CHAPTER 14

Statistics

EXAM DRILL

SOLUTIONS

1. **(a)**

2. **(d)**

3. **(b) :** We know, mean $= a + \frac{\sum f_i d_i}{\sum f_i}$

Here, $a = 55.5$, $n = \sum f_i = 100$, $\sum f_i d_i = 60$

$\therefore$ Mean $= 55.5 + \frac{60}{100} = 55.5 + 0.6 = 56.1$

4. **(a) :** Given, Mode – Median = 24

Also, we know, Mode = 3 Median – 2 Mean

$\Rightarrow$ 24 + Median = 3 Median – 2 Mean

$\Rightarrow$ 2(Median – Mean) = 24

$\Rightarrow$ Median – Mean = 12

5. Since, sum of n natural numbers $= \frac{n(n+1)}{2}$ ...(i)

Now, mean of n natural numbers $= \frac{5n}{9}$ [Given]

$\Rightarrow \frac{n(n+1)}{2n} = \frac{5n}{9}$ [From (i)]

$\Rightarrow \frac{n^2+n}{2n} = \frac{5n}{9} \Rightarrow 9n^2 + 9n = 10n^2 \Rightarrow n(n-9) = 0$

$\Rightarrow$ Either $n = 0$ or $n = 9 \Rightarrow n = 9$ ($\because$ n can't be zero)

6. We know, Mean $= \frac{\text{Sum of the quantities}}{\text{Number of the quantities}}$

$\Rightarrow x = \frac{7+8+x+11+14}{5}$

$\Rightarrow 5x = 40 + x \Rightarrow 4x = 40 \Rightarrow x = 10$

7. Given, $\sum f_i u_i = 20$, $\sum f_i = 100$, $u_i = \frac{x_i - 25}{10} = \frac{x_i - a}{h}$

So, $a = 25$, $h = 10$

$\therefore$ Mean, $\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h = 25 + \frac{20}{100} \times 10 = 27$

8. The abscissa of the point of intersection of less than type and of the more than type cumulative frequency curves of a grouped data gives its median.

9. We know, mean $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

$\Rightarrow \frac{(x-5y)+(x-3y)+(x-y)+(x+y)+(x+3y)+(x+5y)}{6} = 12$

$\Rightarrow \frac{6x}{6} = 12 \Rightarrow x = 12$

10. Given, Mode = 1000, Median = 1250

Now, Mode = 3 Median – 2 Mean

$\Rightarrow$ 1000 = 3(1250) – 2 Mean

$\Rightarrow$ 2 Mean = 2750 $\Rightarrow$ Mean = 1375

11. Let us consider the following table :

Class	Class marks (x_i)	$d_i = x_i - A$	Frequency (f_i)	$f_i d_i$
30-40	35	–20	80	–1600
40-50	45	–10	110	–1100
50-60	55 = A	0	120	0
60-70	65	10	70	700
70-80	75	20	40	800
Total			$\sum f_i = 420$	$\sum f_i d_i = -1200$

(i) **(d) :** Clearly, the possible values of assumed mean (A) are 35, 45, 55, 65, 75.

(ii) **(c) :** The values of $|d_i|$ are 0, 10, 20

Thus, the minimum value of $|d_i|$ is 0.

(iii) **(b) :** Required Mean $= A + \frac{\sum f_i d_i}{\sum f_i} = 55 - \frac{1200}{420}$

= ₹ 52.14

(iv) **(a) :** Mean by direct and assumed mean method are always equal.

(v) **(d) :** Average toll tax received by a vehicle = ₹ 52.14

Total number of vehicles = 420

$\therefore$ Average toll tax received in a day = ₹(52.14 × 420)

= ₹ 21898.80

12. Given frequency distribution table can be drawn as :

Class interval	Class marks (x_i)	Frequency (f_i)	$x_i f_i$	c.f.
100-120	110	7	770	7
120-140	130	12	1560	19
140-160	150	18	2700	37
160-180	170	13	2210	50
Total		50	7240	

(i) **(d) :** Clearly, average mileage

$= \frac{7240}{50} = 144.8$ km/charge

(ii) **(b) :** Since, highest frequency is 18, therefore, modal class is 140-160.

Here, $l = 140$, $f_1 = 18$, $f_0 = 12$, $f_2 = 13$, $h = 20$

$\therefore$ Mode $= 140+\frac{18-12}{36-12-13}\times 20 = 140+\frac{6}{11}\times 20$

$= 140+\frac{120}{11} = 140+10.91 = 150.91$

(iii) (b) : Here, $\frac{N}{2}=\frac{50}{2}=25$ and the corresponding class whose cumulative frequency is just greater than 25 is 140-160.

Here, $l = 140$, $c.f. = 19$, $h = 20$ and $f = 18$

$\therefore$ Median $= l+\left(\frac{\frac{N}{2}-c.f.}{f}\right)\times h$

$= 140+\frac{25-19}{18}\times 20 = 140+\frac{60}{9} = 146.67$

(iv) (a) : Assumed mean method is useful in determining the mean.

(v) (a) : Since, Mean = 144.8, Mode = 150.91 and Median = 146.67 and minimum of which is 144 approx, therefore manufacturer can claim the mileage for his scooter 144 km/charge.

13. (i) (d) : Required number of persons = 9 + 6 = 15

(ii) (c) : Required number of persons = 6 + 8 + 2 = 16

(iii) (a) : 50-60 is the modal class as the maximum frequency is 9.

(iv) (b) : The cumulative frequency distribution table for the given data can be drawn as :

Salaries received (in percent)	Number of employees (f_i)	Cumulative Frequency ($c.f.$)
50-60	9	9
60-70	6	9 + 6 = 15
70-80	8	15 + 8 = 23
80-90	2	23 + 2 = 25
Total	$\Sigma f_i = 25$	

Here, $\frac{N}{2}=\frac{25}{2}=12.5$

The cumulative frequency just greater than 12.5 lies in the interval 60-70.

Hence, the median class is 60-70.

(v) (a) : We know, Mode = 3 Median – 2 Mean

$\therefore$ 3 Median = Mode + 2 Mean

14. (i) (d) : We know that,

Class mark $= \frac{\text{Lower limit + Upper limit}}{2}$

$\Rightarrow m = \frac{\text{Lower limit} + b}{2} \Rightarrow$ Lower limit = $2m - b$

(ii) (a) :

Lifetime (in hours)	Class mark (x_i)	f_i	$d_i = x_i - A$	$f_i d_i$
150-200	175	14	–150	–2100
200-250	225	56	–100	–5600
250-300	275	60	–50	–3000
300-350	325 = A	86	0	0
350-400	375	74	50	3700
400-450	425	62	100	6200
450-500	475	48	150	7200
Total		400		6400

$\therefore$ Average lifetime of a packet

$= A+\frac{\sum f_i d_i}{\sum f_i} = 325+\frac{6400}{400} = 341$ hrs

(iii) (b) : Here, $N = 400 \Rightarrow \frac{N}{2} = 200$

Also, cumulative frequency for the given distribution are 14, 70, 130, 216, 290, 352, 400

$\therefore$ $c.f.$ just greater than 200 is 216, which is corresponding to the interval 300-350.

$l = 300$, $f = 86$, $c.f. = 130$, $h = 50$

$\therefore$ Median $= l+\left(\frac{\frac{N}{2}-c.f.}{f}\right)\times h = 300+\left(\frac{200-130}{86}\right)\times 50$

= 300 + 40.697 = 340.697 ≈ 340 hrs (approx.)

(iv) (a) : We know that Mode = 3 Median – 2 Mean

= 3(340.697) –2(341)

= 1022.091 – 682 = 340.091 ≈ 340 hrs

(v) (c) : Since, minimum of mean, median and mode is approximately 340 hrs. So, manufacturer should claim that lifetime of a packet is 340 hrs.

15. Let us construct the cumulative frequency distribution table :

Marks obtained	Frequency	Cumulative frequency
0 - 10	8	8
10 - 20	10	10 + 8 = 18
20 - 30	12	18 + 12 = 30
30 - 40	22	30 + 22 = 52
40 - 50	30	52 + 30 = 82
50 - 60	18	82 + 18 = 100

Here, $n = 100 \Rightarrow n/2 = 50$

Cumulative frequency just greater than 50 is 52, which lies in the interval 30-40. Therefore, 30-40 is the median class.

16. We have, $m = \frac{1+3+4+5+7}{5} = \frac{20}{5} \Rightarrow m = 4$

Also, $m - 1 = \frac{3+2+2+4+3+3+p}{7}$

$\Rightarrow$ $7m - 7 = 17 + p \Rightarrow 7 \times 4 - 7 = 17 + p$ $(\because m = 4)$

$\Rightarrow$ $28 - 7 = 17 + p \Rightarrow p = 21 - 17 = 4$

$\therefore$ The numbers are 3, 2, 2, 4, 3, 3, 4

Here, $n = 7$, which is odd.

$\therefore$ Median, $q = \left(\frac{n+1}{2}\right)^{\text{th}}$ observation $= \left(\frac{7+1}{2}\right)^{\text{th}}$ observation

$= 4^{\text{th}}$ observation $= 4$ $\therefore$ $p + q = 4 + 4 = 8$

17. Let us construct the following table from the given data:

Class-interval	Frequency (f_i)	Class-mark (x_i)	$\Sigma f_i x_i$
0 - 6	7	3	21
6 - 12	5	9	45
12 - 18	10	15	150
18 - 24	12	21	252
24 - 30	6	27	162
Total	$\Sigma f_i = 40$		$\Sigma f_i x_i = 630$

$\therefore$ Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{630}{40} = 15.75$

18. Let the ten numbers be $x_1, x_2, \ldots\ldots, x_5, x_6, x_7, \ldots\ldots, x_{10}$.

Mean of the first six numbers = 15

$\therefore$ $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 90$ ...(i)

Mean of the last five numbers = 10

$\therefore$ $x_6 + x_7 + x_8 + x_9 + x_{10} = 50$ ...(ii)

Mean of 10 numbers $= \frac{x_1 + x_2 + \ldots + x_9 + x_{10}}{10}$

$$\Rightarrow 12.5 = \frac{(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) + (x_6 + x_7 + x_8 + x_9 + x_{10}) - x_6}{10}$$

$\Rightarrow$ $12.5 \times 10 = 90 + 50 - x_6$ [Using (i) and (ii)]

$\Rightarrow$ $125 = 140 - x_6 \Rightarrow x_6 = 140 - 125 = 15$

Hence, the sixth number is 15.

19. Given, mean = 5.5

x_i	f_i	$f_i x_i$
2	3	6
4	5	20
6	6	36
8	y	$8y$
	$\Sigma f_i = 14 + y$	$\Sigma f_i x_i = 62 + 8y$

Mean $= \frac{\sum f_i x_i}{\sum f_i} \Rightarrow 5.5 = \frac{62 + 8y}{14 + y}$

$\Rightarrow$ $5.5(14 + y) = 62 + 8y \Rightarrow 77 + 5.5y = 62 + 8y$

$\Rightarrow$ $8y - 5.5y = 77 - 62 \Rightarrow 2.5y = 15 \Rightarrow y = 6$

20. We have, $\frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n} = \bar{x}$

$\Rightarrow$ $x_1 + x_2 + \ldots\ldots + x_n = n\bar{x}$ ...(i)

Also, $\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \ldots\ldots + x_n \Rightarrow \sum_{i=1}^{n} x_i = n\bar{x}$

[Using (i)]

$\Rightarrow$ $\sum_{i=1}^{n} x_i - n\bar{x} = 0$

21. Let us construct the following table from the given data :

Class-interval	Frequency (f_i)	Class-mark (x_i)	$f_i x_i$
0 - 20	17	10	170
20 - 40	p	30	$30p$
40 - 60	32	50	1600
60 - 80	24	70	1680
80 - 100	19	90	1710
Total	$\Sigma f_i = 92 + p$		$\Sigma f_i x_i = 5160 + 30p$

Now, Mean $= \frac{\sum f_i x_i}{\sum f_i} \Rightarrow 50 = \frac{5160 + 30p}{92 + p}$

$\Rightarrow$ $4600 + 50p = 5160 + 30p \Rightarrow 20p = 560 \Rightarrow p = 28$

22. It is given that mode = 8, which lies in the interval 7 – 10.

Therefore, 7 – 10 is the modal class.

So, $l = 7, f_1 = 35, f_0 = 25, f_2 = x$ and $h = 3$

$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$\Rightarrow$ $8 = 7 + \left(\frac{35 - 25}{2 \times 35 - 25 - x}\right) \times 3 \Rightarrow 1 = \left(\frac{10}{70 - 25 - x}\right) \times 3$

$\Rightarrow$ $45 - x = 30 \Rightarrow x = 45 - 30 \Rightarrow x = 15$

Hence, the missing frequency (x) is 15.

OR

23. Let us construct the cumulative frequency distribution table for the given data :

Family size	Number of families	Cumulative frequency
1 - 3	4	4
3 - 5	6	10
5 - 7	2	12
7 - 9	2	14
9 - 11	2	16
Total	$\Sigma f_i = 16$	

Here, $n = 16 \Rightarrow \frac{n}{2} = 8$

The cumulative frequency just greater than 8 is 10 and corresponding interval is 3-5. $\therefore$ Median class is 3-5.

So, $l = 3$, $c.f. = 4$, $f = 6$ and $h = 2$

$\therefore$ Median $= l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h = 3 + \left(\frac{8-4}{6}\right) \times 2$

$= 3 + \frac{4}{6} \times 2 = 3 + 1.33 = 4.33$

Hence, median family size is 4.33.

24. Here, $h = 2$

Let us construct the following frequency distribution table:

Daily pocket allowance (in ₹)	Number of children (f_i)	Mid-point (x_i)	$u_i = \frac{x_i - 18}{2}$	$f_i u_i$
11 - 13	3	12	–3	–9
13 - 15	6	14	–2	–12
15 - 17	9	16	–1	–9
17 - 19	13	$18 = a$ (let)	0	0
19 - 21	k	20	1	k
21 - 23	5	22	2	10
23 - 25	4	24	3	12
Total	$\Sigma f_i = 40 + k$			$\Sigma f_i u_i = k - 8$

Mean $(\bar{x}) = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h \Rightarrow 18 = 18 + \left(\frac{k-8}{40+k}\right) \times 2$

$\Rightarrow 0 = \frac{2(k-8)}{40+k} \Rightarrow k - 8 = 0 \Rightarrow k = 8$

OR

We prepare the following cumulative frequency distribution table:

Salary (in thousand ₹)	Number of persons	Cumulative frequency
5 - 10	49	49
10 - 15	133	182
15 - 20	63	245
20 - 25	15	260
25 - 30	6	266
30 - 35	7	273
35 - 40	4	277
40 - 45	2	279
45 - 50	1	280
Total	$\Sigma f_i = 280$	

We have, $n = 280 \Rightarrow \frac{n}{2} = \frac{280}{2} = 140$

Cumulative frequency just greater than 140 is 182 and corresponding interval is 10–15. Therefore, 10–15 is the median class.

So, $l = 10$, $f = 133$, $c.f. = 49$ and $h = 5$.

$\therefore$ Median $= l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$

$= 10 + \left(\frac{140 - 49}{133}\right) \times 5 = 10 + \frac{455}{133} = 10 + 3.42 = 13.42$

Hence, median salary is ₹ 13.42 (in thousand).

24. Here, $h = 10$

Let us construct the following table :

Class-interval	Frequency (f_i)	Class-mark (x_i)	$u_i = \frac{x_i - 55}{10}$	$f_i u_i$
10 - 20	8	15	–4	–32
20 - 30	7	25	–3	–21
30 - 40	12	35	–2	–24
40 - 50	23	45	–1	–23
50 - 60	11	$55 = a$ (let)	0	0
60 - 70	13	65	1	13
70 - 80	8	75	2	16
80 - 90	6	85	3	18
90 - 100	12	95	4	48
Total	$\Sigma f_i = 100$			$\Sigma f_i u_i = -5$

Now, Mean $(\bar{x}) = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$

$= 55 + \left(\frac{-5}{100}\right) \times 10 = 55 - \frac{50}{100} = 55 - 0.5 = 54.5$

Hence, mean is 54.5.

25. Let us construct the following cumulative frequency distribution table :

Class-interval	Frequency	Cumulative frequency
0 - 100	2	2
100 - 200	5	7
200 - 300	x	$7 + x$
300 - 400	12	$19 + x$
400 - 500	17	$36 + x$
500 - 600	20	$56 + x$
600 - 700	15	$71 + x$
700 - 800	9	$80 + x$
800 - 900	7	$87 + x$
900 - 1000	4	$91 + x$

Given, median = 525, which lies in the interval 500 - 600.

$\therefore$ Median class is 500 - 600.

So, $l = 500$, $f = 20$, $c.f. = 36 + x$ and $h = 100$

Median $= l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$

$\Rightarrow 525 = 500 + \left(\frac{50 - 36 - x}{20}\right) \times 100$

$\Rightarrow 25 = 70 - 5x \Rightarrow 5x = 45 \Rightarrow x = \frac{45}{5} \Rightarrow x = 9$

Thus, the missing frequency is 9.

26. Let us construct the following frequency distribution table:

Daily savings (in ₹)	Number of children (f_i)	Class mark (x_i)	$f_i x_i$
1 - 3	7	2	14
3 - 5	6	4	24
5 - 7	x	6	$6x$
7 - 9	13	8	104
9 - 11	y	10	$10y$
11 - 13	5	12	60
13 - 15	4	14	56
Total	$\Sigma f_i = 35 + x + y$		$\Sigma f_i x_i = 258 + 6x + 10y$

Given, sum of frequencies = 64

$\Rightarrow \quad 35 + x + y = 64 \quad \Rightarrow \quad x + y = 29 \Rightarrow \quad y = 29 - x \qquad$ ...(i)

Now, Mean = $\dfrac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow \quad 8 = \dfrac{258 + 6x + 10y}{64} \qquad (\because$ Given, mean = 8)

$\Rightarrow \quad 8 \times 64 = 258 + 6x + 10y$

$\Rightarrow \quad 512 = 258 + 6x + 10(29 - x) \qquad$ [Using (i)]

$\Rightarrow \quad 512 = 258 + 6x + 290 - 10x$

$\Rightarrow \quad 512 = 548 - 4x \Rightarrow 4x = 548 - 512$

$\Rightarrow \quad 4x = 36 \quad \Rightarrow \quad x = 9 \qquad$ ...(ii)

From (i) and (ii), we obtain $y = 29 - 9 = 20$

Hence, the missing frequencies are $x = 9$ and $y = 20$.

OR

Let us construct the following cumulative frequency distribution table:

Class-interval	Frequency	Cumulative frequency
0 - 5	12	12
5 - 10	a	$12 + a$
10 - 15	12	$24 + a$
15 - 20	15	$39 + a$
20 - 25	b	$39 + a + b$
25 - 30	6	$45 + a + b$
30 - 35	6	$51 + a + b$
35 - 40	4	$55 + a + b$

It is given that total frequency = 70

$\Rightarrow \quad 55 + a + b = 70 \qquad$ ...(i)

$\Rightarrow \quad a + b = 70 - 55 = 15$

Given, median is 16 which lies in the class interval 15 - 20.

So, $l = 15,\ f = 15,\ c.f. = 24 + a,\ \dfrac{n}{2} = \dfrac{70}{2} = 35$ and $h = 5$

Now, Median = $l + \left(\dfrac{\dfrac{n}{2} - c.f.}{f}\right) \times h$

$\Rightarrow \quad 16 = 15 + \left(\dfrac{35 - (24 + a)}{15}\right) \times 5 = 15 + \left(\dfrac{11 - a}{3}\right)$

$\Rightarrow \quad 1 = \dfrac{11 - a}{3} \quad \Rightarrow 3 = 11 - a \Rightarrow a = 11 - 3 = 8$

Now, from (i), we get $8 + b = 15 \Rightarrow b = 15 - 8 = 7$

Hence, the missing frequencies are 8 and 7.

27. Let us construct the following frequency distribution table:

Marks	Number of students (f_i)	Class-mark (x_i)	$f_i x_i$
0 - 10	1	5	5
10 - 20	3	15	45
20 - 30	7	25	175
30 - 40	10	35	350
40 - 50	15	45	675
50 - 60	x	55	$55x$
60 - 70	9	65	585
70 - 80	27	75	2025
80 - 90	18	85	1530
90 - 100	y	95	$95y$
Total	$\Sigma f_i = 90 + x + y$		$\Sigma f_i x_i = 5390 + 55x + 95y$

Total number of students = 120 [Given]

$\therefore \quad 90 + x + y = 120$

$\Rightarrow \quad x + y = 120 - 90 = 30 \quad \Rightarrow \quad y = 30 - x \qquad$ ...(i)

Now, mean $(\bar{x}) = \dfrac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow \quad 59 = \dfrac{5390 + 55x + 95y}{120}$

$\Rightarrow \quad 59 \times 120 = 5390 + 55x + 95y$

$\Rightarrow \quad 7080 = 5390 + 55x + 95(30 - x) \qquad$ [Using (i)]

$\Rightarrow \quad 7080 - 5390 = 55x + 2850 - 95x \Rightarrow 1690 = 2850 - 40x$

$\Rightarrow \quad 40x = 2850 - 1690 \Rightarrow 40x = 1160 \quad \Rightarrow x = \dfrac{1160}{40} = 29$

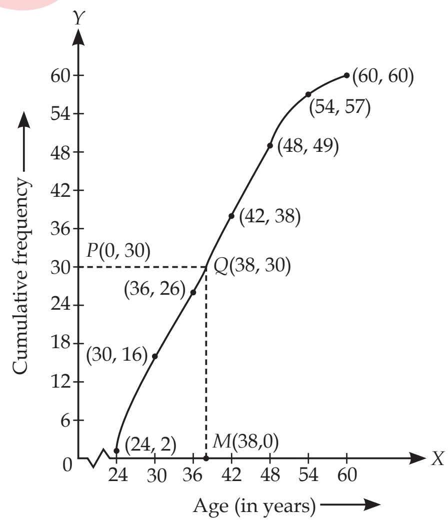
Substituting $x = 29$ in (i), we get $y = 30 - 29 = 1$

Hence, the missing frequencies are $x = 29$ and $y = 1$.

OR

The given frequency distribution is less than type cumulative frequency distribution.

$\therefore$ We plot the points (24, 2), (30, 16), (36, 26), (42, 38), (48, 49), (54, 57) and (60, 60) on a graph paper and join the points with a free hand to get smooth curve to get the cumulative frequency curve or a 'less than type' ogive.



We have $n = 60 \Rightarrow \dfrac{n}{2} = 30$. Take a point $P(0, 30)$ on y-axis and draw $PQ \parallel x$-axis, meeting the curve at Q. Draw $QM \perp$ to x-axis, intersecting x-axis at M. Then, $OM = 38$.

Hence, median = 38 years.

28. Let us construct the cumulative frequency distribution table :

Ages (in years)	Number of persons (f_i)	Cumulative frequency ($c.f.$)	Class-mark (x_i)	$u_i = \dfrac{x_i - 35}{10}$	$f_i u_i$
0 - 10	50	50	5	–3	–150

10 - 20	400	450	15	–2	–800
20 - 30	108	558	25	–1	–108
30 - 40	530	1088	35 = a (let)	0	0
40 - 50	47	1135	45	1	47
50 - 60	10	1145	55	2	20
60 - 70	5	1150	65	3	15
Total	Σf_i = 1150				$\Sigma f_i u_i$ = –976

Mean, $\bar{x} = a + \left(\dfrac{\Sigma f_i u_i}{\Sigma f_i}\right)h$

$= 35 + \dfrac{(-976)}{1150} \times 10 = 35 - \dfrac{976}{115} = 35 - 8.49 = 26.51$

From the table, $n = 1150 \Rightarrow \dfrac{n}{2} = \dfrac{1150}{2} = 575$

Cumulative frequency just greater than 575 is 1088 and corresponding interval is 30 - 40.

Therefore, 30 - 40 is the median class. So, $l = 30$, $f = 530$, $c.f.$ = 558 and $h = 10$

$\therefore$ Median $= l + \left(\dfrac{\frac{n}{2} - c.f.}{f}\right) \times h$

$= 30 + \left(\dfrac{575 - 558}{530}\right) \times 10 = 30 + \dfrac{17}{53} = 30 + 0.32 = 30.32$

Hence, the mean and median of the given distribution is 26.51 years and 30.32 years.

29. We have, $h = 2$. Let us construct the following frequency distribution table :

Age (in years)	Number of students (f_i)	Class-mark (x_i)	$u_i = \dfrac{x_i - 12}{2}$	$f_i u_i$
5 - 7	67	6	–3	–201
7 - 9	33	8	–2	–66
9 - 11	41	10	–1	–41
11 - 13	95	12 = a (let)	0	0
13 - 15	36	14	1	36
15 - 17	13	16	2	26
17 - 19	15	18	3	45
Total	Σf_i = 300			$\Sigma f_i u_i$ = –201

$\therefore$ Mean, $(\bar{x}) = a + \left(\dfrac{\Sigma f_i u_i}{\Sigma f_i}\right)h = 12 + \dfrac{(-201)}{300} \times 2$

$= 12 - \dfrac{67 \times 2}{100} = 12 - \dfrac{134}{100} = 12 - 1.34 = 10.66$

Since, the class 11-13 has the maximum frequency 95, therefore, 11-13 is the modal class.

So, $l = 11, f_1 = 95, f_0 = 41, f_2 = 36$ and $h = 2$

$\therefore$ Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 11 + \left(\dfrac{95 - 41}{2 \times 95 - 41 - 36}\right) \times 2 = 11 + \left(\dfrac{54}{190 - 77}\right) \times 2$

$= 11 + \dfrac{108}{113} = 11 + 0.96 = 11.96$

Hence, mean = 10.66 and mode = 11.96 of the given data.

OR

Since, the maximum frequency is 29 which corresponds to the class 60-80. Therefore, 60-80 is the modal class.

So, $l = 60$, $f_1 = 29$, $f_0 = 21$, $f_2 = 17$ and $h = 20$

$\therefore$ Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 60 + \left(\dfrac{29 - 21}{2 \times 29 - 21 - 17}\right) \times 20 = 60 + \dfrac{8}{20} \times 20 = 60 + 8 = 68$

So, mode of the distribution = 68

Given, mean = 53

Empirical relationship between the three measures of central tendency is

3 Median = Mode + 2 Mean

$\Rightarrow$ 3 Median = 68 + 2 × 53 = 68 + 106 = 174

$\Rightarrow$ Median $= \dfrac{174}{3} = 58$

30. Let us construct the following cumulative frequency distribution table:

Class	Frequency	Cumulative frequency
0 - 10	5	5
10 - 20	x	5 + x
20 - 30	6	11 + x
30 - 40	y	11 + x + y
40 - 50	6	17 + x + y
50 - 60	5	22 + x + y

It is given that $\Sigma f_i = 40$

$\therefore$ $22 + x + y = 40$

$\Rightarrow$ $x + y = 40 - 22 = 18 \Rightarrow y = 18 - x$ ...(i)

The median is 31, which lies in the class-interval 30–40

So, $l = 30$, $f = y$, $c.f. = 11 + x$, $\dfrac{n}{2} = \dfrac{40}{2} = 20$ and $h = 10$

$\therefore$ Median $= l + \left(\dfrac{\frac{n}{2} - c.f.}{f}\right) \times h$

$\Rightarrow$ $31 = 30 + \left(\dfrac{20 - (11 + x)}{y}\right) \times 10$

$\Rightarrow 31 - 30 = \dfrac{9 - x}{y} \times 10 \Rightarrow 1 = \dfrac{9 - x}{18 - x} \times 10$ [Using (i)]

$\Rightarrow$ $18 - x = 90 - 10x$ $\Rightarrow$ $10x - x = 90 - 18$

$\Rightarrow$ $9x = 72$ $\Rightarrow$ $x = 8$

Substituting $x = 8$ in (i), we get $y = 18 - 8 = 10$

Hence, the missing frequencies are $x = 8$ and $y = 10$

31. We prepare the less than type cumulative frequency distribution table :

Marks	Frequency	Cumulative frequency
Less than 5	4	4
Less than 10	6	10
Less than 15	10	20
Less than 20	10	30
Less than 25	25	55
Less than 30	22	77

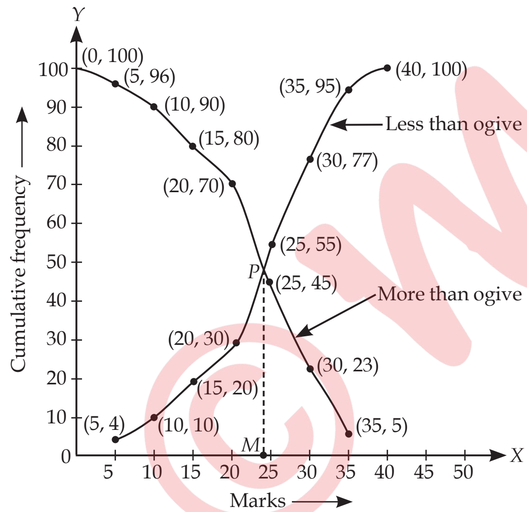
Less than 35	18	95
Less than 40	5	100

We, plot the points (5, 4), (10, 10), (15, 20), (20, 30), (25, 55), (30, 77), (35, 95), (40, 100) on a graph paper and join them by a free hand to get smooth curve. The curve we get is called an ogive of less than type (see figure).

Now, we prepare the more than type cumulative frequency distribution table :

Marks	Frequency	Cumulative frequency
More than or equal to 0	4	100
More than or equal to 5	6	96
More than or equal to 10	10	90
More than or equal to 15	10	80
More than or equal to 20	25	70
More than or equal to 25	22	45
More than or equal to 30	18	23
More than or equal to 35	5	5

Now, we plot the points (0, 100), (5, 96), (10, 90), (15, 80), (20, 70), (25, 45), (30, 23), (35, 5) on the same graph paper and join them by a free hand to get smooth curve. The curve we get is called an ogive of more than type (see figure).



We find that the two types of cumulative frequency curves intersect each other at point P. Perpendicular PM is drawn on x-axis. The value of marks corresponding to M is 24. Hence, the median is 24.

32. We prepare the following table from given data :

Marks	Number of students (f_i)	Cumulative frequency ($c.f.$)
80 - 90	150–141 = 9	9
90 - 100	141–124 = 17	9 + 17 = 26
100 - 110	124–105 = 19	26 + 19 = 45
110 - 120	105–60 = 45	45 + 45 = 90
120 - 130	60–27 = 33	90 + 33 = 123
130 - 140	27–12 = 15	123 + 15 = 138
140 - 150	12–0 = 12	138 + 12 = 150
More than or equal to 150	0	150 + 0 =150

Here, $n = 150 \Rightarrow \frac{n}{2} = 75$

Now, 110-120 is the class whose cumulative frequency is greater than 75.

$\therefore$ 110-120 is the median class.

So, $f = 45$, $c.f. = 45$, $h = 10$, $l = 110$ and $h = 10$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

$$= 110 + \left(\frac{75-45}{45}\right) \times 10 = 110 + \frac{30}{45} \times 10 = 110 + \frac{20}{3}$$

$= 110 + 6.67 = 116.67$

OR

Let us construct the following cumulative frequency distribution table :

Class-interval	Frequency (f_i)	Cumulative frequency ($c.f.$)
0 - 20	6	6
20 - 40	8	6 + 8 = 14
40 - 60	10	14 + 10 = 24
60 - 80	12	24 + 12 = 36
80 - 100	6	36 + 6 = 42
100 - 120	5	42 + 5 = 47
120 - 140	3	47 + 3 = 50
Total	$\Sigma f_i = 50$	

We have, $n = 50 \Rightarrow \frac{n}{2} = 25$

Cumulative frequency just greater than 25 is 36, which lies in the interval 60-80.

$\therefore$ 60-80 is the median class. So, $l = 60$, $f = 12$, $c.f. = 24$ and $h = 20$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

$$= 60 + \left(\frac{25-24}{12}\right) \times 20 = 60 + \frac{20}{12} = 60 + 1.67 = 61.67$$

Since, the class 60-80 has the maximum frequency 12, therefore, 60-80 is the modal class.

So, $l = 60$, $f_0 = 10$, $f_1 = 12$, $f_2 = 6$ and $h = 20$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 60 + \left(\frac{12-10}{2 \times 12 - 10 - 6}\right) \times 20$$

$$= 60 + \frac{2}{8} \times 20 = 60 + 5 = 65$$