = \frac{27}{2} \Rightarrow h = 13.5$$

Hence, height of water in cylindrical vessel is 13.5 cm

25. Given, radius of sphere (r) = 6 cm

$\therefore$ Volume of the sphere

$$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = 288\,\pi \text{ cm}^3 \quad \ldots\text{(i)}$$

Let the internal radius of cylinder be r cm.

External radius of cylinder (R) = 5 cm

Height of cylinder (h) = 32 cm

$\therefore$ Volume of hollow cylinder

$= \pi(R^2 - r^2)h = \pi(5^2 - r^2)32$ ...(ii)

According to the question,

Volume of the hollow cylinder = Volume of sphere

$\Rightarrow$ $32(25 - r^2)\pi = 288\pi$ [Using (i) and (ii)]

$$\Rightarrow (25 - r^2) = \frac{288}{32} \Rightarrow 25 - r^2 = 9$$

$\Rightarrow$ $r^2 = 16 \Rightarrow r = 4$ cm

$\therefore$ Uniform thickness of the cylinder $= R - r$

$= 5 - 4 = 1$ cm

26. Height of the bucket $(h) = 15$ cm

Radius of one end of bucket $(R) = 14$ cm

Radius of the other end of bucket $= r$ cm

Volume of the bucket $= 5390\text{ cm}^3$

$$\Rightarrow \frac{1}{3}\pi(R^2 + r^2 + Rr)h = 5390$$

$$\Rightarrow \frac{1}{3}\times\frac{22}{7}\times(14^2 + r^2 + 14r)\times 15 = 5390$$

$$\Rightarrow 196 + r^2 + 14r = \frac{5390\times 7}{22\times 5} = 343$$

$\Rightarrow r^2 + 14r - 147 = 0 \Rightarrow r^2 + 21r - 7r - 147 = 0$

$\Rightarrow r(r + 21) - 7(r + 21) = 0$

$\Rightarrow (r + 21)(r - 7) = 0 \quad \Rightarrow \quad r = 7, -21$

$\therefore \quad r = 7$ $\quad$ ($\because$ Radius can't be negative)

Hence, radius of other end of bucket is 7 cm.

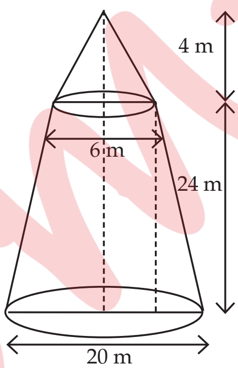
27. Given, diameter of the base and top of frustum are 20 m and 6 m respectively.

$\therefore$ Radius of the base of frustum $(r_1) = 10$ m and

Radius of the top of frustum (r_2) = radius of the base of cone $(r_2) = 3$ m

Height of the frustum $(h) = 24$ cm

Slant height of frustum (l)



$$= \sqrt{h^2 + (r_1 - r_2)^2}$$

$$= \sqrt{(24^2) + (10 - 3)^2}$$

$$= \sqrt{576 + 49} = \sqrt{625} = 25\text{ cm}$$

Curved surface area of the frustum

$= \pi(r_1 + r_2)l$

$$= \frac{22}{7}(10 + 3)\times 25$$

$$= \frac{22}{7}\times 13\times 25 = 1021.43\text{ m}^2$$

Height of the cone $(H) = 28 - 24 = 4$ m

Slant height of the cone $(l_1) = \sqrt{r_2^2 + H^2}$

$$= \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5\text{ m}$$

Curved surface area of the cone

$$= \pi r_2 l_1 = \frac{22}{7}\times 3\times 5 = 47.14\text{ m}^2$$

Area of the canvas required = Curved surface area of frustum + Curved surface area of cone

$= 1021.43 + 47.14 = 1068.57\text{ m}^2$

28. Diameter of a spherical lead shot = 4.2 cm

$\therefore$ Radius of a spherical lead shot $(r) = \frac{4.2}{2} = 2.1$ cm

So, volume of spherical lead shot

$$= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times(2.1)^3 = \frac{4}{3}\times\frac{22}{7}\times 2.1\times 2.1\times 2.1$$

$$= \frac{4\times 22\times 21\times 21\times 21}{3\times 7\times 1000}\text{ cm}^3$$

Now, length of solid lead piece $(l) = 66$ cm

Breadth of solid lead piece $(b) = 42$ cm

Height of solid lead piece $(h) = 21$ cm

$\therefore$ Volume of a solid lead piece (cuboid)

$= l\times b\times h = 66\times 42\times 21\text{ cm}^3$

Since, spherical lead shots are made from a solid rectangular lead piece.

$\therefore$ Number of spherical lead shots

$$= \frac{\text{Volume of solid rectangular lead piece}}{\text{Volume of a spherical lead shot}}$$

$$= \frac{66\times 42\times 21}{4\times 22\times 21\times 21\times 21}\times 3\times 7\times 1000$$

$= 3\times 500 = 1500$

Hence, the required number of spherical lead shots is 1500.

29. Given, length of the wall $(l) = 24$ m,

Thickness of the wall $(b) = 0.4$ m,

Height of the wall $(h) = 6$ m

So, volume of the wall constructed $= l\times b\times h$

$$= 24\times 0.4\times 6 = \frac{24\times 4\times 6}{10}\text{ m}^3$$

Now, $\frac{1}{10}^{\text{th}}$ volume of a wall

$$= \frac{1}{10}\times\frac{24\times 4\times 6}{10} = \frac{24\times 4\times 6}{100}\text{ m}^3$$

Also, length of a brick $(l_1) = 25\text{ cm} = \frac{25}{100}$ m

Breadth of a brick $(b_1) = 16\text{ cm} = \frac{16}{100}$ m

Height of a brick $(h_1) = 10\text{ cm} = \frac{10}{100}$ m

So, volume of a brick $= l_1\times b_1\times h_1$

$$= \frac{25}{100}\times\frac{16}{100}\times\frac{10}{100} = \frac{25\times 16}{10^5}\text{ m}^3$$

Now, number of bricks

$$= \frac{(\text{Volume of wall}) - \left(\frac{1}{10}\times\text{Volume of wall}\right)}{\text{Volume of a brick}}$$

$$= \frac{\left(\frac{24\times 4\times 6}{10} - \frac{24\times 4\times 6}{100}\right)}{\left(\frac{25\times 16}{10^5}\right)} = \frac{\frac{24\times 4\times 6}{100}(10 - 1)}{\frac{25\times 16}{10^5}}$$

$$= \frac{24\times 4\times 6}{100}\times 9\times\frac{10^5}{25\times 16} = \frac{24\times 4\times 6\times 9\times 1000}{25\times 16}$$

$= 24\times 6\times 9\times 10 = 12960$

Hence, the required number of bricks used in constructing the wall is 12960.