EXAM DRILL

Real Numbers

CHAPTER 1

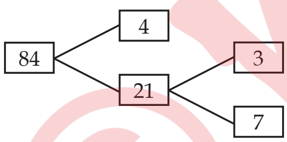
SOLUTIONS

1. **(a) :** Here, 125 < 240
So, applying Euclid's division lemma to 125 and 240, we get
$240 = 125 \times 1 + 115$
$125 = 115 \times 1 + 10$
$115 = 10 \times 11 + 5$
$10 = 5 \times 2 + 0$
Since, remainder = 0, when divisor is 5.
$\therefore$ HCF of 240 and 125 is 5.

2. **(b) :** On dividing n by 9, let q be the quotient and 7 be the remainder.
Then, $n = 9q + 7$
$\Rightarrow$ $3n = 27q + 21 \Rightarrow 3n - 1 = 27q + 20$
$\Rightarrow$ $(3n - 1) = 9 \times (3q + 2) + 2$
$\therefore$ When $(3n - 1)$ is divided by 9, the remainder is 2.

3. **(c) :** If the sum of 3 prime numbers is even, then one of the numbers must be 2.
Let the second number be x.
$x + (x + 36) + 2 = 100$
$\Rightarrow$ $2x = 62 \Rightarrow x = 31$
So, the numbers are 2, 31 and 67.
Hence, largest number is 67.

4. **(b) :** Using factor tree method, we have



$\therefore$ $x = 21;\ y = 84$

5. **(d) :** $12 - 7 = 5$, $15 - 10 = 5$ and $16 - 11 = 5$
Hence, the desired number is 5 less than the LCM of 12, 15, 16.
LCM of 12, 15 and 16 is 240.
Hence, the least number = 240 – 5 = 235

6. Given that, $a = x^5y^3 = x \times x \times x \times x \times x \times y \times y \times y$
and $b = x^3y^4 = x \times x \times x \times y \times y \times y \times y$
$\therefore$ HCF of a and b = HCF (x^5y^3, x^3y^4)
$= x \times x \times x \times y \times y \times y = x^3y^3$

7. Clearly, 2 is neither a factor of p nor that of q
$\therefore$ p and q are both odd.
So, $(p + q)$ must be an even number, which is divisible by 2. Hence, the least prime factor of $(p + q)$ is 2.

8. Since, p is prime, then p and $p + 1$ has no common factor other than 1.
$\therefore$ HCF of p and $(p + 1)$ is 1 and LCM of p and $(p + 1)$ is $p(p + 1)$.

9. $\sqrt{27} = 3\sqrt{3} = 3 \times 1.7320508...$
$\therefore$ It is non-terminating decimal expansion.

10. **(i) (d) :** For a number to end in zero it must be divisible by 5, but $4^n = 2^{2n}$ is never divisible by 5.
So, 4^n never ends in zero for any value of n.

(ii) (c) : We know that product of two rational numbers is also a rational number.
So, $a^2 = a \times a$ = rational number
$a^3 = a^2 \times a$ = rational number
$a^4 = a^3 \times a$ = rational number
....
....
$a^n = a^{n-1} \times a$ = rational number.

(iii) (d) : Let $x = 2m + 1$ and $y = 2k + 1$
Then $x^2 + y^2 = (2m + 1)^2 + (2k + 1)^2$
$= 4m^2 + 4m + 1 + 4k^2 + 4k + 1 = 4(m^2 + k^2 + m + k) + 2$
So, it is even but not divisible by 4.

(iv) (a) : Let three consecutive positive integers be n, $n + 1$ and $n + 2$.
We know that when a number is divided by 3, the remainder obtained is either 0 or 1 or 2.
So, $n = 3p$ or $3p + 1$ or $3p + 2$, where p is some integer.
If $n = 3p$, then n is divisible by 3.
If $n = 3p + 1$, then $n + 2 = 3p + 1 + 2 = 3p + 3 = 3(p + 1)$ is divisible by 3.
If $n = 3p + 2$, then $n + 1 = 3p + 2 + 1 = 3p + 3 = 3(p + 1)$ is divisible by 3.
So, we can say that one of the numbers among n, $n + 1$ and $n + 2$ is always divisible by 3.

(v) (d) : Any odd number is of the form of $(2k + 1)$, where k is any integer.
So, $n^2 - 1 = (2k + 1)^2 - 1 = 4k^2 + 4k$
For $k = 1$, $4k^2 + 4k = 8$, which is divisible by 8.
Similarly, for $k = 2$, $4k^2 + 4k = 24$, which is divisible by 8.
And for $k = 3$, $4k^2 + 4k = 48$, which is also divisible by 8.
So, $4k^2 + 4k$ is divisible by 8 for all integers k, *i.e.*, $n^2 - 1$ is divisible by 8 for all odd values of n.

11. **(i) (b) :** Here $80 = 2^4 \times 5$, $85 = 17 \times 5$ and $90 = 2 \times 3^2 \times 5$
L.C.M of 80, 85 and 90 = $2^4 \times 3 \times 3 \times 5 \times 17 = 12240$
Hence, the minimum distance each should walk when they at first time is 12240 cm.

(ii) (c) : Here $594 = 2 \times 3^3 \times 11$ and $189 = 3^3 \times 7$
HCF of 594 and 189 = $3^3 = 27$
Hence, the maximum number of columns in which they can march is 27.

(iii) (c) : Here $768 = 2^8 \times 3$ and $420 = 2^2 \times 3 \times 5 \times 7$
HCF of 768 and 420 = $2^2 \times 3 = 12$
So, the container which can measure fuel of either tanker exactly must be of 12 litres.

(iv) (b) : Here, Length = 825 cm, Breadth = 675 cm and Height = 450 cm
Also, $825 = 5 \times 5 \times 3 \times 11$, $675 = 5 \times 5 \times 3 \times 3 \times 3$ and $450 = 2 \times 3 \times 3 \times 5 \times 5$
HCF = $5 \times 5 \times 3 = 75$
Therefore, the length of the longest rod which can measure the three dimensions of the room exactly is 75 cm.

(v) (a) : LCM of 8 and 12 is 24.
∴ The least number of pack of pens = 24/8 = 3
∴ The least number of pack of note pads = 24/12 = 2

12. (i) (b) : Here $\sqrt{8} = 2\sqrt{2} =$ product of rational and irrational numbers = irrational number

(ii) (c) : Here, $\sqrt{9} = 3$
So, $2 + 2\sqrt{9} = 2 + 6 = 8$, which is not irrational.

(iii) (b) : Here $\sqrt{15}$ and $\sqrt{10}$ are both irrational and difference of two irrational numbers is also irrational.

(iv) (c) : As $\sqrt{5}$ is irrational, so its reciprocal is also irrational.

(v) (d) : We know that $\sqrt{6}$ is irrational.
So, $15 + 3\sqrt{6}$ is irrational.
Similarly, $\sqrt{24} - 9 = 2\sqrt{6} - 9$ is irrational.
And $5\sqrt{150} = 5 \times 5\sqrt{6} = 25\sqrt{6}$ is irrational.

13. (i) (b) : LCM of x and $y = p^3q^3$ and HCF of x and $y = p^2q$
Also, LCM = $pq^2 \times$ HCF.

(ii) (d) : Number of marbles = $5m + 2$ or $6n + 2$.
Thus, number of marbles, p = (multiple of 5×6) + 2
$= 30k + 2 = 2(15k + 1)$
= which is an even number but not prime

(iii) (d) : Here, required numbers
= HCF (398 – 7, 436 – 11, 542 –15)
= HCF (391, 425, 527) = 17

(iv) (b) : LCM of 126 and 600 = $2 \times 3 \times 21 \times 100 = 12600$
The least positive integer which on adding 1 is exactly divisible by 126 and 600 = 12600 – 1 = 12599

(v) (a) : Here $85C - 340A = 109$ and $425A + 85B = 146$
On adding them, we get
$85A + 85B + 85C = 255 \Rightarrow A + B + C = 3$, which is divisible by 3.

14. The greatest possible speed of the bird is the HCF of 45 and 336.
By Euclid's division lemma, we have
$336 = 45 \times 7 + 21$,
$45 = 21 \times 2 + 3$,
$21 = 3 \times 7 + 0$
Here, remainder is 0 when divisor is 3.
∴ HCF (45, 336) = 3
∴ Required speed is 3 km/h.

15. (i)

3	429
11	143
13	13
	1

∴ Prime factorisation of 429 = $3 \times 11 \times 13$

(ii)

5	7325
5	1465
293	293
	1

∴ Prime factorsiation of 7325 = $5 \times 5 \times 293$

16. We have, $3 \times 5 \times 13 \times 46 + 23$
$= 3 \times 5 \times 13 \times 2 \times 23 + 23 = 23\,(3 \times 5 \times 13 \times 2 + 1)$
$= 23 \times 391$, which is a product of two numbers.
So, the given number is composite.

17. We have, $\dfrac{543}{225} = \dfrac{3 \times 181}{3 \times 75} = \dfrac{181}{75}$
Denominator, $75 = 3 \times 5^2$
∵ The denominator is not of the form $2^m \times 5^n$.
Hence, the rational number $\dfrac{543}{225}$ has non-terminating recurring decimal expansion.

18. Divisors of 99 are 1, 3, 9, 11, 33 and 99
Divisors of 101 are 1 and 101
Divisors of 176 are 1, 2, 4, 8, 16, 11, 22, 44, 88 and 176
Divisors of 182 are 1, 2, 7, 13, 14, 26, 91 and 182
Hence, 176 has the most number of divisors.

19. Applying Euclid's division lemma to 721 and 595, we get
$721 = 595 \times 1 + 126$,
$595 = 126 \times 4 + 91$,
$126 = 91 \times 1 + 35$,
$91 = 35 \times 2 + 21$,
$35 = 21 \times 1 + 14$,
$21 = 14 \times 1 + 7$,
$14 = 7 \times 2 + 0$
Here, remainder is 0 when divisor is 7.
∴ By Euclid's division algorithm, HCF (595, 721) = 7.

20. Applying Euclid's division lemma to 4182 and 15540, we get
$15540 = 4182 \times 3 + 2994$
$4182 = 2994 \times 1 + 1188$
$2994 = 1188 \times 2 + 618$
$1188 = 618 \times 1 + 570$
$618 = 570 \times 1 + 48$
$570 = 48 \times 11 + 42$

$48 = 42 \times 1 + 6$
$42 = 6 \times 7 + 0$
Here, remainder is zero when 6 is the divisor.
$\therefore$ By Euclid's division algorithm
HCF (15540, 4182) is 6.

21. Let $x = 1.3\overline{7} = 1.37777....$
Multiplying both sides by 10, we get
$10x = 13.7777...$ ...(i)
Multiplying (i) by 10, we get
$100x = 137.777...$ ...(ii)
Subtracting (i) from (ii), we get
$90x = 124$

$$\Rightarrow \quad x = \frac{124}{90} = \frac{62}{45} = \frac{62}{3^2 \times 5} = \frac{p}{q}$$

Hence, $q = 45$ has prime factors 3 and 5 (or is not of the form $2^m \times 5^n$).

22. LCM of n and p is 21879
$21879 = 3^2 \times 11 \times 13 \times 17$
Since p is a prime, LCM of n, p is np and HCF $(n, p) = 1$
If $p = 13$, $n = 9 \times 11 \times 17 = 1683 \Rightarrow n + p \neq 2000$
If $p = 17$, $n = 9 \times 11 \times 13 = 1287 \Rightarrow n + p \neq 2000$
If $p = 11$, $n = 9 \times 13 \times 17 = 1989 \Rightarrow n + p = 2000$
$\therefore$ $p = 11$, $n = 1989$.

23. We know that any positive integer is of the form $3q$ or $3q + 1$ or $3q + 2$ for some integer q and one and only one of these possibilities can occur.
So, we have following cases:
Case I : When $n = 3q$, which is divisible by 3
Now, $n = 3q \Rightarrow n + 2 = 3q + 2$,
$\Rightarrow$ $n + 2$ leaves remainder 2, when divided by 3.
$\Rightarrow$ $n + 2$ is not divisible by 3
Again, $n = 3q$
$\Rightarrow$ $n + 4 = 3q + 4 = 3(q + 1) + 1$
$\Rightarrow$ $n + 4$ leaves remainder 1, when divided by 3.
$\Rightarrow$ $n + 4$ is not divisible by 3
Thus, only n is divisible by 3 and $(n + 2)$ and $(n + 4)$ are not divisible by 3.
Case II : When $n = 3q + 1$
$\Rightarrow$ n leaves remainder 1 when divided by 3
$\Rightarrow$ n is not divisible by 3
Now, $n = 3q + 1$
$\Rightarrow$ $n + 2 = (3q + 1) + 2 = 3(q + 1) \Rightarrow n + 2$ is divisible by 3
Again, $n = 3q + 1$
$\Rightarrow$ $n + 4 = 3q + 1 + 4 = 3q + 5 = 3(q + 1) + 2$
$\Rightarrow$ $n + 4$ leaves remainder 2 when divided by 3
$\Rightarrow$ $n + 4$ is not divisible by 3
Thus, only $(n + 2)$ is divisible by 3 and n and $n + 4$ are not divisible by 3
Case III : When $n = 3q + 2$
$\Rightarrow$ n leaves remainder 2 when divided by 3
$\Rightarrow$ n is not divisible by 3
Now, $n = 3q + 2$
$\Rightarrow$ $n + 2 = 3q + 2 + 2 = 3(q + 1) + 1$
$\Rightarrow$ $n + 2$ leaves remainder 1 when divided by 3
$\Rightarrow$ $n + 2$ is not divisible by 3
Again, $n = 3q + 2$
$n + 4 = 3q + 2 + 4 = 3(q + 2)$
$\Rightarrow$ $n + 4$ is divisible by 3
Thus, only $(n + 4)$ is divisible by 3 and n and $(n + 2)$ are not divisible by 3.

24. Applying Euclid's division lemma to 56 and 72, we get
$72 = 56 \times 1 + 16$ ...(i)
$56 = 16 \times 3 + 8$ ...(ii)
$16 = 8 \times 2 + 0$ ...(iii)
Since, remainder is zero, when divisor is 8.
$\therefore$ HCF (72, 56) = 8
From (ii), we get
$8 = 56 - 16 \times 3$
$= 56 - (72 - 56 \times 1) \times 3$ [Using (i)]
$= 56 - 3 \times 72 + 56 \times 3$
$\Rightarrow$ $8 = 56 \times 4 + (-3) \times 72$
$\therefore$ $a = 4$ and $b = -3$
Now, $8 = 56 \times 4 + (-3) \times 72$
$8 = 56 \times 4 + (-3) \times 72 - 56 \times 72 + 56 \times 72$
$\Rightarrow$ $8 = 56 \times 4 - 56 \times 72 + (-3) \times 72 + 56 \times 72$
$\Rightarrow$ $8 = 56 \times (4 - 72) + \{(-3) + 56\} \times 72$
$\Rightarrow$ $8 = 56 \times (-68) + (53) \times 72$
$\therefore$ $a = -68$ and $b = 53$
Hence, a and b are not unique.

25. If possible, let there be a positive integer n for which $\sqrt{n+1} + \sqrt{n-1}$ is rational and equal to $\frac{a}{b}$ (say), where a, b are positive integers. Then,

$$\frac{a}{b} = \sqrt{n+1} + \sqrt{n-1} \quad \text{...(i)}$$

$$\Rightarrow \frac{b}{a} = \frac{1}{\sqrt{n+1} + \sqrt{n-1}}$$

$$= \frac{\sqrt{n+1} - \sqrt{n-1}}{\{\sqrt{n+1} + \sqrt{n-1}\}\{\sqrt{n+1} - \sqrt{n-1}\}}$$

$$= \frac{\sqrt{n+1} - \sqrt{n-1}}{(n+1) - (n-1)} = \frac{\sqrt{n+1} - \sqrt{n-1}}{2}$$

$$\Rightarrow \frac{2b}{a} = \sqrt{n+1} - \sqrt{n-1} \quad \text{...(ii)}$$

Adding (i) and (ii) and subtracting (ii) from (i), we get

$$2\sqrt{n+1} = \frac{a}{b} + \frac{2b}{a} \text{ and } 2\sqrt{n-1} = \frac{a}{b} - \frac{2b}{a}$$

$$\Rightarrow \sqrt{n+1} = \frac{a^2 + 2b^2}{2ab} \text{ and } \sqrt{n-1} = \frac{a^2 - 2b^2}{2ab}$$

$\Rightarrow$ $\sqrt{n+1}$ and $\sqrt{n-1}$ are rationals.

$\left[\because\ a \text{ and } b \text{ are integers} \therefore \frac{a^2+2b^2}{2ab} \text{ and } \frac{a^2-2b^2}{2ab} \text{ are rationals.}\right]$

$\Rightarrow$ $(n + 1)$ and $(n - 1)$ are perfect squares of positive integers.
This is not possible as any two perfect squares differ at least by 3.
Hence, there is no positive integer n for which $(\sqrt{n-1} + \sqrt{n+1})$ is rational.